Optimal Discretionary Monetary Policy under Downward Nominal Wage Rigidity *

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Abstract

We study the implications for optimal monetary policy when declining nominal wages is not a viable margin for adjustment to adverse economic conditions. To this end, we develop a New Keynesian model where downward nominal wage rigidity arises endogenously. We show that the optimal policy response to changing economic conditions is asymmetric. Another important finding is that downward nominal rigidity actually increases welfare. The reason is that downward nominal rigidity is a constraint that changes the choice set, which opens up for a more effective stabilization in terms of welfare losses. Thus, downward nominal rigidity is not just an additional constraint on the policy makers problem.

Keywords: Monetary Policy, Downward Nominal Wage Rigidity.

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Introduction

A robust empirical finding is that money wages don't fall during a recession, at least not to a significant degree. A large number of studies report substantial downward nominal wage rigidity in the U.S. as well as in Europe. The empirical evidence ranges from studies using data from personnel files presented in Baker, Gibbs and Holmstrom, 1994, Wilson, 1999, Altonji and Devereux, 2000 and Fehr and Goette, 2005, survey data in Altonji and Devereux, 2000, Akerlof, Dickens and Perry, 1996 and Fehr and Goette, 2005 to interviews or surveys with wage setters like Bewley, 1999 and Agell and Lundborg, 2003 just to mention a few. Also, this conclusion does not change when one considers a low inflation environment with persistent labor-market slack (see e.g. Agell and Lundborg, 2003 or Fehr and Goette, 2005). Overall, the evidence points towards there being a sharp asymmetry in the distribution of nominal wage changes around zero. Money wages rises but they very seldom falls.

The purpose of this paper is to study the implications for optimal monetary policy when declining nominal wages is not a viable margin for adjustment to adverse economic conditions. To this end, we develop a New Keynesian model that can account for downward nominal wage rigidity. More specifically, this is achieved by introducing wage bargaining between firms and unions in the model. Under reasonable conditions on conflict costs, downward nominal wage rigidity will arise as a rational endogenous outcome.

In the model, price and wage setting is staggered. The main difference with our approach, relative to standard models including a labor market; see Erceg, Henderson and Levin, 2000, is that we model wages as being determined in bargaining between firms and unions (households). To do this we have to modify the simplifying assumption of Erceg et al., 2000 that workers work at all firms. Otherwise, each worker works an infinitesimal amount at each firm, implying that the effect of the workers on firm surplus is zero and there is no surplus to be negotiated over, hence rendering bargaining irrelevant in their setup. This setup implies that wage and price setting are interdependent in our model.

We assume that wage bargaining is not a continuous process. Instead, bargaining is opened with some fixed probability each period, akin to Calvo, 1983. Bargaining is non-cooperative as in the Rubinstein-Ståhl model, with the addition that if there is disagreement but no conflict is called, work takes place according to the old contract. As argued by Holden, 1994, this is in line with the institutions of most western European labor market, as well as the U.S.labor market. Moreover, as in Holden, 1994, there are costs associated with conflicts in addition to costs stemming from impatience.
These costs sometimes renders threats of conflict incredible, leading to agreement on the same wage as in the old contract. As it is reasonable to assume that these costs are larger for firms than workers, workers can credibly threaten firms with conflict, while firms cannot. Since workers only use their threat when they can bid up wages, downward nominal wage rigidity will result.

In the model a non-linear restriction on the evolution of aggregate wages thus arises endogenously. Given the constraints from market behavior, the central bank solves for the optimal monetary policy. We focus on the discretionaty case, since it seems to be most in line with what central banks actually do.

We show that the optimal policy response to changing economic conditions is asymmetric. This asymmetry is not only present in the wage inflation dimension. Most notably, the optimal path for the output gap goes in opposite directions in response to a negative shock to the natural real wage in the cases with and without nominal downward wage rigidity.

Another important finding is that downward nominal wage rigidity can actually increase welfare. The reason is that downward nominal wage rigidity changes the choice set for the policymaker and opens up for a more effective stabilization in terms of welfare. With our calibration, welfare increases by about 0.5 percentage units.

In sections 1 and 2 we outline the model and discuss the equilibrium of the model, respectively. In section 3 we characterize the optimal monetary policy problem when nominal wages cannot fall. Section 4 discuss optimal policy responses and welfare implications of downward nominal wage rigidity and section 5 concludes.

1 The Economic Environment

Goods are produced by monopolistically competitive producers who set prices in staggered contracts as in Calvo, 1983. To each firm a group of workers is attached. Thus, in contrast to Erceg et al., 2000, firms do not perceive workers as atomistic. In each period, bargaining over wages takes place with a fixed probability. Thus, wages are staggered as in Calvo, 1983 but, in contrast to Erceg et al., 2000, they are determined in bargaining between the union and the firm. The assumptions made in their paper generates independence between the price and wage decisions. In this paper this is not the case. Bargaining is non-cooperative as in Rubinstein, 1982. When bargaining, there are costs of conflict as in Holden, 1994. In this paper the conflict costs endogenously generates downward nominal
rigidity. Below, we present the model and derive key relationships; for a full derivation see Carlsson and Westermark, 2006.

1.1 Households

The economy is populated by a continuum of households, indexed on the unit interval, which each supply labor to a single firm. This setup can also be interpreted as a unionized economy. In such a framework, each household can be thought of as the representative union member. Let \( c^h(f) \) denote the consumption household \( h \) of the good firm \( f \) produces. To state the payoff function of a household we first need to introduce the consumption index \( C^h_t \), which is defined as a Dixit-Stiglitz aggregator, i.e.,

\[
C^h_t = \left[ \int_0^1 c^h_t(f) \frac{\sigma - 1}{\sigma - 1} df \right]^{\frac{\sigma}{\sigma - 1}}.
\]

where \( \sigma > 1 \). Let \( p_t(f) \) denote the price of good \( f \) in period \( t \). Then, let \( P_t \) denote the relevant price index. That is, \( P_t \) is defined as the minimum expenditure required to purchase goods resulting in the consumption index of \( C^h_t \) such that \( C^h_t = 1 \),

\[
P_t = \left[ \int_0^1 p_t(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}}.
\]

