COURNOT DUOPOLY AND INSIDER TRADING WITH TWO INSIDERS

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ABSTRACT. We study the effect of competition among insiders in an extension of the static Kyle (1985) model of insider trading introduced by Jain-Mirman (JMC) (2002). In the JMC model competition in the real sector is introduced. In this paper we introduce competition in the stock sector in the JMC model by assuming that there is a manager who is responsible for making the real decisions of the firm as well as an ‘owner’ who has the same information as the manager but has no managerial responsibilities. In this way we can study the interaction between competition in the real sector and competition in the financial sector. We show that the stock price set by the market makers reveals more information than in the JMC model and that the expected equilibrium values of the manager’s profits sometimes decline and sometimes increase depending on the exogenous parameters of the model. Moreover, we prove that due to the competition in the financial sector, the level of output produced by the firm is less than in JMC. Finally, we also study the effect of financial competition in the case in which the market makers receive only one signal and analyze the comparative statics in this case.

Keywords: Cournot Duopoly, Insider Trading, Stock prices, Correlated signals, Kyle model

JEL Classification: G14, D82

1. INTRODUCTION

Economic models that study the effect of financial decisions of the firm should also include the effect of these decisions on the real aspects of the firm, since real and financial effects are inextricably intertwined. Ignoring the financial aspect of a firm that depends on external financing, like a public firm, misses a significant constraint on the performance of that firm in the real sector. For example, the work of Kyle (1985) in studying insider trading does not account for the effect of the insider trades on the real part of the firm. Although many insights about the trades of the insider and the informational content of these trades are possible in the Kyle model, the impact on the real sector is missing and, thus, an important ingredient of the effect of insider trading on the firm is ignored. This point is also apparent when studying the effects of competition, with more than one trader, on the financial side of the firm, as was studied by Tighe (1989), (see also Holden and Subrahmanyam (1992), Foster and Viswanathan (1993)). In these cases the influence

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\footnote{For more details on this literature, see O’hara (1995).}
of the insider on the profitability of the firm is not studied, and, thus, the informational content of the real decisions of the firm and the financial aspects of the firm are also missing.

On the other hand, some recent work incorporates real, as well as financial sectors in their models of insider trading. Leland (1992), for instance, examines the effect of insider trading on real investment without explicitly modeling the role of the insider in linking real and financial decisions. Manove (1989), Dow and Rahi (2003) also incorporate real and financial decisions but focus on the question of fairness rather than the relationship of the real and financial decisions. Finally, Ausebel (1990) studies real aspects of insider trading but without financial markets.

This relationship between the real and financial effects of insider trading was addressed in several papers by Jain and Mirman (2000,2002). In Jain and Mirman (2000, henceforth JM) two basic changes to the Kyle model are made. The first is that the real sector of the economy is added, so that the insider can make real decisions of the firm as well as financial decisions. Second, the market makers can see two signals, a real signal and the order flow signal. The model is designed to yield a linear equilibrium so that the comparative statics are both simple and workable. In JM there is no competition either in the financial sector or the real sector. Competition in the real sector was added to this model in Jain and Mirman (2002, henceforth JMC, where ”C” refers to ”Cournot” due to the Cournot competition in the real sector). In these models the results of Kyle are shown to be altered by the insider’s (the manager of the firm) ability to influence the real output and, thus, the profitability of the firm. This connection between the real and the financial decisions also has a profound effect on the information that is revealed by insider trading. Indeed, it is shown, in JM and JMC, that the amount of information incorporated in the stock price, which is the same in both papers, is greater than in Kyle (1985) and Rochet and Vila (1994), and, more importantly, it is a function of the variables of the model rather than a constant, as in Kyle.

Moreover, in JM and JMC, a compensation scheme for the manager (determined endogenously) was constructed to ensure the existence of linear equilibrium. In fact, there are two parts to the compensation of the manager. The first deals with compensation per share, since it depends on the output decisions of the manager and influences the per share profits of the firm. The second corresponds to a constant compensation, which is a constant times the trade of the manager and does not depend on the number of shares of the firm. The latter is used to influence the trading of the manager in the stock market. In order to align the interests of the firm, the coefficient of these two elements of the compensation of the manager must be set equal. This ensures that the second order condition of the manager is satisfied and, thus, yields a linear equilibrium, as in the Kyle model. Throughout the paper, the term ”compensation scheme” refers to this coefficient. It is shown that the profits of the manager, as well as the compensation received by the
manager in JMC, are less than in JM.

In Daher and Mirman (2005, henceforth DM), competition in the financial sector was added to JM in the form of a second inside trader. It is shown in DM that competition on the financial side of the firm has a profound effect on the results of JM. Indeed, adding another informed trader in the stock market increases the amount of information revealed by the stock price, relative to JM or JMC. In contrast, the expected profits of the manager are not always less than in JM but depend on the variances of the exogenous variables. Moreover, the trading level of the owner in DM resembles the trading level in Tighe (1989) and the trading level of the manager resembles the trading level of the manager in JM. These results are a direct consequence of the presence of the compensation scheme received by the manager in JM and DM.

In this paper we combine the effect of JMC (competition among firms) and DM (competition among insiders) and study the effect of competition in both the real and the financial sectors, simultaneously. This adds richness and depth to the model and allows us to study the relationship between the real and financial sectors. In particular, this paper extends the model in JM by adding competition in both the real as well as in the financial sector, allowing us to study how the different market structures depend on each other. Indeed, we show that the decisions made in the financial market depend on the industrial structure in the real market, i.e., the results depend on the type of competition in the real sector. We also show that the decisions in the real market are influenced by the competition in the financial market. We compare the equilibrium of this paper to JMC and DM. First, we show that adding a second firm to DM, i.e., adding Cournot competition, decreases the value of the firm and, thus, the coefficients of the stock price set by the market makers, decrease. Moreover, the amount of information revealed by the stock price remains unchanged by the competition in the real sector. Second, relative to JMC, we show that adding a second informed trader increases the amount of information incorporated in the stock price. We also show that the profits of the manager depend on the variances of the exogenous variables. Next, we show that the intercept coefficient of the stock price is affected by competition in the financial sector.

The profits of the manager are also affected by the financial competition. Indeed, compared to JM, one would expect that the profits of the manager are less than in JM, due to Cournot competition. But this is not the case since the profits of the firm depend on the number of insiders trading. Hence, we show that the profits, relative to JM, sometimes increase and sometimes decrease, depending on the variances of the exogenous variables. Finally, the coefficient of the real signal in the pricing rule of the market makers is affected by competition in both sectors. Indeed, going from JM to DM or from JM to JMC, the real signal coefficient decreases. Thus, going from JM to this paper, the real signal coefficient set by the market makers decreases as in JMC or in DM. In contrast, the total order flow signal coefficient is affected by competition in the financial sector. In particular, relative
to JM, the order flow coefficient sometimes decreases and sometimes increases similar to
the case going from JM to DM. Indeed, the effect of competition in the real sector on the
stock price coefficients depends crucially on competition in the financial sector.

2. The Model

We consider a version of the Jain-Mirman model (henceforth JMC, 2002). The
economy consists of one real good and one financial asset. The market for the real good
is assumed to be a Cournot duopoly with two firms, each firm produces the real good at
no cost. The inverse demand function facing this industry is stochastic and linear, i.e.,
\[ q' = (a - bY)\tilde{z}, a, b > 0. \]

The prior distribution of $\tilde{z}$ is normal with mean $\bar{z}$ (assumed to be positive) and variance $\sigma^2_z$, $q'$ is the real price of the output and $Y = y_1 + y_2$, the aggregate quantity produced.

Both firms are standard neoclassical firms with respect to the real sector. With respect
to the financial sector, firm 2 is privately owned and privately financed, while firm 1 is
publicly owned. In particular, firm 1 is managed by an insider whose knowledge can be
used in the stock market. In the financial market, the asset is the stock of firm 1, the value
of the asset is the net profits of the firm per share. The stock of firm 1 is publicly traded
in a competitive stock market. We add a second insider to the JMC model, an ”owner”. The owner is assumed to have the same private information about the financial asset as
the manager. The two insiders trade in the stock market based on their inside information.

