The Strategic Value of Incomplete Contracts for Competing Hierarchies*

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Abstract

We explore the strategic value of incomplete contracts in a competing hierarchies environment under adverse selection and moral hazard. We show that principals dealing with (exclusive) competing agents, may prefer to leave contracts silent on some (potentially) verifiable performance measures whenever certain other aspects of agents’ activity remain noncontractible. Two effects are at play once one moves from a complete to an incomplete contract. First, restricting the number of screening instruments has a detrimental effect on principals’ profits as it makes information revelation more costly. This increases information rents associated to informational asymmetry. Second, such restriction may provide principals with strategic power in that it allows to force competing hierarchies to behave in a more friendly manner on the downstream market. Within this framework, we also investigate the welfare impact of vertical arrangements based on resale price maintenance. We find that vertical price control may be either detrimental or beneficial to welfare depending upon the type of non-market externalities that retailers impose on each other.

1 Introduction

It is commonly believed that an optimal contract must limit as much as possible agents’ discretion within agency relationships. According to this view, mechanism designers should profitably exploit all available screening and monitoring instruments in order to prevent agents’ misbehavior. Nevertheless, contractual rules seldom display such high degree of complexity. In practice, contracts appear rather simple and, more strikingly, quite often fail to specify verifiable obligations of the parties. Such examples of arm’s length relationships are widespread in business practices as well as in many other aspects of social life. Manufacturers often delegate marketing and advertising activities to retailers; managerial contracts are typically vague or silent on the competitive and organizational objectives that managers should pursue; lenders usually leave entrepreneurs free to perform

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certain tasks that affect the profitability of their ventures; insurance companies monitor only to a limited extent the behavior of insurees, to name only a few.

So far a few rationales behind this incompleteness puzzle have been provided by the contracting literature. The first, based on the notion of transaction costs, holds that individuals' ability to enforce complete contracts can be limited when certain aspects of agents' performance are not verifiable at the time of enforcing contracting obligations. The second, relying on bounded rationality, argues that complete contracting is not viable when agents are unable to foresee all possible future contingencies.

Although appealing, these views are clearly inadequate to explain why there are circumstances under which sophisticated agents deliberately choose to write incomplete contracts even though transaction costs are low. In this paper we explore this issue by taking a standard agency perspective. Within a competing hierarchies framework, we investigate the extent to which the incompleteness of contracts can be attributed to their strategic use. Specifically, we argue that once one moves from the isolated principal-agent framework to games played by competing hierarchies, the design of incomplete contracts can be rationalized by the interplay between agency costs associated to alternative incentive schemes and the externalities that competing organizations exert on each other. In a setting encompassing both adverse selection and moral hazard, we show that when some aspects of the individuals' actions are nonverifiable, leaving contracts even more incomplete or silent on certain (potentially) verifiable agents' performance measures has a strategic value. More precisely, by committing to leave some verifiable actions unspecified, a principal can influence the nonverifiable actions of her agent so as to induce competitors to behave in a more friendly manner at subsequent stages of the game.

The idea that incomplete contracts have a strategic value is quite general. It can be applied to any competing hierarchies model involving contractual externalities, be it procurement contracting, executive compensations, patent licensing, insurance or credit relationships.\footnote{Examples of games of competing hierarchies are in fact widespread in real life. Just to name a few, consider a procurement model where different authorities regulate economic sectors characterized by technological spillovers; a credit economy where lenders finance competing entrepreneurs under exclusivity rules; or an insurance economy where risk-neutral principals share risk with individuals imposing consumption externalities on each other.} To take an archetypical example, throughout we consider manufacturers-retailers relationships.\footnote{The applied literature on this topic (Lafontaine and Slade, 1997, among many others) has recently argued that the empirical evidence, fairly consistent across industries and firms, quite often appears to be inconsistent with some aspects of the theoretical predictions based on agency theory.} Particular emphasis has indeed been stressed by the recent IO literature on the very incomplete nature of the arrangements regulating trade between vertically related firms. In practice, not only manufacturers often delegate marketing activities to retailers, but they also frequently give up vertical control by refusing to impose contractual restraints that would reduce agency costs and, in turn, potentially improve upon allocative efficiency. We assume that two retailers (agents or dealers) selling differentiated products compete on a downstream market by setting quantities. Production of final outputs requires an essential raw input which is supplied by a pair of exclusive, upstream suppliers (principals or manufacturers). Retailers are selected from a very large population of agents, so that upstream firms dictate the terms of trade in the wholesale-retail relationship. Downstream demands are uncertain and only agents observe a pay-off relevant signal which realizes before contracts are designed: an adverse selection problem. Moreover, an unverifiable...
demand-enhancing activity\(^3\) (effort) is exerted by downstream agents: a moral hazard issue. Principals and agents are risk-neutral. Each manufactures hires a retailer before production occurs, but after uncertainty about demand is realized. In this environment, we consider two alternative wholesale-retail trade regimes. Specifically, each principal can either commit to an incomplete contract, referred to as a quantity fixing scheme (QF), or to a more sophisticated arrangement, comparable to resale price maintenance (RPM). These arrangements typically differ in the underlying information structures which in turn support different types of contractual terms. Specifically, a QF contract is incomplete relative to RPM in the sense that, beyond fixing the quantity supplied to final consumers, it leaves the downstream firm free to choose its most preferred level of promotional effort. Instead, a RPM mechanism also restrains the retail price charged to final consumers in addition to fixing the quantity supplied in the final market.\(^4\)

We examine a three-stage game. In the first stage, before choosing their competitive strategies, upstream suppliers (simultaneously) make a public announcement about the type of mechanism that they will enforce at the contracting stage. In the second stage, the specific trade terms are secretly negotiated after all players have observed the first stage announcements. Finally, in the third stage, product market competition takes place and payments are made according to the settled contracting rules.

Within this framework, the equilibrium determinants of incompleteness rest upon three somewhat natural aspects of information asymmetries: (i) the way different screening instruments shape agency costs; (ii) the type of externalities that bilateral negotiations between principal-agent pairs impose on competing organizations; (iii) the principals’ ability to precommit to certain contractual rules. Intuitively, two contrasting effects are at play once one moves from a complete (RPM) to an incomplete contract (QF). On the one hand, an incomplete contract leaves more information rents simply because mimicking becomes more profitable with fewer screening instruments. On the other hand, limiting principals’ control variables might have a strategic value in that, by allowing agents to respond more efficiently to competition, it can induce a more friendly behavior by rivals at the market stage. Yet, while the former effect has been widely discussed in previous work, the second is novel and fully driven by our focus on competition between vertical hierarchies.\(^5\)

We provide simple conditions under which incomplete contracting may endogenously emerge when agents impose either positive or negative externalities on each other. The basic point is that when downstream demands are independent, that is retailers are monopolists in their own markets, a complete contract is beneficial

\(^3\)Distributors can indeed provide a wide range of services that affect the demand for products being offered. Services such as free delivery, pre-sales advice to potential buyers, show rooms, and after-sales services can play a key role in enhancing demand. Looking at supermarket data relative to the Chicago area, the importance of the retailer activity in price determination and the role of the retailer advertising as a way of competing for customers are documented by Chevalier, Kashyap, and Rossi (2003).

\(^4\)In practice, suppliers can credibly commit to use retail price restrictions in several ways. First, one can imagine that investments in specific monitoring technologies and/or delegation to third parties (such as intermediaries or experts) the task of monitoring retail prices, have the goal of signaling in a credible way the use of retail price restrictions. Second, in a dynamic perspective, suppliers can credibly induce their competitors to believe that retail price restrictions will be used through reputation. Third, a simple instrument, commonly used in business practice, which makes this commitment credible is to stipulate very high penalties for breaching any announced contractual mode.