Assuming that there exists complete contingent claims markets except for leisure implies that households are homogeneous with respect to consumption. Thus, \( C^h_t = C_t \) for all \( t \) and similarly for money holdings. The expected life time utility of the household working at firm \( f \) in period \( t \), \( U_t(f) \), is given by

\[
U_t(f) = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[ u(C_s, Q_s) + l \left( \frac{M_s}{P_s} \right) - v(L_s(f), Z_s) \right] \right\},
\]

where \( \beta \in (0, 1) \) is the households discount factor and \( L_s(f) \) is the labor supply of the household in period \( s \). The period \( S \) utility is additively separable in three arguments, consumption \( u(C_s, Q_s) \), subject to a consumption shock \( Q_s \) common to all households, real money balances \( l \left( \frac{M_s}{P_s} \right) \) and the disutility of working \( v(L_s(f), Z_s) \), subject to a labor supply shock \( Z_s \) common to all households. The
budget constraint of the household is

\[
\frac{B_t(f)}{P_t} + \frac{M_t}{P_t} + C_t \leq \frac{M_{t-1} + I_{t-1}B_{t-1}(f)}{P_t} + (1 + \tau_w) \frac{W_t(f)L_t(f)}{P_t} + \Gamma_t + \frac{T_t(f)}{P_t}
\]

(3)

where \(B_t(f)\) is the nominal value at in period \(t\) of the bonds yielding nominal interest \(I_{t-1}\) held by household working at firm \(f\). Moreover, \(W_t(f)\) denotes the nominal wage of the household and \(\tau_w\) is the tax rate on labor income. As in Erceg et al., 2000, each household own an equal share of all firms and of the aggregate capital stock. Then, \(\Gamma_t\) is the household’s aliquot share of profits and rental income and \(T_t(f)\) denotes the nominal lump sum transfers from the government.

As in Erceg et al., 2000, the subsidy \(\tau_w\) is determined so as eliminate the distortions on the labor market arising from monopoly power. That is, monetary policy is being used only to stabilize the economy. If distortions due to monopoly power were not eliminated, optimal monetary policy would in part try to eliminate these distortions as well. To focus on the stabilization role of monetary policy, we assume that distortions due to monopoly power are eliminated by the fiscal authority.

By standard arguments, the demand function for the generic good \(f\), is

\[
c_t(f) = \left(\frac{p_t(f)}{P_t}\right)^{-\sigma} C_t.
\]

(4)

Besides private sector demand for the product of the firm, given by expression (4), there is also demand from the public sector

\[
g_t(f) = \left(\frac{p_t(f)}{P_t}\right)^{-\sigma} G_t,
\]

(5)

where \(G_t\) is total government consumption. As in Erceg et al., 2000, we assume that \(G_t\) is zero in steady state.

1.2 Firms

In the economy, there is a continuum of monopolistically competitive firms, indexed on the unit interval, which each produce a differentiated good. The production of firm \(f\) in period \(t\), \(y_t(f)\), is given by the following constant returns technology

\[
y_t(f) = A_tK_t(f)^\gamma L_t(f)^{1-\gamma},
\]

(6)
where $A_t$ is the technology level common to all firms and $K_t(f)$ and $L_t(f)$ denote the firms capital and labor input in period $t$, respectively. Since firms have the right to manage, $K_t(f)$ and $L_t(f)$ are chosen optimally, taking the rental cost of capital and the bargained wage $W_t(f)$ as given. Moreover, as in Erceg et al., 2000, the aggregate capital stock is fixed at $\bar{K}$. Standard cost-minimization arguments then imply that the marginal cost in production is given by\(^1\)

$$MC_t(f) = \frac{W_t(f)}{MPL_t(f)},$$

where $MPL_t(f)$ is the firm marginal product of labor.

1.3 **Wage and price choices**

Instead of assuming fully flexible prices we assume that a firm is allowed to change prices in a given period with probability $1 - \alpha$ and renegotiate wages with probability $1 - \alpha_w$. Any firm that is allowed to change wages, is also allowed to change prices, see the discussion below.\(^2\)

1.3.1 **Wage determination**

Bargaining takes place in a setup similar to the model by Holden, 1994. There are two key features of this model. First, there are costs of invoking a conflict. These costs are different from the standard costs in bargaining due to impatience. Instead they are caused by e.g., disrupting business relationships, startup costs and deteriorating management-employee relationships, see Holden, 1994. Second, there is an old contract in place at the firm and if no conflict is called and no new contract is signed, the workers work according to the old contract. As pointed out by Holden, 1994, this is a common feature of many western European countries and the U.S. As soon as a conflict is called, payoffs are determined in a standard Rubinstein-Ståhl bargaining game. However, the costs of conflict implies that it is sometimes not credible in equilibrium to threaten with a conflict. Specifically, if the difference between the old contract and the Rubinstein-Ståhl solution is small, then a party cannot credibly threaten with a conflict and force the new contract into place. The agreement is then on the old contract, resulting in nominal rigidity. If the difference is large enough, then conflict is a

\(^1\) In contrast to Erceg et al., 2000, marginal cost is not generally equal among firms, since firms face different wages out of steady state.

\(^2\) Assuming that prices and wages that are not changed optimally in a given time period are indexed to the steady state inflation and wage inflation rates gives similar results as assuming a zero steady state inflation, as can be seen in Carlsson and Westermark, 2006.
credible threat. Since players are impatient, it is optimal to agree on the Rubinstein-Ståhl solution immediately, rather than to wait and endure a conflict.  

A full explanation for downward nominal wage rigidity are likely to include several other mechanisms that may or may not be complementary. Studies like Bewley, 1999 and others indicate that the underlying causes for this fact is connected to psychological mechanisms involving fairness considerations and managers concern over workplace morale. Moreover, the workers yardstick for fairness seem to be what happens to nominal wages rather than real wages. However, here we focus on the explanation in Holden, 1994, since it is a fully micro-founded rational explanation.

To derive only downward nominal rigidity, asymmetries in conflict costs are required. Specifically, if the costs are large for the firm and negligible for the union, the firm never calls a conflict (at least close to the steady state) and the union calls a conflict when the Rubinstein-Ståhl solution is larger than the old contract. In reality, conflict costs for the workers are probably not zero, but small. Then wages would be adjusted only if the Rubinstein-Ståhl solution exceeds some threshold value \( \omega > W_{t-1} \). For simplicity, we restrict attention to the case when conflict costs are zero for workers.