There are three types of agents. First, there are two risk-neutral rational traders: the
manager and the owner of firm 1, each of whom knows the realization $z$ of $\tilde{z}$. Second, there
are the (nonrational) noise traders, representing small investors with no information on $z$.
The aggregate noise trade is assumed to be a random variable $\tilde{u}$, which is normally dis-
tributed with mean zero and variance $\sigma^2_u$. Finally, there are $K (K \geq 2)$ risk-neutral market
makers who act like Bertrand competitors. We assume, as in JMC, that the market makers
observe two signals, a noisy signal from the real market, denoted by the random variable
\[ \tilde{q} = (a - bY)(\tilde{z} + \tilde{\varepsilon}), \]
where $\tilde{\varepsilon}$ is normally distributed with mean zero and variance $\sigma^2_{\varepsilon}$, and
the total order flow signal. We assume that $\tilde{z}, \tilde{u}$ and $\tilde{\varepsilon}$ are independents.

Following Kyle (1985), the trading mechanism is organized in two steps. In step one,
a linear pricing rule and optimal order rules are determined by the market makers and
the insiders, respectively, as a Bayesian Nash equilibrium. The market makers determine
a (linear) pricing rule $p$, based on their a priori beliefs, where $p$ is a measurable function
\[ p : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}. \]
Each insider chooses a stock trade function $\tilde{x}_i = x_i(\tilde{z})$, where $x_i : \mathbb{R} \longrightarrow \mathbb{R}$

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2 A description of the model is contained in section 3 of JMC.

3 In our model, the owner is another informed agent in the firm. For example a majority shareholder
or CEO, who does not have any managerial responsibilities. However, the owner could be also thought as
insider in the competitive firm, that is firm 2, with the same information as the manager of firm 1.
is a measurable function. In the second step, the insiders observe the realization $z$ of $\tilde{z}$ and submit their stock orders to the market makers based on the equilibrium stock trade functions. The market makers also receive orders from the noise traders, all these orders arrive as a total order flow signal $\tilde{r} = x_1(\tilde{z}) + x_2(\tilde{z}) + \tilde{u} = x(\tilde{z}) + \tilde{u}$. The two signals, the real signal and the order flow signal, are used by the market makers to set the price $p = p(\tilde{q}, \tilde{r})$, based on the equilibrium price function, to clear the market. The insiders know only the value of $\tilde{z}$ and do not know the values of $\tilde{u}, \tilde{\varepsilon}, \tilde{r}, \tilde{z} + \tilde{\varepsilon}$ before their order flow decisions are made. Moreover, each market maker does not know the realization $z$ of $\tilde{z}$ but only knows its distribution. Finally, the market makers cannot observe either $x_1, x_2, u$ or $\varepsilon$.

The value per share of the firm is defined to be the net profits of the firm per share. Hence, the values per share of firm 1 and 2 are, respectively, $v' = (a - bY)y_1\tilde{z}$ and $v_2 = (a - bY)y_2\tilde{\varepsilon}$. Profits for each of the two rational traders are given, respectively, by

$$\Pi_1 := (v' - B - p) \cdot \tilde{x}_1 + B \tilde{x}_1$$
$$\Pi_2 := (v' - B - p) \cdot \tilde{x}_2$$

where $B$ is the compensation scheme of the manager. The compensation scheme of the manager is determined endogenously in order to satisfy the second order conditions of the maximization problem of the manager, or as stated in JM, in order to bring the objective of the firm and the manager into line. The compensation scheme used in JM and JMC is the unique compensation scheme that satisfies the second order conditions. However, the description of the compensation scheme in JM is not complete. In fact, there are two parts to the compensation of the manager. The first deals with compensation per share, as would any other general cost of production per share, and the second corresponds to compensation based on the trading in the stock market of the manager. The coefficient of these two elements are the same and, thus, for the manager offset each other and consequently the compensation of the manager does not belong in the manager’s objective function. However, for the owner the per share part of the compensation scheme remains, since it influences the value of the firm and, thus, must be taken account of by the owner when making trading decisions. Thus, after adjusting for the compensation scheme of the manager, the value of firm 1 becomes, $\tilde{v} = v' - B$.

The need for such a compensation scheme for the manager is due to the fact that the model is designed to produce a linear equilibrium as in the Kyle model. This linearity

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4 Random variables are denoted with a tilde. Realized values lack the tilde. The mean of the random variable is denoted with bar.

5 The compensation scheme used in JM and JMC has only one purpose, i.e., to satisfy the second order condition of the manager’s maximization problem and to ensure the existence of a linear equilibrium. This argument is also applicable in this paper. Indeed, we show, in the next section, that with no compensation scheme the second order condition is not always satisfied for all $z$. The compensation is used to satisfy the second order condition for all $z$. There are other reasons for a compensation scheme for managers, e.g., to reward the manager in a Principle-Agent context. This was done by Holmstrom -Tirole (1993) and Baimain-Verrecchia (1995) in a similar context but these papers do not study the output decisions of the insider, which is the object of our model. Moreover, our model is a general equilibrium model with asymmetric information and is not a Principal-Agent model.
enables us to derive the results and to study them in a simple and straightforward context. This objective gives rise to a model that is tractable and at the same time does not destroy its economic content, thus making it very delicate to the assumptions that are consistent with this objective.

This is a game of incomplete information because the market makers unlike the insiders do not know the realization of $\tilde{z}$. Hence, we seek for a Bayesian-Nash equilibrium. A Bayesian-Nash equilibrium is a vector of five functions $[y_1(\cdot), y_2(\cdot), x_1(\cdot), x_2(\cdot), p(\cdot)]$ such that:

(a) Profit maximization of firm 2,

$$((a - b(y_1 + y_2))y_2 \tilde{z} \geq ((a - b(y_1 + y_2))y_2' \tilde{z})$$

for any level of output $y_2'$ produced by the firm 2;

(b) Profit maximization of the manager,

$$E[((a - b(y_1' + y_2))y_1' \tilde{z} - p(x_1(\tilde{z}) + x_2(\tilde{z}) + \tilde{u}))x_1'(\tilde{z})]$$

(3)

for any level of output $y_1'$ produced by the firm 1 and any alternative trading strategy $x_1'(\tilde{z})$;

(c) Profit maximization of the owner,

$$E[((a - b(y_1 + y_2))y_1 \tilde{z} - B - p(x_1(\tilde{z}) + x_2(\tilde{z}) + \tilde{u}))x_2'(\tilde{z})]$$

(4)

for any alternative trading strategy $x_2'(\tilde{z})$;

(d) Semi-Strong Market Efficiency: The pricing rule $p(\cdot, \cdot)$ satisfies,

$$p(q, r) = E[\tilde{v} | q, r]$$.  

(5)

An equilibrium is linear if there exists constants $\mu_0, \mu_1, \mu_2$ such that,

$$\forall q, r, p(q, r) = \mu_0 + \mu_1 q + \mu_2 r$$.  

(6)

Note that conditions (2), (3) and (4) define optimal strategies of the two firms and the owner while condition (5) guarantees the zero expected profits for the market makers. The stock price, set by the market makers, is equal to the conditional expectation of the asset value given their information. We restrict our study to linear equilibrium. The normal distributions of the exogenous random variables, together with the particular expression of the demand, enable us to derive and to prove the existence of a unique linear equilibrium.

3. Derivation of Equilibrium

In order to derive the unique linear equilibrium, we start by solving the maximization problem of the two firms as well as the maximization problem of the owner of firm 1. The objective of firm 2 is to produce an optimal quantity of outputs, $y_2(\cdot)$, to maximize,

$$v_2 = (a - bY)y_2 \tilde{z}$$.  