\(^5\)In his discussion of the sources of incomplete contracts, Tirole (1999) classifies them in three categories: (i) unforeseen contingencies; (ii) cost of writing contracts; and (iii) cost of enforcing contracts. The strategic use of incomplete contracts clearly does not belong to any of them.
to principals because it reaches the best trade-off between efficiency and rent extraction.\textsuperscript{6} Games of competing hierarchies, though, bring about a novel channel through which ignorance can affect principals’ profits. Specifically, by committing to leave downstream agents free to set some aspects of their performance, each principal can influence in her own interest the subsequent market game played by agents.

A key feature of our contracting environment is the link between market and non-market (effort) externality that downstream retailers impose on each other. When both kinds of externalities are negative (resp. positive), that is goods are substitutes (resp. complements) and effort has a selfish (resp. cooperative) value, the game of contractual choices has multiple equilibria. By contrast, in a free-riding environment, that emerges when goods are substitutes and effort has a cooperative value, multiple equilibria may obtain only if retail competition is not very intense; complete contracts arise at the unique equilibrium otherwise. We also show that when there are multiple equilibria, incomplete contracts are always Pareto superior relative to complete ones whenever effort has a cooperative value, and that the converse holds otherwise.

Besides being a novel theoretical contribution per se, our characterization can be also helpful for understanding the rationale behind incomplete contracting in actual, possibly regulated markets. For instance, it may provide a practical tool for evaluating the recent concerns raised by the UK Office of Fair Trading about the anticompetitive role of actual vertical arrangements which seem not to include retail price instruments in a wide range of circumstances.\textsuperscript{7} In this perspective, our analysis contributes to the IO literature dealing with vertical control. In fact, following the approach taken in Rey and Tirole (1986), we illustrate the welfare implications of resale price maintenance by explicitly modeling the delegation problem between vertical related firms contracting under asymmetric information. In this regard, one peculiar feature of our work is that the outcome of this delegation problem, and in turn its welfare effects, is studied vis-à-vis product market rivals. By doing so, we show that the impact of vertical price control on consumers’ well being and total welfare is ambiguous depending upon the kind of the non-market externalities that retailers impose on each other. This result is in contrast with the past antitrust authorities’ tendency to sentence retail price restrictions and protect simple arrangements such as quantity discounts contracts.\textsuperscript{8} Even more importantly, the main merit of our work is to provide simple testable implications making a step forward towards a careful classification of circumstances under which certain kinds of vertical restraints are socially undesirable relative to others.

Section 2 relates our work to the literature on incomplete contracting and Section 3 sets up the model. By introducing a simple example, Section 4 motivates our approach arguing that when all aspects of agents’ performance are verifiable contracting incompleteness will never emerge at equilibrium. Section 5 introduces the notion of strategic value of incomplete contracts within an illustrative example. Section 6 generalizes the analysis to the case of a fully symmetric competing hierarchies model. Within this framework, we also discuss

\textsuperscript{6}Intuitively, any incentive feasible allocation under an incomplete contract must be also feasible under a complete one.

\textsuperscript{7}The report on the supermarkets code of conduct (2004) shows evidence of the scarce use of vertical arrangements based on retail prices.

\textsuperscript{8}Over the last decades the antitrust authorities in the United States has argued unambiguously against the use of resale price maintenance. Refiners, for instance, have been a favorite target of antitrust arrangements. Courts decisions have pronounced as unlawful contractual schemes through which the retail price was controlled by the upstream refiners.
some important policy implications of our results by illustrating how vertical price control affects welfare. Finally, Section 8 concludes. All proofs are relegated to an Appendix.

2 Related Literature

The common wisdom identifies the major sources of contracting incompleteness as the presence of transaction costs and bounded rationality. As argued by Mookherjee (2005), apart from the usual noncooperative incentive and participation constraints, these approaches search for additional constraints on contracts that rationalize simple real-world mechanisms. On the one hand, as suggested by the property rights approach (Grossman and Hart, 1986, and Hart and Moore, 1990, among others), the agents’ inability to describe future (uncertain) events might severely prevent them from signing complete contracts. On the other hand, even in the presence of very sophisticated agents, enforcement and monitoring costs could limit the possibility of writing complete contracts as well (Battigalli and Maggi, 2002-2003, Maskin, 2002, Maskin and Tirole, 1999, and Segal, 1999, among others). Although based on different presumptions, both approaches support the view that the standard principal-agent analysis is unsatisfactory in building convincing theoretical foundation for contracting incompleteness. Even though these approaches are quite valid, more “strategic” reasons for incompleteness may have been overlooked by the existing literature. Incompletely written contracts can be more easily rationalized in the competing hierarchies environment studied by this paper where such strategic concerns are pervasive.

Few previous studies have addressed the issue of contracting incompleteness by taking a perspective somewhat more in line with the principal-agent paradigm.

**Information Asymmetries, Dynamics and Renegotiation:** Dewatripont and Maskin (1990, 1995) investigate the set of optimal contingencies on which an incentive contract should be based whenever renegotiation is possible. They show that, within a repeated principal-agent set-up, where renegotiation issues produce dynamic externalities, a principal might voluntarily limit the set of publicly observable screening devices in order to relax future renegotiation constraints. Similarly, Crémer (1995) shows that, in a repeated moral hazard setting, principals may voluntarily weaken the effectiveness of their (ex post) monitoring technology in order to make less likely renegotiation threats. Olsen and Torsvik (1993) and Martimort (1999) demonstrate also how moving away from a centralized regulation by introducing competing regulators, another form of incompleteness, may improve commitment in an intertemporal context. Rather than looking at the effects of incomplete contracting on renegotiation constraints with the same principal, we focus on their strategic value and analyze the extent to which those arrangements weaken the market behavior of agents’ dealing with competing principals.

Our work is also related to Bernheim and Whinston (1998). They show that when some aspects of individuals’ behavior are observable but not verifiable, before playing a dynamic game, two individuals may profitably agree on designing contracts which are silent on some potentially contractible aspects of the relationship. Bernheim and Whinston analyze a class of complete information games where the possibility for players to sign (ex ante) binding contracts allows to limit in a mutually beneficial way the space of verifiable actions.
in their subsequent interactions. Two major aspects differentiate our work from theirs. First, we analyze an asymmetric information set-up and incentive constraints arise here from the need for inducing information revelation whereas incentive constraints are deduced from the self-enforceability of nonverifiable actions in their framework. Second, differently from them we examine a competing organizations set-up.

**Information Asymmetries and Signaling:** Aghion and Hermalin, (1990), Allen and Gale (1992) and Spier (1992) present principal-agent models in which asymmetric information leads to contractual incompleteness. With an informed principal, incomplete contracts may be profitably used to signal the principal’s type when transactions costs are sufficiently high.

**Ignorance in Vertical Contracting:** Caillaud and Rey (1995) analyze the optimal information structure of producers with respect to their retailers. They show that ignorance on the retailer’s cost function might create a strategic advantage that could outweigh the associated agency costs. While they rather focus on the value of ignorance in environments where principals can choose whether to acquire (at no additional costs) the relevant market information or, alternatively, to be (strategically) uninformed, in our set-up full extraction is prevented by the moral hazard component. As we shall discuss, this assumption is key for equilibria to display incomplete contracting. Finally, Kessler (1999) examines an agency model where the agent’s information structure is endogenous and the possibility of remaining uninformed for the agent has a positive strategic value. The crucial difference between our paper and Kessler’s analysis is that, while she analyzes the strategic value of ignorance from the agents’ perspective, we rather focus on principals’ incentives to create contracting incompleteness.