Note that downward nominal rigidity implies that there is a potential relationship between wage negotiations today and in the future. This interdependence comes from two sources. First, the wage contract is a state variable in future negotiations and second, the wage set today affects prices set in the future, which, in turn, affects future wage negotiations. The first interdependence is eliminated since we analyze a linearized model close to steady state, as can be seen in the section “Wage inflation and real wages” in Carlsson and Westermark, 2006. The second interdependence is eliminated by the assumption that prices can be changed when wages are allowed to change. Thus, each wage negotiation can be analyzed separately. Moreover, there is no intertemporal interdependence in price setting decisions for a given firm. To see this, note that, since prices can be adjusted in any direction, the current price is not a state variable in future price setting. Any interdependence over time in price setting must thus come via wage negotiations, but such interdependence is ruled out by the assumption that prices change whenever wages change.

The discussion in the previous paragraph is about dependence between current and future wage/price negotiations. Some interdependence remains, though. Specifically, when negotiating a given wage contract, wage setters take into account that the contract affect price setting during the entire period the wage contract is in force. If for example, the price is changed three periods ahead, given the current

\[^3\text{That agreement is immediate is a standard result in bargaining games, see e.g., Rubinstein, 1982.}\]
wage contract, the optimal price is of course affected by the wage. Thus, wage and price setting are interdependent, but only within a given wage contract.

An important implication of the assumptions above is that it allows us to describe goods market equilibrium by the same type of forward looking new Keynesian Phillips curve as in Erceg et al., 2000; see (13).

Unions at a firm represent all workers at the firm and maximizes the welfare of all members. Union utility $U_u$ is then

$$U_u = E_t \sum_{k=0}^{\infty} (\alpha_w \alpha \beta)^k (u (C_{t+k}, Q_{t+k}) - v (L_{t+t+k} (f), Z_{t+k}))$$

$$+ E_t \sum_{k=1}^{\infty} \left( \prod_{i=1}^{k-1} (\alpha_w + (1 - \alpha_w) F_{t+i} (d_{t+i})) \right) (\alpha_w (1 - \alpha) + (1 - \alpha_w) F_{t+k} (d_{t+k})) \beta^k$$

$$\times \sum_{j=0}^{\infty} (\alpha_w \alpha \beta)^j (u (C_{t+k+j}, Q_{t+k+j}) - v (L_{t+k,t+k+j} (f), Z_{t+k+j}))$$

where, letting $W_{t+k}^{opt} (f)$ denotes the unconstrained optimal wage in period $t + k$ for firm $f$,

$$d_{t+k} = \frac{W_{t+k}^{opt} (f)}{W_t (f)}$$

Then $F_{t+k} (d_{t+k})$ is the probability that, when renegotiation takes place, wages are not adjusted. The term $L_{t+k,t+k+j} (f)$ is labor demand in period $t + k + j$ when prices was last changed in period $t + k$. The first sum is the terms when the prices are not allowed to be changed in the future. The last sum is the terms when prices but not wages are changed in the future.

Letting the optimal price as a function of wages be denoted as $p_t (f, W (f))$, firm payoff $U_f$ is

$$U_f = E_t \sum_{k=0}^{\infty} (\alpha_w \alpha \beta)^k (\psi_{t,t+k} ((1 + \tau) p_{t,t} (f, W (f)) y_{t,t+k} (f) - TC (W (f), y_{t,t+k} (f))))$$

$$+ E_t \sum_{k=1}^{\infty} \left( \prod_{i=1}^{k-1} (\alpha_w + (1 - \alpha_w) F_{t+i} (d_{t+i})) \right) (\alpha_w (1 - \alpha) + (1 - \alpha_w) F_{t+k} (d_{t+k}))$$

$$\times \sum_{j=0}^{\infty} (\alpha_w \alpha \beta)^j \psi_{t,t+k+j} ((1 + \tau) p_{t+k,t+k} (f, W (f)) y_{t+k,t+k+j} (f) - TC (W (f), y_{t+k,t+k+j} (f)))) .$$

The first term is the term where the firm change price today and is not selected to change prices in the future. The second term is terms where the firm has changed prices in the future at least once.
The first term is asymmetric, since all firms are allowed to change prices when wages are rewritten. The term $\psi_{t,t+k}$ is how the households value profits in period $t+k$ when in period $k$. This will in general depend on the discount factor $\beta$ and on the marginal utility of income in a given period, i.e., $u_C(C_{t+k}, Q_{t+k})$; see Carlsson and Westermark, 2006 and Erceg et al., 2000 for a definition.

Since the Rubinstein-Ståhl solution can be found by solving the Nash Bargaining problem, we can solve for the unconstrained wage from

$$\max_{W(f)} (U_u - U_o)^\varphi U_f^{1-\varphi}$$

where $\varphi$ is the bargaining power of the union and $U_o$ the outside option for workers. The outside option is the payoff when on strike. Then, workers are assumed to receive a share of average income and not spend any time working. For simplicity, we assume that $U_o$ is independent of shocks. 4 The first-order condition is

$$\varphi U_f \frac{\partial U_u}{\partial W(f)} + (1 - \varphi)(U_u - U_o) \frac{\partial U_f}{\partial W(f)} = 0$$

If $W(f)$ is the solution to the problem above, the wage at the firm that renegotiates the wage with old contract $W_{t-1}(f)$, the resulting wage at the firm is

$$\max\{W(f), W_{t-1}(f)\}.$$

In case the optimal wage is lower than the present contract, the old contract prevails.

1.3.2 Wage evolution

Wage inflation is defined as

$$\pi^\omega_t = \frac{W_t}{W_{t-1}}.$$
and the average wage is

$$W_t = \alpha_w \int_0^1 W_{t-1} (f) df + (1 - \alpha_w) \int_{W_{t-1}(f) > W_t^{opt}} W_{t-1} (f) df + (1 - \alpha_w) \int_{W_{t-1}(f) \leq W_t^{opt}} W_{t,t} (f) df$$

(9)

### 1.3.3 Prices

The producers choose prices to maximize

$$\max_{p_t(f)} E_t \sum_{k=0}^{\infty} (\alpha w)^k \psi_{t,t+k} \left( (1 + \tau) p_t (f) y_{t,t+k} (f) - TC (W_{t+k}, y_{t,t+k} (f)) \right)$$

(10)

s.t. $y_{t,t+k} (f) = \left( \frac{p_t (f)}{P_{t+k}} \right)^{-\sigma} (C_{t+k} + G_{t+k})$

where $TC (W_{t+k}, y_{t,t+k} (f))$ denotes the cost function. The last term is just firm profits in period $t+k$, given that prices are last reset in period $t$. The first-order condition is

$$E_t \sum_{k=0}^{\infty} (\alpha w)^k \psi_{t,t+k} \left( \frac{\sigma - 1}{\sigma} (1 + \tau) p_t (f) - MC_{t+k} \right) y_{t,t+k} (f) = 0$$

(11)

As for wages, the subsidy $\tau$ is determined as to set $\frac{\sigma - 1}{\sigma} (1 + \tau) = 1$. That is, we again assume that fiscal policy is used to alleviate distortions due to monopoly price setting.