(7)
The first order condition is,
\[ y_2 = \frac{(a - by_1)}{2b}. \]  

The decision rule of the manager of firm 1 is the pair \((y_1(\tilde{z}), x_1(\tilde{z}))\). The manager’s expected profits are,
\[ G_1 = E[((a - b(y_1 + y_2))y_1\tilde{z} - p(x_1(\tilde{z}) + x_2(\tilde{z}) + \hat{u}))x_1(\tilde{z})]. \]

The first order conditions are,
\[ x_1(\tilde{z}) = \frac{(a - bY)(y_1 - \mu_1)\tilde{z} - \mu_0 - \mu_2x_2(\tilde{z})}{2\mu_2} \quad \text{and} \quad y_1 = \frac{(a - by_2 + \mu_1 b)}{2b}. \]

Computing the Cournot equilibrium in the real sector from (8) and (10), we get,
\[ y_1 = \frac{(a + 2\mu_1 b)}{3b} \quad \text{and} \quad y_2 = \frac{(a - \mu_1 b)}{3b}. \]

The owner of firm 1 chooses the decision rule \(x_2(\tilde{z})\) to maximize expected profits,
\[ G_2 = E[((a - b(y_1 + y_2))y_1\tilde{z} - B - p(x_1(\tilde{z}) + x_2(\tilde{z}) + \hat{u}))x_2(\tilde{z})]. \]

The first order condition is,
\[ x_2(\tilde{z}) = \frac{(a - bY)(y_1 - \mu_1)\tilde{z} - \mu_0 - B - \mu_2x_2(\tilde{z})}{2\mu_2}. \]

Computing the equilibrium in the stock market from (10) and (13), we have,
\[ x_1(\tilde{z}) = \frac{(a - bY)(y_1 - \mu_1)\tilde{z} - \mu_0 + B}{3\mu_2} \quad \text{and} \quad x_2(\tilde{z}) = \frac{(a - bY)(y_1 - \mu_1)\tilde{z} - \mu_0 - 2B}{3\mu_2}. \]

**Remark 3.1.** First, note that the outputs produced by the two firms are deterministic. This is due to the fact that the variable \(\tilde{z}\) is multiplicative,\(^6\) and implies, in turn, that the trading level of each insider is a linear function of \(\tilde{z}\). Thus, the trading strategies of the insiders are normal random variables. Second, the two signals received by the market makers, i.e. the real signal \(\tilde{q}\) and the total order flow signal \(\tilde{r}\), are both normal random variables (since \(\tilde{z}, \tilde{u}\) and \(\tilde{\varepsilon}\) are independents). Third, since the outputs are deterministic, the value of firm 1 is also a normal random variable. Consequently, the normality of these random variables allows us to derive an explicit linear form for their conditional expected values. Hence, the stock price set by the market makers, which satisfies the zero expected profits condition, is linear.\(^7\)

\(^6\) It would, perhaps, be more interesting to have a model with output that depends on the value of \(\tilde{z}\). However, this would be inconsistent with the normality of the trading functions as well as the signals and thus inconsistent with the linearity of the pricing function.

\(^7\) It should be pointed out that adding another informed trader to JMC does not change the distributions of the equilibrium outcome and, thus, Theorem 1 in the appendix, applies as in JMC. Hence, the stock price set by the market makers has a linear conditional expectation form.
Lemma 1. Under competition in the real and financial sectors, the coefficients of the price function are,

(i) \[ \mu_0 = \bar{v} - \mu_1 \bar{q} - \mu_2 \bar{r}, \]

(ii) \[ \mu_1 = \frac{a \sigma^2}{b(\sigma^2_g + 9 \sigma^2_u)}, \]

(iii) \[ \mu_2 = \frac{\sqrt{2} a^2 \sigma \epsilon \sqrt{(1 - k)^{3/4}}}{3 \epsilon}, \]

where \( k = \frac{\sigma^2}{(\sigma^2_g + 9 \sigma^2_u)}. \)

Proof: See the Appendix.

The second order conditions for the manager are,

\[ 2b x_1(\tilde{z}) \tilde{z} > 0 \quad \text{and} \quad \mu_2 > 0, \]

and the Hessian determinant is positive. For the owner, \( \mu_2 > 0. \) By Lemma 1, the second order condition for the owner is satisfied. In contrast, the first condition in (14) is not always satisfied since the support of the normal random variables is the entire real line. Specifically, substituting the value of \( x_1 \) in the left hand condition in (14), we find,

\[ b(y_1 - \mu_1)^2(\bar{z} - \tilde{z}) + 3B \tilde{z} \]

(15)

Since the second order condition in (15) is quadratic, there exits values of \( \tilde{z} \) for which the second order condition is not satisfied. Now, we present a necessary and sufficient condition for the existence of the linear equilibrium for all \( \tilde{z}. \)

Proposition 1. A linear equilibrium exists if and only if,

\[ B = \mu_0 = \frac{(a - bY)(y_1 - \mu_1)}{3} \tilde{z}. \]

Proof: We start by proving sufficiency. If \( B = \mu_0, \) then the expression for \( x_1 \) becomes,

\[ x_1(\tilde{z}) = \frac{(a - bY)(y_1 - \mu_1)}{4\mu_2} \tilde{z}. \]  

Thus, the second order condition is satisfied. Hence, a linear equilibrium exists. Next, we prove the necessity. Indeed, since there exists a linear equilibrium, the second order condition is satisfied for all \( \tilde{z}. \) The second order condition has the form, \( cx^2 + dx > 0. \) Here \( c = b(y_1 - \mu_1)^2 > 0 \) and \( d = 3B - b(y_1 - \mu_1)^2 \tilde{z}. \) So, we must have \( d = 0 \) to satisfy the second order condition and thus the result is proved.

Remark 3.2. Note that if \( \tilde{z} \) is centered, i.e. \( \tilde{z} = 0, \) then the profits of firm 2 are identically zero and the model reduces to the Daher-Mirman (henceforth DM, 2005) model which studies only the competition in the stock sector. Note that in the “centered” case, the equilibrium always exists (the second order condition is satisfied) and is symmetric (the
trading levels of the two insiders exchanged in the stock market are the same). Moreover, the effect of the compensation received by the manager, as in JMC, is to center the profits of the manager around zero. Since the owner has no managerial tasks, no compensation scheme is necessary and, thus, his profits function, as in Kyle (1985), is always centered around $\bar{z}$, the mean of $\tilde{z}$.

Remark 3.3. First, note that $B$ is positive. Second, the intercept coefficient of the price function, $\mu_0$, is different from zero. In contrast to JMC, this result is similar to DM. This result is due to the presence of competition in the stock sector.

The unique linear equilibrium of this model is presented and characterized in the next Proposition.

Proposition 2. A linear equilibrium exists. The equilibrium is unique and is characterized by,

(i) $y_1 = \frac{a + 2\mu_1 b}{3b}$ and $y_2 = \frac{a - \mu_1 b}{3b}$,

(ii) $x_1(\tilde{z}) = \frac{(a - bY)(y_1 - \mu_1)\tilde{z}}{3\mu_2}$ and $x_2(\tilde{z}) = \frac{(a - bY)(y_1 - \mu_1)\tilde{z} - 3B}{3\mu_2}$,

(iii) $p(\tilde{q}, \tilde{r}) = \mu_0 + \mu_1\tilde{q} + \mu_2\tilde{r}$,

(iv) $\mu_0 = B = \frac{(a - bY)(y_1 - \mu_1)}{3}\bar{z}$,

(v) $\mu_1 = \frac{a}{b}k$ and $\mu_2 = \frac{\sqrt{2\sigma_\epsilon^2\epsilon}\sqrt{(1 - k)^3k}}{9b\sigma_\epsilon}$,

where $k = \frac{\sigma_\epsilon^2}{(\sigma_\epsilon^2 + 9\sigma_\epsilon^2)}$, $\mu_1 \in (0, \frac{a}{b})$ and $\mu_2 > 0$.