**Miscellaneous:** Our analysis is also closely related to the literature on vertical restrains (D’Amato et al., 2006, Gal-Or, 1991, 1992, 1999, Martimort, 1996, Rey and Stiglitz, 1995 and Rey and Tirole, 1986, among others). The fundamental difference with this literature is that while previous contributions have adopted a complete contracting approach and mainly taken the set of control instruments as given, we endogenize this set. From an organizational design viewpoint, our results also provide a simple explanation for the evidence showing that often, in real life, upstream manufacturers delegate non-market decisions, such as advertising and marketing activities, to downstream retailers (see for instance Laffontaine and Slade, 1997, and Sheppard, 1993). In this respect, besides providing a theory of delegation based on information asymmetries, we also contribute to assess the welfare properties of vertical arrangements based on vertical price control (RPM) in imperfectly competitive industries.

Finally, from a methodological viewpoint, it is worth noting that our analysis relaxes two standard assumptions of the literature dealing with strategic delegation and decentralized decision making. First, we consider a contract space much broader than simple linear contracts (an ad-hoc restriction often used to rationalize delegation in complete information environments). Second, unlike the body of literature dealing with managerial incentives (Fershtman and Judd, 1987, and Sklivas, 1987, among many others), we focus on unobservable contracting. Specifically, in our set-up allocation proposals, i.e., the specific trade terms of the vertical contract,
are secret and cannot be used for strategic purposes.

3 The Model

Environment: Consider a downstream industry where two retailers, $R_1$ and $R_2$, producing two symmetrically differentiated products compete by setting quantities. The production of final output requires an essential raw input which is supplied by a pair of exclusive upstream suppliers, $S_1$ and $S_2$. The inverse market demand facing the good produced by the retailer-$i$ is uncertain and is defined by:

$$p_i(\theta, e_i, q_{-i}, q_i, \rho, \sigma) = \tilde{\theta} + e_i + \sigma e_{-i} - q_i + \rho q_{-i}, \text{ for each } i = 1, 2,$$

where $q_i$ is the quantity produced of good-$i$, $p_i$ is the retail price level charged for this product, and $\tilde{\theta}$ is a common shock affecting both demands.

This parameter is drawn on the compact support $\Theta \equiv [\bar{\theta}, \bar{\theta}]$ according to the cumulative distribution function $F(\theta)$ having a (strictly) positive density, $f(\theta) = F'(\theta)$, with $|f'(\theta)|$ being bounded. We assume also that the hazard rate $h(\theta) = (1 - F(\theta))/f(\theta)$ is monotonically decreasing, $h(\theta) < 0$ for all $\theta \in \Theta$. The realization of $\tilde{\theta}$ is private information of retailers at the time contracts are signed, and $e_i$ denotes an unverifiable demand-enhancing activity (effort) performed by each retailer-$i$.

Within this framework, the effort variable is meant to capture all retailers’ non-market activities such as production of indivisible services, investment in advertising or pre-sale advice to potential buyers. Such kind of effort has two effects on the demand system. Clearly, it enhances own consumers’ willingness to pay, but it may also influence the competitor’s demand. This assumption seems reasonable in at least two natural cases. First, when effort is interpreted as production of indivisible services, bundled with the final product, it might have a negative impact on competitors’ demand if goods are substitutes; and the opposite obtains otherwise. Differently, as argued by Mathweson and Winter (1984), when effort captures pre-sale services or generic advertising, it could be well the case that information on the product’s existence benefits also competitors: a free-riding story.

Following Che and Hausch (1999), we shall distinguish these alternative scenarios by assuming that efforts have a cooperative nature if $\sigma \geq 0$, and a selfish value otherwise. Throughout we shall assume that $|\sigma| \leq 1$ in order to guarantee that own-effort effects are larger than cross ones, i.e., $\partial p_i(.)/\partial e_i \geq \partial p_i(.)/\partial e_{-i}$. Furthermore, $\rho$ denotes the measure of products’ differentiation and it also satisfies $|\rho| \leq 1$ in order to guarantee that own-price

$$V(q_1, q_2, I, \theta) = \sum_{i=1,2} e_i(q_i + \sigma q_{-i}) + \theta \left( \sum_{i=1,2} q_i \right) - \frac{1}{2} \left( \sum_{i=1,2} q_i^2 \right) + \rho q_1 q_2 + I.$$

The sign of $\rho$ determines whether produced goods are complements, independents or substitutes. The sign of $\sigma$, though, determines whether retailers impose positive ($\sigma \geq 0$) or negative externalities ($\sigma < 0$) on each other through their non-market activities.

Most of our results below hold under the assumption $\Delta \theta = \bar{\theta} - \bar{\theta}$ small enough. Alternatively, they hold on broader supports as soon as $F(.)$ is uniform, the disutility of effort is quadratic and we focus on a linear equilibrium.
effects are larger than cross-price ones in the direct demand system, \( \partial q_i(\cdot)/\partial p_i \geq \partial q_i(\cdot)/\partial p_{-i} \). For expositional purposes, in the analysis we shall focus on two cases. The first where market and effort externalities have the same sign on demands, that is \( \sigma \rho > 0 \). The second describes a free-riding set-up, that is \( \sigma \geq 0 \) and \( \rho \leq 0 \).

Providing effort is costly and we denote by \( \psi(e_i) \) the disutility function satisfying \( \psi'(e_i) \geq 0 \) and standard Inada conditions \( \psi'(0) = 0, \lim_{e_i \to +\infty} \psi'(e_i) = +\infty \). Moreover, in order to guarantee well behaved optimization programs we also impose \( \psi''(e_i) > 1/2 \) for all \( e_i \in \mathbb{R}_+ \). Finally, we assume that both upstream and downstream firms produce at constant marginal costs normalized to zero.

For the sake of simplicity, most of our results will be derived by using Taylor expansions, hence they hold for \( \Delta \theta = \bar{\theta} - \theta \) small enough. Alternatively, they also hold more generally when \( \psi(e) \) is quadratic and \( F(\cdot) \) is uniform since Taylor expansions are then exact.

**Mechanisms:** We consider two alternative mechanisms available to each supplier: a complete mechanism, which will be referred to as RPM, and an incomplete one denoted QF. If a QF arrangement has been chosen, such mechanism is of the form \( \{t_i(\hat{\theta}_i), q_i(\hat{\theta}_i)\}_{\hat{\theta}_i \in \Theta} \) where \( \hat{\theta}_i \) is retailer-i’s report on the demand parameter, \( q_i(\hat{\theta}_i) \) is the corresponding input level supplied by \( S_i \) and \( t_i(\hat{\theta}_i) \) is the fixed-fee paid by \( R_i \) to \( S_i \). Similarly, if RPM is preferred, an incentive mechanism is of form \( \{t_i(\hat{\theta}_i), q_i(\hat{\theta}_i), p_i(\hat{\theta}_i)\}_{\hat{\theta}_i \in \Theta} \) where now \( p_i(\hat{\theta}_i) \) denotes the retail price of good-i following report \( \hat{\theta}_i \).

A RPM arrangement is more complete relative to QF because this latter contract restricts the set of screening instruments by leaving unspecified the retail market price. Therefore, it is reasonable to think of a QF arrangement as being equivalent to a vertically decentralized organizational structure or to an arm’s length relationship. Under this contractual scheme, the upstream manufacturer does not have enough instruments to monitor the promotional effort level exerted by the retailer. Instead, RPM will be seen to replicate the constrained vertical integration outcome, since by dictating the retail price and the quantity sold to the retailer, the upstream manufacturer is able to control directly the retailer’s effort level.