### 1.3.4 Price evolution

From expression (1), using that $\frac{P_t}{P_{t-1}} = \pi_t$, the price level is determined by

$$P_t^{1-\sigma} = \alpha w \int_0^1 (\pi p_{t-1} (f))^{1-\sigma} df + (1 - \alpha w) \int (p_{t,t} (f))^{1-\sigma} df$$

### 1.4 Steady state

In the non-stochastic steady state, the shocks $A_t$, $Z_t$ and $Q_t$ are equal to their steady-state values $\bar{A}$, $\bar{Z}$ and $\bar{Q}$. and the shock $G_t$ equal to zero at all dates. Moreover, all firms produce the same (constant) amount of output, i.e. $\bar{y}(f) = Y$, using the same (constant) quantity of labor and all households supply the same amount of labor, i.e. $\bar{L}(f) = L$. Moreover, we will have that $\bar{C} = Y$ and that $B = 0$. Also, $M$ and $P$ are constant.
To find the steady state of the model we use the production function (6) together with the condition \( MPL = MRS \) (which holds due to the tax scheme outlined above) to solve for \( L \) and, in turn, \( Y \) and \( C \).

2 Equilibrium

First, let the superscript * denote variables in the flexible price and wage equilibrium, which we below refer to as the natural equilibrium, and a hat above a small letter variable denote log-deviations from the variables steady-state level. Linearizing around the steady state then gives the following system of equations, where the constants are given in section B,

\[
\begin{align*}
\hat{x}_t &= E_t \left( \hat{x}_{t+1} + \frac{1}{\rho_G} \left( \hat{\pi}_t - \hat{\pi}_{t+1} - \hat{\pi}_t^* \right) \right), \\
\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \frac{\gamma}{1 - \gamma} \Pi \hat{x}_t + \Pi (\hat{w}_t - \hat{w}_t^*), \\
\hat{w}_t &= \hat{w}_{t-1} + \hat{\pi}_t^w - \hat{\pi}_t, \\
\hat{\pi}_t^w &= \max\{0, \frac{1 + \alpha_w}{2\alpha_w} \beta E_t \hat{\pi}_{t+1}^w - \Omega_x \hat{x}_t - \Omega_w (\hat{w}_t - \hat{w}_t^*) - \Omega_x^0 \hat{\pi}_t + \Omega_x^1 E_t \hat{\pi}_{t+1} + \Omega_x^2 E_t \hat{\pi}_{t+2} \}. \quad (15)
\end{align*}
\]

Equation (12) is the goods demand equation which relates the output gap \( \hat{x}_t \), i.e. the log-deviation between output and the natural output level, to the expected future output gap and the expected real interest rate gap \( (\hat{r}_t - \hat{\pi}_{t+1} - \hat{\pi}_t^*) \), where \( \hat{r}_t \) denotes the log-deviation of the nominal interest rate from steady state and \( \hat{\pi}_t^* \) is the log-deviation of the natural real interest rate from it’s steady state. This relation is derived using the households first order condition with respect to consumption, i.e., the consumption Euler equation (see the section “Log-linearization of the model” in Carlsson and Westermark, 2006 for details).

The price setting (Phillips) curve, equation (13) is derived using the firms first-order condition (11), (see the section “Log-linearization of the model” in Carlsson and Westermark, 2006 for details). Relative to models with no explicit labor market, like Clarida, Gali and Gertler, 1999, price setting is also affected by the real wage gap, i.e., the log-deviation between the real wage \( \hat{w}_t \) and the natural real wage \( \hat{w}_t^* \). Since \( \Pi > 0 \), both a positive real wage gap as well as a positive output gap feeds in inflationary pressure in price setting decision. The reason is that, both effects raise marginal cost.
above its steady-state level, inducing higher prices and hence higher inflation.

The evolution for the real wage follows from the definition of the aggregate real wage and is described by the identity (14) and states that today’s real wage is equal to yesterday’s real wage plus the difference between wage and price change (\(\hat{\pi}^\omega_t - \hat{\pi}_t\)).

Finally, equation (15) describes the wage setting behavior (see the section “Log-linearization of the model” in Carlsson and Westermark, 2006 for details). From wage setting above, we know that wages are set according to (7) and (8). This implies that wage inflation is non-negative and is set according to the last term in the max operator of (15) when positive. Hence, the max operator captures the restriction from wage setting in (15). Due to the interdependence of wage and price setting, wage inflation also depend on inflation today and in the future. Relative to the price setting curve, there are one future wage inflation term and two future price inflation terms in (15). The reason for this asymmetry is that probability adjusted discounting is different in price and wage setting. Also, the coefficient in front of \(E_t \hat{\pi}^\omega_{t+1}\) is different from the coefficient in front of \(E_t \hat{\pi}_{t+1}\) in (13). The term \(\frac{1+\alpha_w}{2}\) is the unconditional (steady state) probability that wages are unchanged \((1+\alpha_w)\) divided by the conditional probability that wages are unchanged \((\alpha_w)\) when shocks are positive. The reason for the term \(\frac{1+\alpha_w}{\alpha_w}\) is that, in wage negotiations, the probability adjusted discount rate is \(\frac{1+\alpha_w}{\alpha_w}\). The term \(\frac{1}{\alpha_w}\) stems from the way relative wages feeds into wage inflationary pressure. For our baseline calibration, \(\Pi\) and \(\Omega_w\) are positive while \(\Omega_x\), \(\Omega_{\pi 0}\), \(\Omega_{\pi 1}\) and \(\Omega_{\pi 2}\) are negative. Thus, wage inflation depend positively on the output gap and negatively on the real wage gap. In general, the sign of these two coefficients depend on e.g. the relative bargaining power between unions and firms.