Proposition 2 shows that the real signal and the total order flow signal, are respectively,

(16) $\tilde{q} = \alpha(\tilde{z} + \tilde{\epsilon})$

and

(17) $\tilde{r} = \beta + \gamma\tilde{z} + \tilde{u}$,

where $\alpha = (a - bY)$, $\beta = \frac{B}{\mu_2}$ and $\gamma = \frac{2(a - bY)(y_1 - \mu_1)}{3\mu_2}$. Here, $\alpha$ and $\gamma$ are the deterministic parts of $\tilde{q}$ and $\tilde{r}$, respectively, while $(\tilde{z} + \tilde{\epsilon})$ and $\tilde{z}$ are the random parts.
Note that adding a second informed trader to JMC changes the equilibrium outcome. In particular, in contrast to JMC, the compensation scheme of the manager must be equal to $\mu_0$. Moreover, there is an important informational effect, due to the presence of competition in the financial and the real sectors, on the equilibrium outcome of the two signals observed by the market makers. Note also that the noise in the total order flow signal, $\tilde{u}$ (equation 17), is additive and homoskedastic. In particular, an increase in the number of insiders leads to an increase of the deterministic part of the total order flow signal, $\gamma$, i.e., the slope of the total order flow signal increases. In other words, increasing the number of insiders increases the slope of the insider trading function in this paper compared to JMC. Moreover, this is done without influencing the variance of the noise term $\tilde{u}$. Hence, $\tilde{r}$ becomes more informative relative to JMC. In contrast, the deterministic part of the real signal $\alpha$ multiplies the sum of the random part, $(\tilde{z} + \tilde{\varepsilon})$ (equation 16). Therefore, the noise term in the real signal, $\tilde{\varepsilon}$ is heteroskedastic (see Creane 1994) but in such a way that changing the deterministic part of the real signal has no informational effect. In other words, changing the real signal coefficient $(a - bY)$, changes the variance of $\tilde{q}$. Thus, the information change resulting from the change in $(a - bY)$ is offset by the change of the variance of the real signal. Consequently, adding another informed trader, increases the information of the market makers (as in Tighe (1989)) but adding a competitive firm does not change the information incorporated in the stock price.

4. Comparative Statics Analysis

In this section we compare the equilibrium outcome presented in Proposition 2 to the equilibrium outcomes in JMC and DM. Note that, throughout this paper, ”jmc” refers to JMC and ”dm” to DM.

4.1. Equilibrium Outcome. We analyze the effect of both competition in the real and financial sectors on the equilibrium outcome. First, we compare the equilibrium outcome of our model, i.e., with competition in both markets, to the equilibrium outcome in JMC, i.e., with competition only in the real sector. Second, we compare the equilibrium outcome of our model to the equilibrium outcome in DM, i.e., in the presence of competition in the financial sector.

Lemma 2. The effect of competition in both the real and financial markets, as compared to competition only in the real sector (JMC) is as follows,

(i) $\mu_{jmc}^0 = 0 < \mu_0$, $\mu_1 < \mu_{jmc}^1$,

(ii) $\left\{ \begin{array}{l} \mu_2 > \mu_{jmc}^2, \text{ when } \sigma^2_{\tilde{\varepsilon}} \text{ is small relative to } \sigma^2_z, \\ \mu_2 < \mu_{jmc}^2, \text{ otherwise, } \end{array} \right.$

(iii) $y_1 < y_{jmc}^1$, $y_2 > y_{jmc}^2$, $Y < Y_{jmc}$,
(iv) \[
    x_1(z) < x^{jmc}(z),
\]

(v) \[
    B(\mu_1) < B^{jmc}(\mu_1).
\]

**Proof:** Recall that \( \mu^{jmc}_0 = 0 \), which is less than \( \mu_0 \) (by Lemma 1), and
\[
    \mu^{jmc}_1 = \frac{a\sigma_u^2}{b(\sigma_z^2 + 6\sigma_z^2)}.
\]
Comparing \( \mu^{jmc}_1 \) to \( \mu_1 \), the result in (i) is proved. It is easy to prove (ii) (use (i)) and it is straightforward to prove (iii) by using (i) and (ii). Substituting the value of \( \mu_0, \mu_1, \mu_2, y_1 \) and \( y_2 \) in the expressions of \( x_1(.) \) and \( x_2(.) \) from Proposition 2, we find that,
\[
    x_1(z) = \sqrt{\frac{2\sigma_u}{2\sigma_z}} z,
\]
which is less than the level of trading of the manager in JMC, that is,

\[
    x^{jmc}(z) = \frac{\sigma_u}{\sigma_z} z.
\]

Hence, (iv) is proved. It is easy to check that,
\[
    B(\mu_1) = \frac{(a - b\mu_1)^2}{27b} \bar{z} \quad \text{and} \quad B^{jmc}(\mu_1) = \frac{(a - b\mu_1)^2}{18b} \bar{z},
\]
which proves (v).

**Lemma 3.** Comparing the owner’s level of trading, as well as the total level of trading of the two insiders, to the level of trading in JMC where only competition in the real sector is allowed, yields,

(i) there exists a \( z^- \in \mathbb{R}^- \) such that,
\[
    \begin{cases} 
        x_2(z) > x^{jmc}(z), & \text{if } z \in (z^-, 0), \\
        x_2(z) < x^{jmc}(z), & \text{otherwise},
    \end{cases}
\]

(ii) there exists a \( z^+ \in \mathbb{R}^+ \) such that,
\[
    \begin{cases} 
        x(z) < x^{jmc}(z), & \text{if } z \in (0, z^+), \\
        x(z) > x^{jmc}(z), & \text{otherwise}.
    \end{cases}
\]
Proof: Substituting for \(Y, y_1, \mu_1\) and \(\mu_2\) from Proposition 2 into the expression for \(x_2(.)\) yields,

\[
x_2(z) = \frac{\sqrt{2} \sigma_u}{2 \sigma_z} (z - \bar{z}).
\]

Comparing the expressions in (18) and (19), the result in \((i)\) holds. By adding (18) and (19), we obtain,

\[
x(z) = \frac{\sqrt{2} \sigma_u}{\sigma_z} z - \frac{\sqrt{2} \sigma_u}{2 \sigma_z} \bar{z}.
\]

Comparing equation (18) to equation (20), we get the result. ■

Before interpreting these results the relationship between the basic Jain-Mirman model (JM, 2000), JMC, DM and the model of this paper should be reiterated. Our model adds a second informed trader to the model of JMC, so there is competition in both the real and the financial sectors. On the other hand, DM adds a second insider to JM so that there is competition in the financial sector but not in the real sector. In both our model and in DM the second inside trader has no managerial responsibilities but has the same information as the manager. Hence, not only can we study the effect of competition among insiders on the variables of the model, but we can also study the effect that competition in the real sector has on competition in the stock sector and visa versa.

To begin, note that the stock price set by the market makers, is affected by both competition in the real sector and competition in the financial sector. Lemma 2 \((i)\), shows that the intercept coefficient of the stock price, \(\mu_0\), is greater than in JMC. Indeed, to satisfy the second order condition of the maximization problem of the manager of firm 1, and to guarantee the existence of a linear equilibrium, as in DM, competition in the financial sector requires that the compensation scheme of the manager (which is positive) has the value \(\mu_0\). In the absence of this competition in the financial sector, i.e., in JM or JMC, \(\mu_0\) must be equal to zero. In order to satisfy the zero expected profits, the market makers set \(\mu_0\) different than zero due to the presence of competition in the stock market.