We assume that the mechanisms ruling each hierarchy are private, i.e., cannot have any commitment power. However, the choice of the specific contractual mode, namely whether quantity forcing (QF) or resale price maintenance (RPM) is chosen, is itself observable.

The commitment assumption plays a central role in our analysis and deserves motivations. The key issue is whether principals can credibly commit not to exert vertical price control. In practice, there are several ways through which suppliers can achieve this goal. First, as observed by Dewatripont and Maskin (1995), one can imagine that when observability of prices requires setting up some (observable) monitoring procedure in advance, then not acquiring such monitoring technology at the outset may make gathering meaningful information impossible afterwards. Second, suppliers may well achieve the same objective through reputation mechanisms in a dynamic framework. Finally, a simple instrument, commonly used in business practice, which makes this commitment credible is to stipulate very high penalties for breaching any announced contractual mode.

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14 Assuming that produced goods are complements, \( \rho > 0 \), and that efforts create negative externalities, \( \sigma < 0 \), seems unreasonable.
Within this framework, we follow Myerson (1982) and Martimort (1996) to characterize the set of incentive feasible allocations in each hierarchy. Indeed, for any output choice made by $R_i$, there is no loss of generality in looking for $S_i$’s best response to $S_{-i}$’s contractual offer within the class of direct and truthful mechanisms.

**Timing, Strategies and Equilibrium Concept:** Firms play a three-stage game whose sequence of events is as follows:

1. Each supplier $S_i$ either publicly commits to verify the (ex post) realization of the retail price in market-$i$, together with $R_i$’s sales level or, alternatively, she might give up vertical control and use only sales as a screening device.

2. Uncertainty about demand realizes and only $R_1$ and $R_2$ observe it.

3. Each supplier $S_i$ secretly offers a menu of contracts to his own retailer $R_i$. If the contract is accepted, $R_i$ reports a message $\hat{\theta}_i \in \Theta$ to $S_i$ about the realized demand state. Effort is exerted, product market competition takes place and, finally, payments are made after verifiable actions have been observed. If the offer is turned down, $S_i$ and $R_i$ enjoy their outside options which are normalized to zero, and $R_{-i}$ acts as a monopolist in the downstream market.

The equilibrium concept we use is *Perfect Bayesian Equilibrium* with the added “passive beliefs” refinement that, provided $R_i$ receives any unexpected offer from $S_i$, he still believes that $R_{-i}$ will produce the same quantity. We denote by $\mathcal{G}$ the three stages game of contractual choices cum mechanisms offers and market interactions.

**Complete Information Benchmark:** Under complete information the two kinds of contracts QF and RPM implement the same allocation. Both mechanisms can emerge in an equilibrium of $\mathcal{G}$. Let us denote by $\{p^*(\theta), q^*(\theta), e^*(\theta)\}$ the solution to the following equation:

$$
\theta + (1 + \sigma)\phi(q^*(\theta)) - (2 - \rho)q^*(\theta) = 0, \text{ for each } \theta \in \Theta
$$

with $\phi(.) = \psi'(.)^{-1}$, $q^*(\theta) = \psi'(e^*(\theta))$ and $p^*(\theta) = q^*(\theta)$ for all $\theta$.

One can easily check that efforts, outputs and fixed fees are the same under both contractual regimes. Hence, at equilibrium, suppliers are indifferent between a complete and an incomplete contract. This result is based on our assumption of non-linear contracting. As argued by Katz (1991), the use of agents in games of competing hierarchies does not affect the equilibrium outcomes if there exists a contract which perfectly internalizes the vertical externality between the principals and their agents. Furthermore, it should be noticed that, under complete information, both types of contracts produce the same external effect on third parties, i.e., they generate the same consumers’ surplus and total welfare. As we illustrate in the rest of the analysis, this property no longer holds under information asymmetries.
4 Preliminary Remarks

Before presenting our argument in more details, it is worthwhile explaining why the issue of contracting incompleteness must be addressed in a second-best setting where principals cannot achieve full rent extraction. To illustrate this point, we shall now develop a simple example of vertical contracting with adverse selection only. Accordingly, we assume that providing effort is too costly for both retailers, i.e., \( \psi(e) = +\infty \) for all \( e \in \mathbb{R}_+ \), so that each (inverse) demand function is defined by \( p_i(\hat{\theta}, q_i, q_{-i}) = \hat{\theta} - q_i + \rho q_{-i} \) for \( i = 1, 2 \). Merely for the sake of simplicity, we also consider the case where while \( R_1 \) must deal with \( S_1 \) to receive inputs, \( R_2 \) and \( S_2 \) are vertically integrated and produce as a unique entity, labeled \( S_2\cdot R_2 \).

To begin with, observe that when \( S_1 \) monitors both retail price and sales he can fully extract the information rent of his retailer and infer (correctly at the equilibrium) the production of \( S_2\cdot R_2 \).

**Lemma 1** When \( S_1 \) monitors both sales and retail price no information rent is left to \( R_1 \).

At an equilibrium of the game, outcomes must solve the following type-contingent system of first-order conditions equalizing marginal revenues to marginal costs within each pair:

\[
\theta - 2q_i^P(\theta) + \rho q_{-i}^P(\theta) = 0 \quad \text{for} \quad i = 1, 2.
\]

This yields a symmetric equilibrium output:

\[
q_i^P(\theta) = \frac{\theta}{2 - \rho} \quad \text{for} \quad i = 1, 2.
\]

Consider now the optimal contract when \( S_1 \) no longer controls the retail price of his retailer. The upstream supplier does not have enough instruments to achieve full extraction. The underlying idea is standard, high-demand types have an incentive to mimic low-demand ones, and \( S_1 \) has to give up information rents in order to induce information revelation. To trade-off optimally allocative efficiency and rent extraction, the supplier must also distort downward the output relative to the complete information level. In this context, the Revelation Principle applies in \( S_1\cdot R_1 \) hierarchy (keeping as fixed the production of \( R_2 \)). Denoting by \( U_1(\theta) \) retailer \( R_1 \)'s information rent, we have:

\[
U_1(\theta) = p_1(\theta, q_1(\theta), q_2(\theta))q_1(\theta) - t_1(\theta) = \max_{\hat{\theta} \in \Theta} \left\{ p_1(\theta, q_1(\hat{\theta}), q_2(\hat{\theta}))q_1(\hat{\theta}) - t_1(\hat{\theta}) \right\}.
\]

Using standard techniques to study screening in competing hierarchies environments (Martimort, 1996), we obtain the following first- and second-order local conditions for incentive compatibility:

\[
\dot{U}_1(\theta) = (1 + \rho \hat{q}_2(\theta))q_1(\theta) \quad (IC_1),
\]

\[\text{This mechanism is similar to Gal-Or (1991).}\]
which, together with the agent’s participation constraint
\[ U_1(\theta) \geq 0 \quad (PC), \]
define the set of incentive feasible allocations in \( S_1-R_1 \) hierarchy.