3 The monetary policy problem

The central bank is assumed to maximize social welfare. Following the main part of the monetary policy literature we focus on the limiting cashless economy (see e.g. Woodford, 2003 for a discussion) with the social welfare function

\[
E_t \sum_{t=0}^{\infty} \beta^t \left( u(C_t, Q_t) - \int_0^1 v(n_t(h), Z_t) \, dh \right).
\]

Following Rotemberg and Woodford, 1997, Erceg et al., 2000 and others, we take a second order approximation to (16) around the steady state. This yields a standard expression for the welfare gap
(see the section “Welfare” in Carlsson and Westermark, 2006 for details, c.f. Erceg et al., 2000), i.e., the discounted sum of log-deviations of welfare from the natural (flexible price and wage welfare level)

$$E_t \sum_{t=0}^{\infty} \beta^t \left( \theta_x (\hat{x}_t)^2 + \theta_\pi (\hat{\pi}_t)^2 + \theta_{\pi^\omega} (\hat{\pi}_t^{\omega})^2 \right)$$

(17)

where we have omitted higher order terms and terms independent of monetary policy. One can show that $\theta_x < 0$, $\theta_\pi < 0$ and $\theta_{\pi^\omega} < 0$.

We focus on the discretionary case, since this is closest to what central banks do in practice. Note that welfare depends only on the variables $\hat{x}_t$, $\hat{\pi}_t$ and $\hat{\pi}_t^{\omega}$. Also, these three variables can be determined solely from equations (13) to (15). \(^5\)

To find the optimal rule under discretion, the central bank solves the following problem

$$V (\hat{w}_{t-1}, \hat{\pi}_t^\omega) = \max_{\{\hat{x}_t, \hat{\pi}_t, \hat{\pi}_t^\omega, \hat{w}_t\}} \theta_x (\hat{x}_t)^2 + \theta_\pi (\hat{\pi}_t)^2 + \theta_{\pi^\omega} (\hat{\pi}_t^{\omega})^2 + \beta E_t V (\hat{w}_t, \hat{w}_{t+1}^\ast)$$

subject to equations (13) to (15), disregarding that expectations can be influenced by policy.

Note that the last restriction can be replaced by

$$\hat{\pi}_t^\omega \geq \frac{1 + \alpha_w}{2 \alpha_w} \beta E_t \hat{\pi}_{t+1}^{\omega} - \Omega_x \hat{x}_t - \Omega_w (\hat{w}_t - \hat{w}_t^* ) - \Omega_\pi \pi_t + \Omega_\pi^1 E_t \hat{\pi}_{t+1} + \Omega_\pi^2 E_t \hat{\pi}_{t+2} \quad (18)$$

$$\hat{\pi}_t \geq 0 \quad (19)$$

The Lagrangian is

$$L = \theta_x (\hat{x}_t)^2 + \theta_\pi (\hat{\pi}_t)^2 + \theta_{\pi^\omega} (\hat{\pi}_t^{\omega})^2 + \beta E_t V (\hat{w}_t, \hat{w}_{t+1}^\ast)$$

$$- \lambda_t^\pi (\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1} - \Pi_x \hat{x}_t - \Pi_w (\hat{w}_t - \hat{w}_t^*) ) - \lambda_t^w (\hat{w}_t - \hat{w}_{t-1} - \hat{\pi}_t^{\omega} - \hat{\pi}_t ) - \mu_t^\omega (\hat{\pi}_t^{\omega})$$

$$- \mu_t^\pi \left( \frac{1 + \alpha_w}{2 \alpha_w} \beta E_t \hat{\pi}_{t+1}^{\omega} - \Omega_x \hat{x}_t - \Omega_w (\hat{w}_t - \hat{w}_t^*) - \Omega_\pi \pi_t + \Omega_\pi^1 E_t \hat{\pi}_{t+1} + \Omega_\pi^2 E_t \hat{\pi}_{t+2} - \hat{\pi}_t^{\omega} \right).$$

Note that the problem with the original max constraint (15) and the problem with inequality constraints (18) and (19) need not be equivalent. It is obviously true that a solution $(\hat{x}_t, \hat{\pi}_t, \hat{\pi}_t^{\omega}, \hat{w}_t)$ to the problem with the original max constraint also satisfies the two inequality constraints. However, it is possible that there is a solution $(\hat{x}_t, \hat{\pi}_t, \hat{\pi}_t^{\omega}, \hat{w}_t)$ to the problem with inequality constraints so that

\(^5\)To find the optimal path for the interest rate, the solution for $\hat{\pi}_t$, $\hat{\pi}_t^{\omega}$ and $\hat{x}_t$ is plugged into the Euler equation (12) which is then solved for $\hat{\pi}_t$. 

none of the inequality constraints is binding, leading to a violation of the original max constraint. The following Lemma rules out this possibility, implying that the problems are equivalent.

**Lemma 1** At least one of the inequality constraints (18) and (19) must be binding.

**Proof:** See the section “Optimal Discretionary Policy” in Carlsson and Westermark, 2006.

The intuition for the result is the following. Since the constraints (18) and (19) both put lower bounds on $\hat{\pi}_t^\omega$ and welfare, as can be seen from expression (17), is decreasing in $\hat{\pi}_t^\omega$, the central bank sets $\hat{\pi}_t^\omega$ as low as possible, implying that one of the inequality constraints (18) and (19) must bind.

The model can be solved by using the first-order conditions and substituting in the constraints to eliminate the Lagrange multipliers from these; see the section “Optimal Discretionary Policy” in Carlsson and Westermark, 2006 for details. This gives rise to two system depending on whether the inequality constraint binds or not (see the section on the numerical algorithm in the appendix A).

Note that we solve the problem in a different way than Erceg et al., 2000. Instead of postulating the interest rate rule and then choosing parameters to maximize welfare, we find the paths for $\hat{x}_t$, $\hat{\pi}_t$, $\hat{\pi}_t^\omega$ and $\hat{w}_t$ that maximize welfare, as suggested by Woodford, 2003. ⁶

The first issue at hand is to determine the stochastic properties of the natural real wage, $\hat{w}_t^*$. It is easy to show that $\hat{w}_t^*$ is a linear combination of all the exogenous driving forces in the model ($\hat{Q}_t$, $\hat{Z}_t$, $\hat{A}_t$ and $G_t$), i.e.,

$$\hat{w}_t^* = a_Q\hat{Q}_t + a_Z\hat{Z}_t + a_A\hat{A}_t + a_GG_t.$$  

where $a_Q < 0$, $a_Z < 0$, $a_A > 0$ and $a_G < 0$. If we are interested in analyzing for example the effects of a productivity shock we just set the other shocks to zero. If productivity follows an AR(1) process then obviously $\hat{w}_t^*$ follows an AR(1) process. If all shocks follow AR(1) processes, then when analyzing each shock separately as with the productivity shock, $\hat{w}_t^*$ is also AR(1). Thus, to simplify analysis we suppress the shocks $\hat{Q}_t$, $\hat{Z}_t$, $\hat{A}_t$ and $G_t$ and assume that $\hat{w}_t^*$ follows an AR(1) process;

$$\hat{w}_t^* = \eta\hat{w}_{t-1}^* + \varepsilon_t,$$

where $\varepsilon_t$ is an i.i.d. shock with standard deviation $\sigma_\varepsilon$.