Second note that \(\mu_1\), the response of the market makers to the real signal, is always less than in JMC, while the response of the market makers to the total order flow signal, \(\mu_2\) depends on the variances of the exogenous variables. In other words, \(\mu_2\) is lower in this model than in JMC when the variance of the noise from the real sector, i.e. \(\sigma_2^2\), is large relative to the variance \(\sigma_z^2\). This relationship, between \(\mu_1\) and \(\mu_2\), is similar to DM (compared to JM) as well as in the Jain-Mirman (1999, henceforth JMJ, where "J" refers to "Junior" since it does not take into account the effect of the real market) model going from one to two insiders, each with the same information. Indeed it is this result of JMJ that is the basis for understanding the effect of adding another insider. JMJ is a Kyle-type model (with one insider) in which the market makers observe two signals, i.e., the total order flow and \(\bar{z} + \tilde{\varepsilon}\), where \(\bar{z}\) is the liquidation value of the asset and \(\tilde{\varepsilon}\) is a noise term.
Since, the market makers observe the two signals, the price is set in the same way as in JM but with no real market so that the information of each insider is the value of the asset. When competition between two insiders is studied in JMJ, the relationship between \( \mu_1 \) and \( \mu_2 \), is similar to the result in our model. In other words, going from one to two insiders, \( \mu_1 \) decreases while \( \mu_2 \) depends on the variances of the exogenous variables. This result reflects the fact that with two insiders there is more information in the order flow signal and, thus, the order flow signal gets, in general, more weight, except when there is so much noise in the real signal that it becomes like the one signal Kyle model. In this case, the result is then similar to the Kyle model, i.e., \( \mu_2 \) decreases. However, with production, although the results are similar to the JMJ results, the problem is more complicated since the value of the firm has two components, the value of the random variable \( \tilde{z} \) (which is the information in Kyle) and the deterministic, real part of the firm.

In fact, \( \mu_2 \) is lower in this model than in JMC, when \( \sigma_z^2 \) is large relative to \( \sigma_q^2 \). This result is similar to the result in Kyle’s model going from one to two insiders (see in Tighe 1989) and is consistent with the results of JMJ. In order to understand how the coefficient \( \mu_2 \) varies, recall that

\[
\mu_2 = \frac{\sigma_{vr} \sigma_q^2}{\sigma_q^2 \sigma_r^2 - \sigma_{qr}^2}.
\]

Hence the values of \( \mu_2 \) in this paper and in JMC are, respectively,

\[
\mu_2 = \frac{3\sqrt{2}a_2 \sigma_2^2 \sigma_z}{b \sigma_u (\sigma_z^2 + 9 \sigma_q^2)^2} \quad \text{and} \quad \mu_{2}^{\text{JMC}} = \frac{2a_2 \sigma_2^4 \sigma_z}{b \sigma_u (\sigma_z^2 + 6 \sigma_q^2)^2}.
\]

Increasing \( \sigma_q^2 \) relative to \( \sigma_z^2 \) increases the variance of the real signal, \( \sigma_q^2 \), as well as the covariance of the real signal and the total order flow, \( \sigma_{qr} \). Hence, in this case, the numerator increases less than the denominator, thus yielding a lower value for \( \mu_2 \) in this model relative to JMC.

The relationship between \( \mu_2 \) and the variances of the exogenous variables can also be inferred from the zero expected profit condition. The expression of the stock price in JMC, at equilibrium, is

\[
p(\tilde{q}, \tilde{r}) = E[\tilde{v}|\tilde{q}, \tilde{r}] = \mu_{1}^{\text{JMC}} \tilde{q} + \mu_{2}^{\text{JMC}} \tilde{r}.
\]

Taking expectation on both sides of the equality we obtain,

\[
\tilde{v}^{\text{JMC}} = \mu_{1}^{\text{JMC}} \tilde{q} + \mu_{2}^{\text{JMC}} \tilde{r},
\]

which is equivalent to

\[
(a - bY) y_1 \tilde{z} - B^{\text{JMC}} = \mu_{1}^{\text{JMC}} \tilde{q} + \mu_{2}^{\text{JMC}} \tilde{r},
\]

or

(21)

\[
(a - bY) y_1 \tilde{z} = B^{\text{JMC}} + \mu_{1}^{\text{JMC}} \tilde{q} + \mu_{2}^{\text{JMC}} \tilde{r}.
\]
A similar calculation of the equilibrium stock price in this paper, taking account of the facts that $\mu_0$ is different from zero and is equal to the compensation scheme, yields,

\begin{equation}
(a - bY)y_1 \tilde{z} = 2B + \mu_1 \tilde{q} + \mu_2 \tilde{r}.
\end{equation}

When $\sigma_z^2$ is large relative to $\sigma_z^2$, the left hand sides of equations (21) and (22) are close in value ($s$) and, thus, the right hand sides must be close. In this case $\mu_2$ is less than in JMC, since the unconditional profits of the two firms are close and the real signal is relatively noisy. This is similar to the Kyle result that the coefficient of the order flow decreases as the number of traders increases since the total order flow increases and the value of the firm is the same.

In contrast, when $\sigma_z^2$ is small relative to $\sigma_z^2$, the total order flow signal coefficient, $\mu_2$ in this model is greater than in JMC. The intuition for this result is that the same consideration as in JMJ is present here, i.e., the demand for the stock, together with the zero expected profits condition for the market makers, reduce $\mu_1$. However, in our model, $\mu_2$ increases since it is now more informative. Moreover, the unconditional profits of the firm 1 in JMC are less than the unconditional profits of firm 1 in our paper. Since the real signal coefficient is reduced, the more informative order flow coefficient increases to reflect the increased value of the firm. In order to understand this relationship between the stock price coefficients and the unconditional profits of firm 1, consider the expression of the unconditional profits of firm 1 in JMC and in our model given by equations (21) and (22), respectively. These two equations yield the following expressions,

\begin{equation}
(a - bY)y_1 \tilde{z} - B^{jmc} = \mu_1^{jmc} \tilde{q} + \mu_2^{jmc} \tilde{r},
\end{equation}

and

\begin{equation}
(a - bY)y_1 \tilde{z} - B = \mu_0 + \mu_1 \tilde{q} + \mu_2 \tilde{r}.
\end{equation}

Hence, when $\sigma_z^2$ is small relative to $\sigma_z^2$, the left hand side of (23) is less than the left hand side of (24) and, thus, the right hand side of (23) must be less than the right hand side of (24). This increase in profits, given that $\mu_1$ decreases relative to JMC, implies that the more informative signal be given more weight, thus, $\mu_2$ is greater than in JMC.

Third, competition in the financial sector has an effect on the output of firm 1. In fact, the level of output produced by firm 1 is less than in JMC. To see this, recall the expression of the output produced by firm 1,

\[ y_1 = \frac{a + \mu_1 b}{3b}. \]

Hence, the output is a function of $\mu_1$, the coefficient of the real sector signal of the stock price, which is set by the market makers. Lemma 2 (i) shows that competition in the stock sector reduces the value of $\mu_1$ and, thus, the output of firm 1 in our model relative to JMC. Moreover, due to the movement along the reaction curve of firm 2, the level of output of firm 2 is greater than in JMC. Consequently, total output in this model is less than in JMC. Hence, the addition of another informed trader influences the production
decision of firm 1, as well as the production decision of firm 2.

Fourth, competition in the stock sector affects the insiders trades. Indeed, Lemma 2 (iv) shows that the manager’s trading level is less than in JMC. This result is also consistent with Kyle (1985), going from one to two insiders (see in Tighe 1989). Moreover, from equation (18), the manager’s trading level, as in JMC, is a linear function on $\tilde{z}$. This is due to the presence of the compensation scheme that makes the form of the trading function of the manager linear through the origin.$^8$ Moreover, competition in the financial sector also affects the trading level of the owner, as well as the total trade. Both of these trades are affine functions of $z$, since the owner has no managerial responsibilities. Indeed, the trading level of the owner has the same form as the trading level of the insider in a Kyle-type model (see Tighe 1989).