Expressing the fixed-fee as a function of \( U_1(\theta) \) and \( q_1(\theta) \), \( S_1 \)’s problem, denoted thereafter by \((P)\), can be rewritten as:
\[
(P) : \max_{\{q_1(\cdot), U_1(\cdot)\}} \int \{p_1(\theta, q_1(\theta), q_2(\theta))q_1(\theta) - U_1(\theta)\} f(\theta)d\theta, \\
\text{subject to } (IC_1), \ (IC_2) \text{ and } (PC).
\]

A contractual externality similar to the competing-contracts effect studied in Martimort (1996) is at play in this environment. Specifically, information rents have now a different structure relative to those arising in a standard (isolated) principal-agent framework. In fact, the revelation of a good demand state has two impacts. First, it reveals that consumers’ willingness to pay is large. Second, since \( \theta \) affects symmetrically demands, it also provides information on the market behavior of the opponent hierarchy, which (in equilibrium) will produce more in better demand states. More precisely, besides \( R_1 \)’s own quantity schedule, \( q_1(\theta) \), now intensified downstream competition, as reflected by larger levels of \( \rho \), and greater responsiveness of the integrated structure \( S_2-R_2 \) to demand fluctuations, as measured by \( \dot{q}_2(\theta) \), shape his information rents. The steeper \( q_2(\theta) \) is the lower is its level at any given \( \theta \). This implies in turn that, when \( \dot{q}_2(\theta) \) increases, the integrated pair \( S_2-R_2 \) behaves less aggressively at the market stage so as to make \( S_1 \) less willing to grant information rents if goods are substitutes, and the opposite obtains if goods are complements.

Assuming that information rents are increasing in \( \theta \) so that \((PC)\) binds only at \( \theta \), we solve a relaxed program \((P')\), neglecting \((IC_2)\) in \((P)\),
\[
(P') : \max_{\{q_1(\cdot)\}} \int \{p_1(\theta, q_1(\theta), q_2(\theta))q_1(\theta) - h(\theta)(1 + \rho \dot{q}_2(\theta))q_1(\theta)\} dF(\theta).
\]

Optimizing pointwise yields the following first-order condition which is both necessary and sufficient (given our assumptions ensuring concavity of the objective) for optimality:
\[
(4) \quad \theta - 2q_1^Q(\theta) + \rho q_2^Q(\theta) - h(\theta)(1 + \rho \dot{q}_2^Q(\theta)) = 0.
\]

At a best-response to what the integrated structure produces, \( S_1 \) equalizes her virtual marginal revenues to zero at each \( \theta \). Under asymmetric information, everything happens as if the demand parameter \( \theta \) is now replaced by a lower virtual demand parameter, namely \( \theta - h(\theta)(1 + \rho \dot{q}_2^Q(\theta)) \), which captures also the extent of competitive pressure on the downstream market.
On the other hand, the vertically integrated retail pair $S_2$-$R_2$ chooses its output level so as to maximize profit under complete information, i.e., $\theta - 2q_2^Q(\theta) + \rho q_1^Q(\theta) = 0$. From equation (4) one can show that in a Nash equilibrium of the market subgame $R_1$’s output $q_1^Q(\theta)$ must solve the following differential equation:

$$q_1^Q(\theta) = \frac{(2 + \rho)(\theta - (2 - \rho)q_1^Q(\theta) - h(\theta))}{\rho^2 h(\theta)},$$

with the boundary condition $q_1^Q(\bar{\theta}) = q_1^P(\bar{\theta})$.

The next Proposition shows that, in this simple case, complete contracts are always preferable to incomplete ones.

**Proposition 2** The following properties are satisfied at each $\rho \in [-1, 1]$:

- An incomplete contract entails underproduction relative to a complete one, that is $q_1^Q(\theta) \leq q_1^P(\theta)$ for all $\theta$ in $\Theta$ (with equality only at $\bar{\theta}$);
- $S_1$ strictly prefers to control the retail price.

The economic intuition of this result rests upon the idea that when the retail price is not monitored, the upstream manufacturer chooses the quantity schedule so as to equalize her virtual marginal revenues to zero. By doing so, output is reduced below the complete information level for rent extraction reasons. Differently, under vertical price control, $S_1$ can extract $R_1$’s private information at no costs. This implies that $S_1$ can always replicate any allocation implemented when sales are the only screening instrument by means of vertical price control, but this is clearly never optimal.

Summarizing, the lesson of this simple example is that restricting the set of screening instruments is never worthwhile when the supplier can achieve the vertically integrated structure, first-best profit under complete contracting. Intuitively, when the agent’s performance does not include nonverifiable components affecting both the own principal’s and the competitors’ objective functions, a complete contract can always replicate every allocation which is feasible under an incomplete one. As a consequence, there is no strategic value in restricting the set of screening instruments. This is the reason why the question of whether incomplete contracts have a strategic value must be addressed necessarily in second-best environments where information asymmetries prevent upstream suppliers to achieve full extraction even under complete contracting. The rest of the analysis moves in this direction.

**5 An Illustrative Example**

This section analyzes the strategic value of incomplete contracts. In order to illustrate the basic point in a transparent way, let us consider again the set-up where a fully integrated firm $S_2$-$R_2$ competes with a vertically separated supplier-retailer hierarchy $S_1$-$R_1$. Moreover, to further simplify the analysis we also assume that
$S_2-R_2$ does not exert any effort (which formally amounts to impose $\psi_2(e_2) = +\infty$ for all $e_2 \in \mathbb{R}_+$). Once the argument at the core of this example will be clear, it becomes easier to understand the properties of our complete model, which entails symmetric competing hierarchies.

The system of (inverse) market demands is thus defined by:

$$p_1(\hat{\theta}, e_1, q_1, q_2) = \hat{\theta} + e_1 - q_1 + \rho q_2, \quad \text{and} \quad p_2(\hat{\theta}, e_1, q_2, q_1) = \hat{\theta} + \sigma e_1 - q_2 + \rho q_1.$$

It is worthwhile observing that, in this framework, none of the contractual regimes allows $S_1$ to fully extract $R_1$’s information rents since, even when the retail price can be contracted upon, $S_1$ cannot disentangle the impact of the demand parameter $\theta$ and the retailer’s effort on the residual demand the latter faces. The underlying idea is as follows: The possibility for the retailer to claim that large sales are due to a high effort level, whereas they result instead from a high demand, induces the upstream manufacturer to give up some information rent to the high demand retailer (high $\theta$) in order to induce truth-telling. As a result, the second-best allocation will be characterized by a downward distortion of both quantity and effort supplied by the retailer when he faces low demand states. This information rent, of course, depends on the chosen contractual mode.

The choice of the contractual mode has two opposite effects on $S_1$’s profits whenever $R_1$’s effort and output have the same impact on $S_2-R_2$’s demand, namely $\rho \sigma > 0$. On the one hand, restricting the set of screening instruments may lead the upstream supplier to grant more information rents relative to RPM because a screening instrument is given up: an agency cost effect. On the other hand, by changing the rivals’ behavior at the market stage, an incomplete contract may also have a strategic value relative to a complete one: a strategic effect. The relative strength of these two effects will depend upon the severity of the agency problem. As we shall prove, when information rents are small enough, the strategic effect dominates the agency one. The key point is that besides creating a vertical externality between $S_1$ and $R_1$, an incomplete contract also generates an horizontal externality which may drive the integrated structure $S_2-R_2$ to behave in a more friendly manner. By contrast, in the free-riding set-up, the oversupply of effort provided by $R_1$ under QF makes its competitor more aggressive at the market stage so as to make the strategic effect reinforcing the agency one. An incomplete contract is thus always dominated by the complete one.

Below we solve the game in two steps. First, we characterize the market allocation under both contractual regimes. Then, the equilibrium contract will be derived by using a backward induction argument.