⁶ To solve for optimal rules one can use the paths together with the Euler equation and suitable criteria for the shape of the rule; see Woodford, 2003 for a discussion.
Numerically, we solve this system by iteration on the policy functions and updating the value function given the new policy rules. We take into account of how expectations in the constraints are affected when doing this.

A standard reference for algorithms with occasionally binding constraints is Christiano and Fisher, 2000. Unfortunately, we cannot use the algorithm proposed in Christiano and Fisher, 2000, since our model is slightly different. Specifically, our problem include future expectations of the control variables in the constraints. The way we take care of this problem is that we use that the control variables are functions of the state variables and rewrite the constraint set in terms of state variables only. This is related to the method used in e.g., Soderlind, 1999. However, instead of using the policy functions from the previous iteration when doing this as in Soderlind, 1999 we use the current policy function, as in the algorithm used in e.g., Krusell, Quadrini and Rios-Rull, 1996. Note that the algorithm in Krusell et al., 1996 can be thought of as analyzing a one period deviation from a proposed policy. Iteration finishes when there are no gains from deviating from the proposed policy. The full algorithm is outlined in appendix A; see also appendix B for the calibration. 7

4 Optimal policy

We solve the model for the standard calibration as described in appendix B, both in the case with and without downward nominal wage rigidity. In figure 1 we plot the impulse responses to a negative shock (of $-0.025$) to the natural real wage $\hat{w}_t^*$. Starting with the unconstrained case, the negative shock has the effect that the actual real wage is higher than the natural real wage, causing a positive real wage gap. The real wage can be adjusted by changing in

flation and wage in

flation. An increase in in

flation and a decrease in wage in

flation decreases the real wage. But the effects of changes in inflation and wage inflation do not fully compensate for the shock. To gain some intuition for the effect on the output gap, note that expression (13) can be rewritten as

$$\hat{x}_t = \frac{1 - \gamma}{\gamma} \frac{1}{\Pi} (\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1} - \Pi (\hat{w}_t - \hat{w}_t^*))$$

(21)

where $\hat{w}_t$ and $\hat{w}_t^*$ is the real wage and natural real wage, respectively. The effect through the real wage has a negative effect on the output gap. This effect works through marginal costs. Given constant
Figure 1: Impulse responses to a negative shock (of $-0.025$) in the natural real wage.
prices, marginal costs, i.e., $\frac{W_t}{MPL_t}$, have to be the same in equilibrium. Because wages are higher than the flexible equilibrium, the marginal product has to increase and the output gap decreases. However, there are also effects through price setting. Given constant wages, when prices increase, marginal costs must increase, which is achieved by a decrease in the marginal product, which in turn increases the output gap. As can be seen from figure 1, the price effects dominate the effects though real wages. The intuition for the behavior in the constrained case is similar, but with the difference that the real wage gap is larger, since wage inflation cannot be adjusted to change real wages. This leads to opposite effects for the output gap, since now real wage effects dominate price effects.

The inflation response is slightly larger in the constrained case than in the unconstrained case. The reason is that, since the central bank cannot use wage inflation, it will use inflation more aggressively. However, due to the quadratic loss function, this does not full compensate for the inability to use wage inflation and the real wage gap is larger. Wage inflation is zero for three quarters, whereafter the constraint no longer binds and wage inflation is positive.

In figure 2, we plot the resulting impulse responses to a positive shock (of 0.025) to $\hat{\omega}_t^*$. For the unconstrained case, the impulse responses are a mirror image through the horizontal axis of the negative case. In contrast, the constrained case is asymmetric, with the constraint on wage inflation binding at the later stages instead of the early stages. This also affects the responses of the other variables. Thus, policy is asymmetric in several dimensions.

In the constrained case, the impulse responses are similar to the unconstrained as long as the constraint does not bind. Wage inflation is positive for three quarters and decreases over time, whereafter the constraint binds and wage inflation becomes zero. When the wage constraint binds the output gap is smaller and inflation larger than in the unconstrained case.

Next, we turn to welfare analysis. Note first that it need not be the case that the model with a constraint necessarily leads to lower welfare relative to the unconstrained case. The reason is that downward nominal rigidity is not an additional constraint on the problem, i.e., a constraint that makes the feasible set smaller. Instead, it is a constraint that changes the choice set. To compute welfare we construct sequences of shocks for a thousand periods and use these to find paths for the variables $\hat{x}_t$, $\hat{\pi}_t$, $\hat{\pi}^*_t$, and $\hat{\omega}_t$. Then welfare is computed from these paths using the welfare criterion (17),

---

8 Recall that prices are a fixed markup on marginal cost

9 Too see this, consider the relationship between, say $\hat{\pi}^*_t$ and $\hat{x}_t$, treating other variables as constants. Then, in the unconstrained case, the relationship is linear with slope $\Omega_x$. In contrast, in the constrained case, the relationship is piecewise linear with slope 0 below some critical value of $\hat{x}$ and $\Omega_x$ above this value.
Figure 2: Impulse responses to a positive shock (of 0.025) in the natural real wage.
ignoring the periods \( t > 1000 \). This is repeated a thousand times to generate an approximation of the unconditional expectation. The log welfare difference with flexible price welfare is in the unconstrained case \(-0.0314\) versus \(-0.0267\) in the constrained case. Thus, we find that downward nominal rigidities increases welfare. The gain is approximately 0.5 percentage units of the unconstrained welfare level.

5 Concluding remarks

In this paper is to study the implications for optimal monetary policy when declining nominal wages is not a viable margin for adjustment to adverse economic conditions. A New Keynesian model is developed that can account for downward nominal wage rigidity. This is achieved by introducing wage bargaining between firms and unions in the model. Under realistic conditions on conflict costs, downward nominal wage rigidity arises as a rational endogenous outcome.