Finally, the comparison between the compensation schemes in both models depends on the variances of the exogenous variables. Specifically, higher values of the variance of the noise signal of the real sector signal, $\sigma_z^2$, relative to $\sigma_\epsilon^2$, makes the compensation scheme in our model less than in JMC. In contrast, when $\sigma_\epsilon^2$ is small, the compensation in our model is greater than in JMC. The interpretation of this result is that when $\sigma_\epsilon^2$ is small, relative to $\sigma_z^2$, the unconditional net profits of firm 1 in JMC are less than in our paper and, hence, the manager in JMC needs to be compensated less than in our model. In contrast, when $\sigma_z^2$ is large, relative to $\sigma_\epsilon^2$, the total order flow signal becomes more informative relative to the real signal, since its variance is also large. This case is consistent with Kyle-type models, going from one to two insiders.$^9$ It should be pointed out that the comparison of the compensation schemes, as $\sigma_\epsilon^2$ varies relative to $\sigma_z^2$, between JMC and this paper is similar to the comparison between the coefficient of the total order flow signal, $\mu_2$ as $\sigma_\epsilon^2$ varies relative to $\sigma_z^2$, between JMC and this paper (see Lemma 2).

Now, we compare the equilibrium outcome of this model to the equilibrium outcome in DM, i.e., when only competition in the financial sector is studied.

**Lemma 4.** Comparing to DM, where only the competition in the financial sector is allowed,

(i) $\mu_0 < \mu_0^{dm}$, $\mu_1 < \mu_1^{dm}$, $\mu_2 < \mu_2^{dm}$,

(ii) $y_1 < y_1^{dm}$, $Y > y_1^{dm}$,

---

$^8$Note that the compensation scheme of the manager pushes the linear trading function through the origin and, thus, the profits of the manager are independent of the prior beliefs of the market makers.

$^9$When the market makers receive only one signal, i.e., the total order flow signal, as in Kyle, then, the greater is the number of "owners", each with same information, the smaller is the compensation scheme received by the manager.
(iii) \[ x_1(\tilde{z}) = x_{1dm}(\tilde{z}), \quad x_2(\tilde{z}) = x_{2dm}(\tilde{z}), \]

(iv) \[ B < A_{dm}. \]

**Proof:** Recall that \( \mu_{1dm} \) has the same expression as \( \mu_{jmc} \). Hence, by Lemma 2, the inequality in the middle of (i) is proved. Since, \( \mu_0 \) takes the same value as the compensation scheme, we prove (iv), which is equivalent to the left hand side of (i). Indeed, it is easy to check that the values of \( B \) and \( A \) are respectively,

\[
B = \frac{3a^2\sigma_2^4}{b(\sigma_2^2 + 9\sigma_2^2)\epsilon^2} \bar{z} \quad \text{and} \quad A_{dm} = \frac{3a^2\sigma_2^4}{b(\sigma_2^2 + 6\sigma_2^2)\epsilon^2} \bar{z}.
\]

Comparing these two values, (iv) is then proved. It is straightforward to prove the right hand side of (i) (use the middle and the left hand side of (ii)). Comparing the level of output produced by firm 1 and the total outputs in this model to DM, we obtain (ii). Finally, recall that the values of the insiders’ trading orders of DM are the same as in equations (18) and (19). Thus, (iii) is proved.

Lemma 4 shows how competition in the real sector affects the equilibrium outcome when competition in the stock sector is taken into account.

It should be noted that competition in the real sector adds no new information, but does lower the profits of the firm due to Cournot competition in the real sector. These lowered profits must be incorporated into the stock price function of the market makers. Also, the compensation scheme is affected by competition in the financial sector. In particular, without competition in the financial sector adding a second firm in the real sector has no effect on the value of \( \mu_0 \), since \( \mu_0 = 0 \), in both JM and JMC. But the addition of a second insider changes the value of \( \mu_0 \) by making it equal to the compensation scheme. Thus, the effect of adding competition in the real sector depends on the financial aspects of the firm, in particular, it depends on the number of insiders trading on the stock market. However, the major effect of competition in the real sector is to decrease profits without increasing the amount of information.

Note that Lemma 4 shows that the coefficients of the stock price function, in our model, are all less than DM. In order to understand this result, recall the expression of the ”net” unconditional expected value of the firm 1 in each model. For DM,

\[
\bar{v}^{dm} = (a - by_{dm})y_{dm}z - A_{dm} = \mu_0 + \mu_{1dm}q + \mu_{2dm}r.
\]

The unconditional expected value of firm 1 in our model is,

\[
\bar{v} = (a - bY)y_1z - B = \mu_0 + \mu_1q + \mu_2r.
\]

\footnote{It is shown after Proposition 2, that the noise term in the real signal is heteroskedastic and multiplicative and, thus, the informational content of the stock price does not change in this model, relative to DM.}
The left hand sides in equations (26) and (25) are, respectively,
\[ \frac{3a^2\sigma_z^2(\sigma_z^2 + 2\sigma_z^2)}{b(\sigma_z^2 + 6\sigma_z^2)^2} \quad \text{and} \quad \frac{3a^2\sigma_z^2(\sigma_z^2 + 2\sigma_z^2)}{b(\sigma_z^2 + 9\sigma_z^2)^2}. \]

This means that the unconditional net profit of firm 1 in this model is less than in DM. Hence, the right hand side of (25) must be less than the right hand side of (26). In this paper, the manager is compensated less than in DM, and hence the value of \( \mu_0 \) in this paper is less than in DM. Together with the fact that \( \bar{r} \) is the same in both models (by Lemma 4(iii)) and the fact that \( \bar{q} \) in this paper is less than in DM, we obtain that \( \mu_1 \) and \( \mu_2 \) are both less than in DM. In sum, competition in the real sector reduces the unconditional expected value of firm 1 and, thus, the coefficients of the stock price signals are less with the presence of real sector competition than in the absence of this competition. This result is similar to the result comparing JMC to JM.\(^{11}\)

Second, the output produced by firm 1 is less than in DM and the total output is greater than in DM. This is true for two reasons. First, since \( \mu_1 \) has an effect on the output, the reduction in \( \mu_1 \) lowers the output of firm 1. Second due to the Cournot duopoly framework, the output of firm 1 also decreases due to the existence of the second firm. Thus the two effects together reduce the output of firm 1. The total output of the two firms increases due to Cournot competition, and, thus the price and the gross profit (revenue) of firm 1 decreases.

Third, the trading function of each insider is the same as in DM, although the financial asset value, i.e., the profits of the firm, is different in both models. In part, this is due to the fact that the market makers take into account the changes in the market structure (since the output produced by firm 1 as well as the total outputs are deterministic). This is also due to the zero expected profit condition of the market makers (which requires that the price should be equal to the conditional expectation of the asset’s value given the signals). In other words, at equilibrium, each insider trades the same quantity as in DM in order to maximize profits.

4.2. Information Revelation. We start this section by evaluating the information revelation in this model relative to JMC and DM. We show that adding another insider increases the informational content of the stock price compared to JMC. In contrast, adding a competitive firm in the real sector, the level of the information revealed by the stock price set by the market makers remains unchanged relative to DM. Hence, the information incorporated in the stock price is affected by the presence of competition in the financial sector and not competition in the real sector. In order to interpret these results we use,\(^{11}\)

\(^{11}\)In JMC the value of \( \mu_2 \) is less than in JM. The difference between the result stated in this paper and the result reported in JMC comes from the calculation of \( \mu_1 \). After correcting the value of \( \mu_1 \) in JMC, the value of \( \mu_1 \) in JMC is less than in JM.
as a measure of informativeness, the conditional variance of $\tilde{z}$ given the information of the market maker.\textsuperscript{12} This can be written as,\textsuperscript{13}

$$Var(\tilde{z}|\tilde{q}, \tilde{r}) = \frac{(y_1 - \mu_1)}{3y_1} \sigma_z^2.$$ 

Substituting the expressions of $y_1$ and $\mu_1$ of Proposition 2, we get,

(27) $$Var(\tilde{z}|\tilde{q}, \tilde{r}) = \frac{\sigma_z^2}{\sigma_z^2 + 3\sigma_z^2} \frac{\sigma_z^2}{\sigma_z^2 + 2\sigma_z^2}.$$ 

**Proposition 3.** The stock price reveals more information in this model than in the presence of competition in the real sector (JMC) and the same amount of information as in the presence of competition in the stock sector (DM).