**$S_2$-$R_2$’s Program:** To begin with, let us briefly analyze the program solved by $S_2$-$R_2$. For each realization of $\hat{\theta}$ and any incentive feasible allocation $\{e_1(\theta), q_1(\theta)\}_{\theta \in \Theta}$ implemented by $S_1$, the vertically integrated structure $S_2$-$R_2$ solves:

$$\max_{q_2 \in \mathbb{R}_+} p_2(\theta, e_1(\theta), q_2, q_1(\theta)) q_2.$$
The corresponding necessary and sufficient condition yields the following reaction function at each $\theta$:

$$q_2(\theta) = \frac{1}{2}(\theta + \rho q_1(\theta) + \sigma e_1(\theta)).$$

From equation (7) one can infer that $S_1$ has an incentive to choose strategically the contractual mode to soften $S_2-R_2$’s behavior at the competitive stage. In fact, the contractual regime maximizing the intercept $\frac{1}{2}(\sigma e_1(\theta) + \rho q_1(\theta))$ in (7) will be the most preferred one either when goods are substitutes or when they are complements, everything else being kept constant.\footnote{Obviously, this effect must be traded oﬀ with the effect of contracting incompleteness on information rents, which may well go in the opposite direction of decreasing $S_1$’s profits.}

**Complete Contracting:** Let us first consider a RPM arrangement. $S_1$ can now contract also the retail market price besides the quantity supplied by the downstream firm to final consumers. The effort level is then indirectly fixed as a function of $\theta$ through the inverse demand, i.e., $e_1 = p_1 + q_1 - \rho q_2 - \theta$. Intuitively, RPM is less flexible than QF simply because when the retailer faces a retail price target, she is indirectly forced to choose the effort level according to a suboptimal rule.\footnote{Indeed, under retail price restrictions the upstream producer has full control of all available instruments. See also Blair and Lewis (1994) and Martimort and Piccolo (2006).}

Let us define $R_1$’s information rent as:

$$U_1(\theta) = p_1(\theta)q_1(\theta) - \psi(p_1(\theta) + q_1(\theta) - \rho q_2(\theta) - \theta) - t_1(\theta),$$

by definition of incentive compatibility, we have:

$$U_1(\theta) = \max_{\hat{\theta} \in \Theta} \left\{ p_1(\hat{\theta})q_1(\hat{\theta}) - \psi(p_1(\hat{\theta}) + q_1(\hat{\theta}) - \rho q_2(\theta) - \theta) - t_1(\hat{\theta}) \right\}.$$

This yields the following first- and second-order local conditions for incentive compatibility:

$$\hat{U}_1(\theta) = (1 + \rho \hat{q}_2(\theta))\psi'(e_1(\theta)) \quad (IC_1),$$

$$(\hat{p}_1(\theta) + \hat{q}_1(\theta))(1 + \rho \hat{q}_2(\theta))\psi''(e_1(\theta)) \geq 0 \quad (IC_2).$$

Then, (IC$_1$) and (IC$_2$) together with the retailer’s participation constraint

$$U_1(\theta) \geq 0 \quad (PC),$$

define the set of incentive feasible allocations in $S_1-R_1$ hierarchy for a fixed output schedule $q_2(\theta)$ chosen by the rival pair $S_2-R_2$. $S_1$’s problem, denoted thereafter by $(P^P)$, is to design a menu of contracts to maximize the expected franchise fee he receives from $R_1$ subject to the participation and incentive compatibility constraints, together with the additional restriction required by the retail price target.

Expressing the fixed fee as a function of the retailer’s revenue and information rent, we can thus write $(P^P)$
as:

\[
(P) : \max_{\{U_1(\cdot), q_1(\cdot), e_1(\cdot)\}} \int_{\theta}^{\overline{\theta}} \{((\theta + e_1(\theta) - \rho q_2(\theta) - q_1(\theta))q_1(\theta) - \psi(e_1(\theta)) - U_1(\theta)) \} f(\theta) d\theta,
\]

subject to \((IC_1), (IC_2)\) and \((PC)\).

We will first assume and check ex post that \(1 + \rho q_2(\theta) \geq 0\) for all \(\theta\). Then \(U_1(\theta)\) is increasing and \((PC)\) binds only at \(\theta\), i.e., the lowest demand parameter. We obtain immediately:

\[
U_1(\theta) = \int_{\theta}^{\overline{\theta}} (1 + \rho q_2(x)) \psi'(e_1(x)) dx.
\]

Integrating by parts into the expression of the maximand of \((P)\) and neglecting \((IC_2)\) (that will be checked ex post also) yields a simplified optimization problem \((P^P)\):

\[
(P^P) : \max_{\{q_1(\cdot), e_1(\cdot)\}} \int_{\theta}^{\overline{\theta}} \{((\theta + e_1(\theta) - q_1(\theta) + \rho q_2(\theta))q_1(\theta) - \psi(e_1(\theta)) + h(\theta)(1 + \rho q_2(\theta))\psi'(e_1(\theta))\} f(\theta) d\theta.
\]

At a best response to the schedule \(q_2(\theta)\) implemented by the competing pair \(S_2-R_2\), the production and effort in \(S_1-R_1\) hierarchy are respectively given by the following first-order conditions:

\[
q_1(\theta) = p_1(\theta) = \theta + e_1(\theta) + \rho q_2(\theta) - q_1(\theta), \tag{8}
\]

\[
q_1(\theta) = \psi'(e_1(\theta)) + h(\theta)(1 + \rho q_2(\theta))\psi''(e_1(\theta)). \tag{9}
\]

Notice that the dichotomy result found in a regulatory environment with a single hierarchy (Laffont and Tirole, 1993, Ch. 3) holds in this setting. Specifically, since under RPM the only screening variable is effort, which is downward distorted relative to the complete information level, output is produced according to the efficient rule that would prevail under complete information.

By using \((7)\) together with \((8)\) and \((9)\), one can also check that the allocation \(\{e_1^P(\theta), q_1^P(\theta)\}_{\theta \in \Theta}\) solves the following system of differential equations:

\[
\dot{q}_1^P(\theta) = \frac{2(q_1^P(\theta) - \psi'(e_1^P(\theta)) + h(\theta)\psi''(e_1^P(\theta))(2 + \rho(1 + \sigma e_1^P(\theta))))}{\rho^2 h(\theta)\psi''(e_1^P(\theta))}, \tag{10}
\]

and,

\[
\dot{e}_1^P(\theta) = \frac{(4 - \rho^2)q_1^P(\theta) - (2 + \rho)}{2 + \rho \sigma}. \tag{11}
\]

\(^{18}\) Given concavity of the objective, these conditions are also sufficient.
with the boundary conditions \( q_1^P(\bar{\theta}) = q_1^*(\bar{\theta}) \) and \( e_1^P(\bar{\theta}) = e_1^*(\bar{\theta}) \).

In the Appendix, we show that, in the limit of small uncertainty, the allocation in any equilibrium satisfies the standard properties of no distortion at the top and underproduction at the bottom. More precisely, the ability of high-demand types to mimic low-demand ones forces the supplier to give up a rent in order to elicit separation. To reduce this costly rent, the supplier lowers the effort schedule below its complete information level, \( e_1^P(\theta) < e_1^*(\theta) \) for all \( \theta < \bar{\theta} \). Although, sales are not used for rent extraction purposes, because effort is downward distorted, output itself has to fall below its complete information level, but this effect is indirect only.

**Incomplete Contracting:** Under QF, \( S_1 \) does not observe the retail price, but she still observes the quantity supplied by \( R_1 \) on the retail market. A QF mechanism can thus be viewed as an incomplete contract relative to RPM since the upstream producer voluntarily gives up one of the possible screening instruments.