Focusing on discretionary monetary policy, we show that when money wages cannot fall, the optimal policy response to changing economic conditions becomes asymmetric. Moreover, this asymmetry is not only present in the wage inflation dimension. Most notably, the optimal path for the output gap goes in opposite directions in response to a negative shock to the natural real wage in the cases with and without nominal downward wage rigidity.

We also find that downward nominal wage rigidity can increase welfare. The reason is that downward nominal wage rigidity changes the choice set and opens up for a more effective stabilization in terms of welfare. With our calibration, welfare increases by about 0.5 percentage units.
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Appendix

A Numerical algorithm

We first define \( N \subset \mathbb{R}^3 \) nodes over the state space. Note that the algorithm described here is based upon the formulation of the optimal policy problem in 4 as stated in the section “Optimal Discretionary Policy” in Carlsson and Westermark, 2006. The main outline of the algorithm follows algorithm 12.2 in Judd, 1998.

Step 0. Guess policy functions \( U_0^0 \), i.e., parameter values in complete polynomials;

\[
\hat{\pi}(\hat{w}_{t-1}, \hat{w}_t^*),
\hat{\pi}(\hat{w}_{t-1}, \hat{w}_t^*),
\hat{x}(\hat{w}_{t-1}, \hat{w}_t^*)
\]

and value function

\[
V(\hat{w}_{t-1}, \hat{w}_t^*)
\]

Note that the decision rules are different depending on whether the inequality constraint is binding or not. Thus, we make guesses for six different polynomials.

Then go to step 2.

Step 1. Find the new guess for the policy functions \( U_1^{l+1} \) in the following way. First solve for the Lagrange multipliers from the first-order conditions with respect to \( \hat{\pi}, \hat{\pi}_\omega \) and \( \hat{x} \). Then solve for the policy functions \( U_1^{l+1} \) from the two systems consisting of the first-order condition with respect to \( \hat{w}_t \) (where the solutions for the Lagrange multipliers has been substituted in) together with the three constraints respectively, with a collocation method; see the section “Optimal Discretionary Policy” in Carlsson and Westermark, 2006 for details. For a given guess \( V^l \) of the value function, compute

\[
E_t\hat{\pi}_{t+1},
E_t\hat{\pi}_\omega_{t+1},
E_t\hat{x}_{t+1},
E_tV^l_1(\hat{w}_t, \hat{w}^*_t)
\]
where transition probabilities are computed using the new guess for the policy functions. (Step 1 in Judd, 1998 p416).

Step 2. Compute current period utility $P^{l+1}_i$ for $i = 1, \ldots, N$, given policy function guesses $U^{l+1}_i$.

Step 3. Update the value function as follows:

First compute the new transition matrix $Q^{l+1}$ by using the flow equation for real wages, the policy functions for inflation and wage inflation, and applying a Newton-Coates approach. Second, compute $V^{l+1}(\hat{\pi}_{t-1}, \hat{w}_t^*)$ using

$$V^{l+1} = (I - \beta Q^{l+1})^{-1} P^{l+1}.$$ 

Finally, compute the derivative of the value function with respect to $\hat{w}_{t-1}$.

Step 4. If $\|V^{l+1} - V^l\| < \varepsilon$ stop. Otherwise, go to step 1.

B Calibration

Following Erceg et al., 2000 we assume that;

$$u(C, Q) = \frac{1}{1-\chi_C} (C - Q)^{1-\chi_C},$$

$$v(L, Z) = \frac{1}{1-\chi_L} (1 - L - Z)^{1-\chi_L}.$$ 

As derived below, we have the following parameter definitions in the inflation and wage inflation constraints,

$$\Pi = \frac{1 - \alpha_w \alpha}{\alpha_w \alpha} (1 - \alpha_w \alpha \beta),$$

$$\Omega_x = \left(1 - \frac{1 + \alpha_w \beta}{2}\right) \frac{1 - \alpha_w \Phi_y}{\alpha_w \Phi_d},$$

$$\Omega_w = \left(1 - \frac{1 + \alpha_w \beta}{2}\right) \frac{1 - \alpha_w \Phi_w}{\alpha_w \Phi_d},$$

$$\Omega_{\pi 0} = \left(1 - \frac{1 + \alpha_w \beta}{2}\right) \frac{1 - \alpha_w \Phi_y}{\alpha_w \Phi_d} \frac{1}{1 - \alpha_w \alpha \beta} \frac{1}{1 - \alpha_w \alpha},$$

$$\Omega_{\pi 1} = \beta \Omega_{\pi 0},$$

$$\Omega_{\pi 2} = \left(1 - \frac{1 + \alpha_w \beta}{2}\right) \frac{1 - \alpha_w \Phi_y}{\alpha_w \Phi_d} \frac{1}{1 - \alpha_w \alpha \beta} \frac{1}{2} \left(\alpha_w (1 - \alpha) + \frac{1 - \alpha_w \alpha}{2}\right) \beta^2,$$

---

*10 In terms of Judd, 1998 p. 416, compute $\pi(y_i, U^{l+1}_i)$ where $U^{l+1}_i$ consists of $\hat{\pi}_t$, $\hat{\pi}_y^*$ and $\hat{x}_t$ and $y_i = (\hat{w}_{t-1}, \hat{w}_t^*)$. This gives $P^{l+1}_i = \pi(y_i, U^{l+1}_i)$. 

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where

\[
\Phi_q = \varphi (\bar{u}_C (1 - \gamma) (1 + \tau_w) \bar{w} \bar{L} + \gamma \bar{v}_L \bar{L}) (\bar{y} ((1 + \tau) (1 - \sigma) + \sigma)) + \varphi \tau \bar{y} \left( \bar{L} ((1 - \gamma) (1 + \tau_w) \bar{w} \bar{u}_C - \gamma (\bar{v}_L \bar{L} + \bar{v}_L)) \right) \\
- (1 - \varphi) (1 - \gamma)(\bar{y} \bar{v}_L \bar{L} \sigma) + (1 - \varphi) (\bar{u} - \bar{v} - \bar{U}_o) (1 - \gamma) \bar{y} \sigma,
\]