**Proof:** Recall that the conditional variance in JMC and DM are, respectively,

$$\frac{\sigma_z^2}{\sigma_z^2 + 2\sigma_z^2} \frac{\sigma_z^2}{\sigma_z^2 + 3\sigma_z^2}.$$ 

Comparing these expressions to equation (27), the result is proved \( \blacksquare \)

The intuition for this result is the following. First, the noise term $\tilde{\varepsilon}$, in the real signal is heteroskedastic and multiplicative (see equation 16) and therefore, the information of the market makers is the same when the deterministic part $\alpha$ of the real signal increases. Hence, adding a second firm to DM, increases the deterministic part of the real signal without changing the deterministic part $\gamma$ of the total order flow signal (Lemma 4) and thus the information transmitted to the market makers through both signals remains unchanged.

Second, note that the noise term $\tilde{u}$, in the total order flow signal is homoskedastic (see equation 17). Hence, the greater the deterministic part $\gamma$ of the total order flow signal, the more informative is the total order flow signal. Adding a second informed trader to JMC, i.e, the model of our paper, changes the values of the deterministic part $\alpha$, of the real signal as well as the deterministic part $\gamma$, of the total order flow signal. Hence, when the market makers observe these two signals, they receive more information about $z$ only through the total order flow since the noise of the real is heteroskedastic and multiplicative. Consequently, adding competition in the financial sector increases the level of information incorporated in the stock price set by the market makers. Note that this result is consistent with the Kyle-type model, going from one to two insiders (see Tighe 1989 or Holden and Subrahmanyam (1992)).

\textsuperscript{12}Since there is no real market in Kyle (1985),(see also JMJ (1999) and Tighe (1989)) the value of the financial asset is the information of the insider. In contrast, in JM, JMC, DM and in this paper, the information of the insider is not the value of the firm.

\textsuperscript{13}See Theorem 1 in the Appendix
5. Insiders’ Profits

In this section, we analyze the profits of the two insiders. First, we show that the profits of the manager are not always less than the profits of the manager in JMC. Second, we prove that the profits of the manager and the owner of this model are less than in DM.

5.1. Manager’s Profits. We show, in this section, that the manager’s conditional profits in our model are always less than in DM. On the other hand, the conditional profits of the manager in JMC are less than our model when \( \sigma_z^2 \) is small, relative to \( \sigma_z^2 + 9\sigma_\epsilon^2 \) and greater than in JMC, otherwise. To interpret these results, we calculate the profits of the manager in this model. Substituting for \( Y, x_1(\cdot), x_2(\cdot), \mu_0, \mu_1, \mu_2 \) from Proposition 2, in the manager’s conditional profit function, we obtain,

\[
G_1(\tilde{z}) = \frac{3\sqrt{2}a^2(\sigma_z^2)^2\tilde{z}^2\sigma_u}{2b\sigma_z(\sigma_z^2 + 9\sigma_z^2)^2}.
\]  
(28)

Recall that the insider’s profits in JMC and DM are respectively,

\[
G_{jmc}^1(\tilde{z}) = \frac{2a^2(\sigma_z^2)^2\tilde{z}^2\sigma_u}{b\sigma_z(\sigma_z^2 + 6\sigma_z^2)^2}.
\]  
(29)

and

\[
G_{dm}^1(\tilde{z}) = \frac{3\sqrt{2}a^2(\sigma_z^2)^2\tilde{z}^2\sigma_u}{2b\sigma_z(\sigma_z^2 + 6\sigma_z^2)^2}.
\]  
(30)

The following Proposition, shows that the manager’s conditional profits in our model are less than in DM.

**Proposition 4.** The profits of the manager in the presence of competition in the real and the stock sectors are less than in the presence of only competition in the stock sector. Formally,

\[
G_1 < G_{dm}^1
\]

The essence of this result is that, relative to DM, competition in the real sector reduces the net unconditional profits \( \bar{v} \), which is the value of the financial asset of firm 1. To understand the relationship between the value of the firm and the profits of the manager, recall that the expressions of the manager’s conditional profits in our model and DM are, respectively,

\[
G_1(\tilde{z}) = \left[ (a - bY)(y_1 - \mu_1)\tilde{z} \right] x_1(\tilde{z}) \quad \text{and} \quad G_{dm}^1(\tilde{z}) = \left[ (a - by_{dm})(y_{dm} - \mu_{1dm})\tilde{z} \right] x_{dm}^1(\tilde{z})
\]

Indeed, the expression of the manager’s conditional profits is a product of the trading level of the manager times a function of \( \tilde{z} \), i.e., \( g(\tilde{z}) = \frac{(a-bY)(y_1-\mu_1)\tilde{z}}{3} \). Since the trading level of the manager is the same in both models (see Lemma 4), the same Lemma shows that \( g(\tilde{z}) \) in our model is less than in DM. Consequently, the Cournot duopoly structure reduces the manager’s profits in this model relative to DM.
We show, in Proposition 5, that relative to JMC, the manager’s profits are not always less his profits in JMC. We have,

**Proposition 5.** The profits of the manager in the presence of competition in the real and financial sectors, relative to the manager’s profits with competition only in the real sector, are

\[
\begin{cases} 
G_1 > G_1^{jmc} & \text{when } \sigma^2_\varepsilon \text{ is small relative to } \sigma^2_z, \\
G_1 < G_1^{jmc} & \text{otherwise}
\end{cases}
\]

*Proof:* The proof is obvious and thus omitted.

Note that for small values of \(\sigma^2_\varepsilon\), relative to \(\sigma^2_z\), the unconditional net profits of firm 1 (equations 21 and 22), in JMC are less than in our model. Thus, the conditional profits of the manager in JMC are less than in our model. The intuition behind this result is that when \(\sigma^2_\varepsilon\) is small, relative to \(\sigma^2_z\), the real signal \(\tilde{q}\) is more informative than the total order flow signal \(\tilde{r}\). In other words, when \(\sigma^2_\varepsilon\) is small, relative to \(\sigma^2_z\), adding a second insider to JMC does not affect the information of the market makers since the real signal dominates. Moreover, the informational content from the real signal has a direct effect on the value of the firm and, thus, on the unconditional profits of the manager through the zero profits condition of the market makers. This direct effect reduces the profits of the manager with competition in only the real sector relative to the profits of the manager in the presence of competition in both the real and the financial sectors. In contrast, when \(\sigma^2_\varepsilon\) is large, relative to \(\sigma^2_z\), the net unconditional profits of firm 1 in our paper is less than in JMC and, thus the conditional profits of the manager in JMC are greater than in our model. Indeed, in this case, increasing the number of insiders, increases the informational content of the total order flow signal \(\tilde{r}\) and thus reduces the value of the firm which in turn reduces the profits of the manager in this model relative to the profits with only competition in the real sector (JMC).\(^{14}\) Note that this case is similar to the Kyle-type model going from one to two insiders (See Tighe (1989) or Holden-Subrahmanyam (1992)).

It should be pointed out that adding a second firm, which does not change the information content of the two signals (Proposition 3), reduces the value of the original firm. This is due, mostly, to Cournot competition. However adding another informed trader

\(^{14}\)The relationship between JM and JMC, going from one to two firms producing in the real sector yields the intuitive result that profits are reduced, due to Cournot competition. However, going from one to two insiders, i.e., JM to DM has an ambiguous effect on profits. However, it would appear that going from one insider and one real firm to two insiders and two real firms, i.e., JM to this paper, that profits should also decline due to Cournot competition, i.e., as going from JM to JMC. This is not the case. In fact, the effect on profits due to the addition of a second firm depends upon the number of insiders trading. Indeed, in this situation profits sometimes decline and sometimes increase, going from JM to DM. Thus, the addition of a second informed trader is more dominant in determining the profits of the manager than the addition of a second firm.
increases the informational content of the total order flow signal but has a minor effect, compared to adding a second firm, on the profitability of the firm. Note that adding a second insider to JMC has the same effect on the profits of the manager (Proposition 5), on the stock price coefficient $\mu_2$ (Lemma 2) and on the compensation scheme of the manager.