Let us now redefine retailer \( R_1 \)'s information rent under a QF regime as:

\[
U_1(\theta) = -t_1(\theta) + \max_{e_1 \in \mathbb{R}_+} \{ (\theta + e_1 - q_1(\theta) + \rho q_2(\theta))q_1(\theta) - \psi(e_1) \}.
\]

By definition of incentive compatibility, we have:

\[
U_1(\theta) = \max_{\bar{\theta} \in \Theta} \left\{ -t_1(\bar{\theta}) + \max_{e_1 \in \mathbb{R}_+} \{ (\theta + e_1 - q_1(\bar{\theta}) + \rho q_2(\theta))q_1(\bar{\theta}) - \psi(e_1) \} \right\}.
\]

From which we obtain the following first- and second-order local conditions for incentive compatibility:

\[
(12) \quad \dot{U}_1(\theta) = (1 + \rho \dot{q}_2(\theta))q_1(\theta),
\]

\[
(13) \quad (1 + \rho \dot{q}_2(\theta))\dot{q}_2(\theta) \geq 0,
\]

to which we must also add the participation constraint:

\[
(14) \quad U_1(\theta) \geq 0.
\]

This leads to state \( S_1 \)'s contracting problem, denoted thereafter as \((P^Q)\), as:

\[
(P^Q) : \max_{\{U_1(.),q_1(.)\}} \int_{\bar{\theta}} \{ (\theta + \phi(q_1(\theta)) - q_1(\theta) + \rho q_2(\theta))q_1(\theta) - U_1(\theta) \} f(\theta) d\theta,
\]

subject to (12), (13) and (14).

\[\text{From Martimort (1996), it is known that the equilibrium is unique when } \rho < 0 \text{ (substitutes) and that there may exist a continuum of equilibria when } \rho > 0 \text{ (complements). Although Martimort (1996) has no effort, the same is true here. All these equilibria have the same slope at } \theta = \bar{\theta}. \text{ Provided we focus on the case of small uncertainty, these equilibria are thus similar up to terms of order greater than two when it comes to assess the principals' expected profits and social welfare. Coming back to footnote 13, note that there always exists a unique linear equilibrium of the game in the case of uniform distribution, with quadratic disutility of effort, even for large uncertainty.}\]
For any given quantity schedule specified by the direct revelation mechanism QF, \( R_1 \) gains flexibility under a quantity-fixing arrangement in the sense that the effort level is chosen to command more information rents than it would be efficient from the manufacturer’s viewpoint. More specifically, while choosing the optimal effort level, the retailer does not internalize the impact of his effort on the information rent given up by the upstream manufacturer. QF introduces a vertical externality between the manufacturer and his retailer. As rents and effort are positively related via quantity, it will be thus profitable to oversupply effort relative to RPM everything else being kept equal.

We will again first assume and check ex post that \( 1 + \rho \hat{q}_2(\theta) \geq 0 \) for all \( \theta \). Then \( U_1(\theta) \) is increasing and (14) binds at \( \theta \) only. We obtain immediately:

\[
U_1(\theta) = \int_{\theta}^{0} (1 + \rho \hat{q}_2(x))q_1(x)dx.
\]

Integrating by parts the above expression and inserting into the maximand of \( (P^Q) \) yields the expression of the relaxed program \( (P'^Q) \):

\[
(P'^Q) : \max_{(q_1(\cdot))} \int_{\theta}^{0} \{ (\theta + \phi(q_1(\theta)) - q_1(\theta) + \rho \hat{q}_2(\theta))q_1(\theta) - \psi (\phi (q_1(\theta))) - h(\theta)(1 + \rho \hat{q}_2(\theta))q_1(\theta) \} f(\theta)d\theta.
\]

At a best-response to the schedule \( q_2(\theta) \) implemented by the \( S_2-R_2 \) pair we get:

\[
(15) \quad \theta + \phi(q_1^Q(\theta)) - 2q_1^Q(\theta) + \rho q_2^Q(\theta) - h(\theta)(1 + \rho \hat{q}_2^Q(\theta)) = 0.
\]

Differentiating equation (7) yields \( 2q_2^Q(\theta) = 1 + \rho q_1^Q(\theta) + \sigma \hat{e}_1^Q(\theta) \), using this condition together with \( \hat{e}_1^Q(\theta) = \phi'(q_1^Q(\theta))q_1^Q(\theta) \), one can immediately show that \( q_1^Q(\theta) \) solves the following differential equation:

\[
(16) \quad \dot{q}_1^Q(\theta) = \frac{(2 + \rho)(\theta - h(\theta)) + (2 + \rho \sigma)\phi(q_1^Q(\theta)) - q_1^Q(\theta)(4 - \rho^2)}{\rho h(\theta)(\sigma \phi'(q_1^Q(\theta)) + \rho)},
\]

with the boundary condition \( q_1^Q(\theta) = q_1^Q(\bar{\theta}) \).

Under the QF regime the dichotomy result no longer holds. In fact, since \( S_1 \) gives up a screening instrument by not controlling the retail price, the only screening device is output which must be downward distorted for rent extraction reasons. Once again, the ability of high-demand types to mimic low-demand ones forces the supplier to give up a costly information rent. Reducing this rent requires a downward distortion of the output level below its complete information level, i.e., \( q_1^Q(\theta) < q_1^Q(\theta) \) for all \( \theta < \bar{\theta} \). Moreover, although effort is chosen according to an efficient rule, because sales are downward distorted, the effort itself falls below its first-best level, but this effect is again indirect only. This lack of dichotomy will be key to prove that complete contracts might not be optimal in our environment of competing hierarchies.

**Choice of the Contractual Mode:** Having characterized the market allocation under both contractual
regimes, we now turn to investigate whether RPM or QF is the chosen contractual mode at equilibrium. As a preliminary result, we state the next Proposition which provides a useful description of how outputs and efforts are ordered under both regimes. In fact, this result will be key for showing, as well as interpreting, the equilibrium characterization that we provide below.

**Proposition 3** Assume that \( \Delta \theta \) is small enough and \( \psi''(e) > \max \left\{ \frac{1}{2}, \frac{2 + \sigma \rho}{4 - 2 \rho (\sigma + \rho)}, \frac{1 + \sigma \rho}{2 - \rho^2} \right\} \) for all \( e \in \mathbb{R}_+ \). Then the following properties hold:

- \( e^Q_1(\theta) \geq e^P_1(\theta) \) for all \( \theta \) with equality holding only at \( \bar{\theta} \);
- \( q^Q_2(\theta) \geq q^P_2(\theta) \) (resp. \(<\)) if and only if \( \sigma \geq 0 \) (resp. \(<\)) for all \( \theta \) with equality only at \( \bar{\theta} \) when \( \sigma \neq 0 \);
- \( q^Q_1(\theta) > q^P_1(\theta) \) (resp. \(\leq\)) if and only if \( \rho \sigma > 0 \) (resp. \(\leq\)) for all \( \theta \) with equality only at \( \bar{\theta} \) when \( \rho \sigma \neq 0 \).

The economic interpretation of the above result rests on the idea that when the upstream manufacturer gives up retail price control, the downstream firm increases his information rent by playing on his effort choice. Intuitively, as effort affects positively quantity, under a QF contract \( R_1 \) finds it profitable to supply more effort relative to RPM in order to enjoy more rents, hence \( e^Q_1(\theta) \) must be larger than \( e^P_1(\theta) \) at each \( \theta \). This difference in efforts shifts in turn the reaction function of the integrated structure when moving from a complete to an incomplete contract. From equation (7) it is easy to check that when \( \sigma \geq 0 \) the output level \( q_2(\theta) \) increases by moving from RPM to QF, and it diminishes otherwise. This explains why besides introducing a vertical externality between \( S_1 \) and \( R_1 \), an incomplete contract also creates an horizontal externality on the integrated structure \( S_2-R_2 \), so that \( q^Q_2(\theta) \) is larger than \( q^P_2(\theta) \) when \( \sigma \geq 0 \), and lower otherwise.