\[
\Phi_d = -\varphi (\bar{u}_C (1 - \gamma) (1 + \tau_w) \bar{w} \bar{L} + \gamma \bar{v}_L \bar{L}) \bar{y} (1 - \gamma) + \varphi \tau \bar{y} \bar{L} \left( (1 - \gamma)^2 (1 + \tau_w) \bar{w} \bar{u}_C - \gamma^2 (\bar{v}_L \bar{L} + \bar{v}_L) \right) - (1 - \varphi) (1 - \gamma) \bar{y} \bar{v}_L \bar{L} \gamma - (1 - \varphi) (\bar{u} - \bar{v} - \bar{U}_o) (1 - \gamma) \bar{y} (1 - \gamma),
\]

\[
\Phi_Y = \varphi (\bar{u}_C (1 - \gamma) (1 + \tau_w) \bar{w} \bar{L} + \gamma \bar{v}_L \bar{L}) \left( (1 + \tau) \bar{y} - \bar{y} \frac{1}{1 - \gamma} \right) + \varphi \tau \bar{y} \bar{L} \left( (1 - \gamma) (1 + \tau_w) \bar{w} \bar{u}_C \bar{C} + (1 + \tau_w) \bar{w} \bar{L} \bar{u}_C + (\bar{v}_L \bar{L} + \bar{v}_L) \bar{L} \right) \frac{1}{1 - \gamma} - (1 - \varphi) (1 - \gamma) \bar{y} \left( \bar{u}_C \bar{C} - \bar{v}_L \bar{L} \frac{1}{1 - \gamma} \right) - (1 - \varphi) (\bar{u} - \bar{v} - \bar{U}_o) \bar{y},
\]

\[
\Phi_w = \varphi \tau \bar{y} \bar{L} (1 - \gamma) (1 + \tau_w) \bar{w} \bar{u}_C
\]

where

\[
\bar{v} = \frac{1}{1 - \chi_L} (1 - \bar{L} - \bar{Z})^{1 - \chi_n},
\]

\[
\bar{v}_L = - (1 - \bar{L} - \bar{Z})^{-\chi_n},
\]

\[
\bar{v}_{LL} = -\chi_n (1 - \bar{L} - \bar{Z})^{-\chi_n - 1},
\]

\[
\bar{u} = \frac{1}{1 - \chi_C} (\bar{C} - \bar{Q})^{1 - \chi_C},
\]

\[
\bar{u}_C = (\bar{C} - \bar{Q})^{-\chi_C},
\]

\[
\bar{u}_{CC} = -\chi_C (\bar{C} - \bar{Q})^{-\chi_C - 1}.
\]

and let

\[
\rho_C = -\frac{\bar{u}_{CC} \bar{C}}{\bar{u}_C},
\]

\[
\rho_L = -\frac{\bar{v}_{LL} \bar{L}}{\bar{v}_L}.
\]
To find a relation between $\chi_C$ and $\chi_n$, on one hand, and $\rho_C$ and $\rho_L$, on the other, note that

$$\rho_C = \frac{\bar{u}_{CC}(\bar{C} - \bar{Q})}{\bar{u}_C \bar{C}} \frac{\bar{C}}{\bar{C} - \bar{Q}} = \chi_C \frac{\bar{C}}{\bar{C} - \bar{Q}}$$

$$\rho_L = -\frac{\bar{v}_{LL}(1 - \bar{L} - \bar{Z})}{\bar{v}_L (1 - \bar{L} - \bar{Z})} = -\chi_n \frac{\bar{L}}{1 - \bar{L} - \bar{Z}}.$$  

Moreover, to ensure efficiency we set

$$\tau = \frac{1}{\sigma - 1}$$

$$\tau_w = \frac{1}{1 - \gamma} \left( \frac{\sigma - 1 - \varphi}{\bar{u}_C y \varphi} (\bar{u} - \bar{v} - \bar{U}) - 1 \right)$$  

Finally, we have the parameters in the welfare function (17)

$$\theta_x = \frac{\bar{C}}{2} \bar{u}_C \left( -\rho_C + \rho_L \frac{1}{1 - \gamma} \frac{\gamma}{1 - \gamma} \right),$$

$$\theta_w = \frac{-\bar{v}_L \bar{L}}{2} \left[ \alpha w \alpha \left( 1 - \beta \alpha w \alpha \right) \left( (1 - \alpha)\alpha w + (1 - \alpha w) \right) \right],$$

$$\theta_{xw} = \frac{-\bar{v}_L \bar{L}}{2} \frac{\alpha w}{1 - \alpha w} \left( \sigma \left( (1 - \alpha)\alpha w + \frac{1 - \alpha w}{\bar{u}_C y} \right) (1 - \gamma)^2 + \gamma^2 + 2\gamma (1 - \gamma) \right) \frac{1}{1 - \beta \frac{1 + \rho}{2}}$$

$$+ \sigma \frac{(1 - \gamma)^2 (1 - \alpha)}{1 - \beta \alpha w \alpha \left( (1 - \alpha)\alpha w + (1 - \alpha w) \right)}.$$  

Again following Erceg et al., 2000 we assume that $Q = 0.3163, Z = 0.03, K = 30Q$ and $\bar{A} = 4.0266$. Using our parameterization of the deep parameters, presented in table 1 below, together with the definitions for the derived parameters above yields the values presented in table 1, below.\(^{11}\)

\(^{11}\)Note that we define the variety parameter for the goods market is slightly different than in Erceg et al., 2000. Our variety parameter is $\sigma$ and theirs is $\theta_p$. We have $\frac{\sigma}{\theta_p} = 1 + \theta_p$, or, solving for $\sigma$, 

$$\sigma = \frac{1}{\theta_p} (\theta_p + 1).$$

Since Erceg et al., 2000 use $\theta_p = \frac{1}{2}$ we get $\sigma = 4$.  

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Table 1: Baseline Calibration of the Model

<table>
<thead>
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<th>Deep Parameters</th>
<th>Baseline values</th>
<th>Derived Parameters</th>
<th>Baseline values</th>
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</thead>
<tbody>
<tr>
<td>$\alpha$</td>
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</tbody>
</table>

Also, when calculating the steady state utility for the household net of the outside option we assume a replacement ratio of 0.25. Moreover, we parametrize $\alpha$ differently from Erceg et al., 2000. In our model, the probability for a firm to change their price in a given period is given by $1 - \alpha \alpha_w$ and not $1 - \alpha$. We set $\alpha = 0.5$. This implies that the expected duration for a price contract is $1/(1 - 0.75 \times 0.5) = 1.6$ quarters or 4.8 months, which is in line with the micro evidence presented by Bils and Klenow, 2004.