**Remark 5.1.** If the market makers receive only one signal (the total order flow), the conditional profits of the manager in our model when there is only one signal are less than JMC, which are less than DM. This result is due to the fact that the unconditional profits of the firm with competition in both real and financial sectors are less than in JMC as well as in DM, when only the total order flow signal is observed.  

**Remark 5.2.** Note that the compensation scheme of the manager makes his profits independent of the prior beliefs, $\bar{z}$, of the market makers. On the other hand, the profits of the owner depend on the prior beliefs of the market makers, a result similar to JMJ, going from one to two insiders, and to Tighe. This is due to the fact that the owner has no managerial activities and, thus, he does not receive a compensation scheme.

### 5.2. Owner’s Profit.

In this section, we compare the owner’s conditional profits in this model to the owner’s conditional profits in DM. We show that under competition in both the real and the financial markets, the conditional profits of the owner are less than the owner’s profits in DM.

In order to interpret these results, we calculate the profits of the manager in this model. Substituting for $Y, x_1(\cdot), x_2(\cdot), \mu_0, \mu_1, \mu_2$ from Proposition 2, in the manager’s profits function, we obtain,

$$G_2(\hat{z}) = \frac{3\sqrt{2} a^2 (\sigma_z^2)^2 (\hat{z} - \bar{z})^2 \sigma_u}{2 b \sigma_z (\sigma_z^2 + 9 \sigma_e^2)^2}.$$  

(31)

That the owner’s profits in DM is,

$$G_{2dm}(\hat{z}) = \frac{3\sqrt{2} a^2 (\sigma_z^2)^2 (\hat{z} - \bar{z})^2 \sigma_u}{2 b \sigma_z (\sigma_z^2 + 6 \sigma_e^2)^2}.$$  

(32)

15Indeed, it is shown in DM that the conditional expected profits of the manager in JMC are less than in DM when the market makers observe only the total order flow. Moreover, it is easy to check that when the market makers observe only the total order flow, the profits of the manager in this model are also less than in JMC.

16It should be pointed out that in JMJ, i.e., there is no real market, with two insiders as well as with two signals, a symmetric linear equilibrium exists, in which the profits function of each insider depends on the prior beliefs of the market makers. This is not the case in this paper, since there is a real market. Thus, a compensation scheme is required to satisfy the second order condition of the manager. Moreover, this compensation scheme implies that the beliefs of the market makers have no effects on the profits of the manager.
A comparison between the expressions of the owner’s profits in this model and in DM, leads to the following result.

**Proposition 6.** The conditional profits of the owner in our model are less than the owner’s profits in the presence of only competition in the financial sector, that is,

\[ G_2 < G_{2m} \]

The intuition for this result, is that adding a competitive firm in the real sector reduces the net value of the firm 1 and then reduces the conditional profits of the owner relative to DM.

**Remark 5.3.** If the market maker receives only the total order flow as in Kyle, the conditional profits of the owner in this paper continue to be less than in DM. This is due to Cournot competition which lowers the profits of firm 1.

**APPENDIX**

We start this appendix by recalling the Theorem that we use to prove Lemma 1. Then we prove this Lemma.

**Theorem 1.** If the \( p \times 1 \) vector \( Y \) is normally distributed with mean \( U \) and covariance \( V \) and if the vector \( Y \) is partitioned into two subvectors such that \( Y = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \) and if \( Y^* = \begin{pmatrix} Y_1^* \\ Y_2^* \end{pmatrix} \)

\[ U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \]

are the corresponding partitions of \( Y^*, U \) and \( V \), then the conditional distribution of the \( m \times 1 \) (\( m < p \)) vector \( Y_1 \) given the vector \( Y_2 = Y_2^* \) is the multivariate normal distribution with mean \( U_1 + V_{12}V_{22}^{-1}(Y_2^* - U_2) \) and covariance matrix \( V_{11} - V_{12}V_{22}^{-1}V_{21} \).

**Proof:** See Graybill, Theorem 3.10 pp 63.

**Proof of Lemma 1:**
We apply Theorem 1 to the normal random vector \( B = (\tilde{v}, \tilde{q}, \tilde{r}) \). First, in this case we have \( p = 3 \) and \( m = 1 \). Second by identification, we have \( Y_1 = \tilde{v} \) and \( Y_2 = \begin{pmatrix} \tilde{q} \\ \tilde{r} \end{pmatrix} \). \( U_1 = \tilde{v}, \)

\[ U_2 = \begin{pmatrix} \tilde{q} \\ \tilde{r} \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} \sigma_v^2 & \sigma_{vq} & \sigma_{vr} \\ \sigma_{vq} & \sigma_q^2 & \sigma_{qr} \\ \sigma_{vr} & \sigma_{qr} & \sigma_r^2 \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \]
Where $V_{11} = \sigma_v^2$, $V_{12} = (\sigma_{vq}, \sigma_{vr})$, $V_{21} = \left( \frac{\sigma_{vq}}{\sigma_{vr}} \right)$ and $V_{22} = \left( \frac{\sigma_q^2}{\sigma_r^2} \frac{\sigma_{qr}}{\sigma_r^2} \right)$.

Note that
\[
V_{22}^{-1} = \frac{1}{D} \begin{pmatrix}
\sigma_r^2 & -\sigma_{qr} \\
-\sigma_{qr} & \sigma_q^2
\end{pmatrix},
\]
where $D$ is the determinant of $V_{22}$, that is $D = \sigma_q^2 \sigma_r^2 - \sigma_{qr}^2$.

So we obtain,
\[
\mu_0 = \bar{v} - \mu_1 \bar{q} - \mu_2 \bar{r}
\]  
(33)
\[
\mu_1 = \frac{\sigma_{vq} \sigma_r^2 - \sigma_{vr} \sigma_{qr}}{D}
\]  
(34)
\[
\mu_2 = \frac{\sigma_{vr} \sigma_q^2 - \sigma_{vq} \sigma_{qr}}{D}
\]  
(35)

Substituting for the variances and covariances in (34) and (35), we get
\[
\mu_1 = \frac{(a - bY)^2 y_1 \sigma_v^2 \sigma_u^2}{D},
\]  
(36)
\[
\mu_2 = \frac{2(a - bY)^2 y_1 (y_1 - \mu_1) \sigma_v^2 \sigma_z^2}{3 \mu_2 D}.
\]  
(37)

Computing (36) and (37), we obtain
\[
3 \mu_2^2 = \frac{2(a - bY)^2 \mu_1 (y_1 - \mu_1) \sigma_v^2}{\sigma_u^2}.
\]  
(38)

Calculating for the expression of $D$, we get
\[
D = \frac{4(a - bY)^4 (y_1 - \mu_1)^2 \sigma_z^2 \sigma_v^2}{9 \mu_2^2} + (a - bY)^2 \sigma_u^2 (\sigma_v^2 + \sigma_z^2).
\]

Substituting the above expression of $D$ in (37), we find
\[
3 \mu_2^2 = \frac{2(a - bY)^2 (y_1 - \mu_1)(y_1 + 2 \mu_1) \sigma_v^2 \sigma_z^2}{3 \sigma_u^2 (\sigma_v^2 + \sigma_z^2)}.
\]  
(39)

Solving (38) and (39), we get
\[
3 \mu_1 (\sigma_v^2 + \sigma_z^2) = \sigma_z^2 (y_1 + 2 \mu_1).
\]

Substituting for $y$ to solve $\mu_1$, we obtain
\[
\mu_1 = \frac{a \sigma_z^2}{b (\sigma_z^2 + 9 \sigma_v^2)}.
\]
To solve for \( \mu_2 \), we substitute the above value of \( \mu_1 \) in (38) and taking the positive root, we get

\[
\mu_2 = \sqrt{2a^2 \sigma_z \sqrt{(1-k)^3}k},
\]

where \( k = \frac{\sigma_z^2}{\sigma^2 + 9\sigma_z^2} \).

**References**