Finally, as for \( q_1(\theta) \), three different effects are at play simultaneously by moving from RPM to QF. First, for any given output level, \( R_1 \) will exert more effort under QF relative to RPM. When the retail price is not controlled, the agent is residual claimant for the full impact of his effort on enhancing demand. This effect raises effort and thus \( R_1 \)'s output: a demand-enhancing effect. Second, since sales is the only screening instrument under QF, one needs to distort it downward for rent extraction reasons: a rent extraction effect. Third, owing to the horizontal externality, the output of the competing structure \( S_2-R_2 \) is shifted upward when goods are substitutes and downward when they are complements: a strategic effect.

When efforts do not create externalities, \( \sigma = 0 \), or goods are independent, \( \rho = 0 \), the strategic effect is absent. The demand enhancing and the rent extraction effects then exactly compensate in the limit of small uncertainty, that is \( q^Q_1(\theta) = q^P_1(\theta) \) for all \( \theta \).

Yet, when products are differentiated and efforts generate demand spillovers, the strategic effect reinforces when market and effort externalities have the same sign, that is \( \rho \sigma > 0 \). In this case, an incomplete contract increases effort and so it moves \( q_2(\theta) \) in a more suitable manner relative to RPM. By contrast, in the free riding

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20 In our linear environment, assuming \( \psi'' \) large enough guarantees the existence of separating equilibria under both regimes.

21 Remember that under QF the downstream retailer chooses his effort according to the efficient rule \( q_1 = \psi'(e_1) \).

22 As a consequence, the competing hierarchies framework at hand displays the same features as the sequential monopolies model studied in Martimort and Piccolo (2006).
case the strategic effect leads the integrated structure $S_2-R_2$ to behave more aggressively at the market stage since the consumers’ willingness to pay increases and $q_2(\theta)$ increases. This in turn lowers $q_1(.)$ as quantities are strategic substitutes.

We can now show that incomplete contracts may have a strategic value under certain conditions related to the severity of the agency problem and to the presence of externalities between agents.

**Proposition 4** Assume that $\Delta \theta$ is small enough and $\psi''(e) > \max\left\{\frac{1}{2}, \frac{2+\sigma}{1-2\rho(\sigma+\rho)}, \frac{1+\sigma}{2-\rho}\right\}$ for all $e \in \mathbb{R}_+$. Then $S_1$ prefers an incomplete contract if $\rho \sigma > 0$, and the converse obtains otherwise.

The intuition for this result hinges again upon the fact that by inducing more effort on the retailer’s side, an incomplete contract changes also the market behavior of the competing structure. Of course, once one moves from a complete to an incomplete contract, information rents increase because a screening instrument is given up and information revelation becomes more costly: an *agency cost effect*. When goods are independent or effort does not create demand externalities, this effect drives the upstream supplier to always prefer RPM. If goods are differentiated and there are effort spillovers, the strategic effect may outweigh the excessive agency costs associated to QF whenever market and non-market externalities have the same sign. In fact, as an incomplete contract allows $S_1$ to force $S_2-R_2$ to behave in a more friendly manner at the market stage, it raises the supplier’s profits by increasing effort and so the expected transfer that can be extracted from $R_1$. In the free-riding case, though, the strategic effect has a negative value as a QF mechanism makes $S_2-R_2$ more aggressive relative to RPM, this adds to the excessive agency costs effect and thus drives $S_1$ to prefer RPM.

### 6 Symmetric Hierarchies

We now extend the example presented in the previous section to a fully-fledged (symmetric) competing hierarchies model. Now both retailers exert market and effort externalities on each other. Accordingly, downstream demands are given by (1). Both effects illustrated above extend to this more general framework and they are key in determining the equilibrium of the game $G$. First, an incomplete contract still commands more information rents. Second, for any given mechanism ruling the competing hierarchy, restricting the set of the screening instruments available to a principal may create a strategic effect influencing the rival’s market behavior. As before, the vertical externality that a QF mechanism creates within each vertical hierarchy is translated horizontally on the competing organization. Increasing retailers’ effort thus provides a beneficial effect on a supplier’s profits to the extent that it weakens the competitive stance of the opposing hierarchy on the downstream market. Of course, since we are now dealing with a full model of competing organizations, multiple equilibria of the game of contracting mode choices might emerge.

**Complete Contracting:** With this hierarchy framework it is straightforward to describe the set of incentive feasible allocations within $S_i-R_i$ pair with the following first- and second-order local conditions for incentive
compatibility:

\[ U_i(\theta) = (1 + \sigma \dot{e}_{-i}(\theta) + \rho \dot{q}_{-i}(\theta)) \psi'(e_i(\theta)) \]  

(17)

\[ (\dot{p}_i(\theta) + \dot{q}_i(\theta))(1 + \sigma \dot{e}_{-i}(\theta) + \rho \dot{q}_{-i}(\theta)) \psi''(e_i(\theta)) \geq 0, \]

(18)

and the participation constraint:

\[ U_i(\theta) \geq 0. \]

(19)

Equipped with this characterization, we may define \( S_i \)'s problem as:

\[ (\mathcal{P}_i^P) : \max_{\{U_i(\cdot), \psi_i(\cdot), \epsilon_i(\cdot)\}} \int_\theta \{ (\theta + \epsilon_i(\theta) + \sigma e_{-i}(\theta) - q_i(\theta) + \rho q_{-i}(\theta))q_i(\theta) - \psi(e_i(\theta)) - U_i(\theta) \} f(\theta) \, d\theta, \]

subject to (17), (18) and (19).

Assuming that the quantity \( (1 + \sigma \dot{e}_{-i}(\theta) + \rho \dot{q}_{-i}(\theta)) \) remains positive for all \( \theta \) (a condition to be checked ex post), \( U_i(\theta) \) is increasing and the participation constraint (19) binds only at \( \theta_i \). This leads to the following expression:

\[ U_i(\theta) = \int_\theta (1 + \sigma \dot{e}_{-i}(x) + \rho \dot{q}_{-i}(x)) \psi'(e_i(x)) \, dx. \]

Inserting \( U_i(\theta) \) into the maximand of \( (\mathcal{P}_i^P) \) and integrating by parts yields the following relaxed program \( (\mathcal{P}_i^{P_0}) \):

\[ \max_{\{q_i(\cdot), \epsilon_i(\cdot)\}} \int_\theta \{ p_i(\theta, e_i(\theta), e_{-i}(\theta), q_i(\theta), q_{-i}(\theta))q_i(\theta) - \psi(e_i(\theta)) - h(\theta)(1 + \sigma \dot{e}_{-i}(\theta) + \rho \dot{q}_{-i}(\theta)) \psi'(e_i(\theta)) \} f(\theta) \, d\theta. \]

Pointwise optimization with respect to output and effort respectively lead to the following first-order conditions:

\[ q_i(\theta) = p_i(\theta) = \theta + e_i(\theta) + \sigma e_{-i}(\theta) - q_i(\theta) + \rho q_{-i}(\theta), \]

(20)

\[ q_i(\theta) = \psi'(e_i(\theta)) + h(\theta)(1 + \sigma \dot{e}_{-i}(\theta) + \rho \dot{q}_{-i}(\theta)) \psi''(e_i(\theta)). \]

(21)

From equations (20) and (21) one can infer that the dichotomy result still holds under complete contracting even in this more general framework. Only effort needs to be distorted for incentive reasons, pricing is not.

**Incomplete Contracting:** Under this contractual regime there is still a lack of dichotomy. The upstream supplier gives up the retail price as a screening instrument and types’ revelation must be elicited only through
sales. Proceeding as before, the retailer $R_i$’s information rent under a QF regime can be rewritten as:

$$U_i(\theta) = -t_i(\theta) + \max_{e_i \in \mathbb{R}_+} \{ (\theta + e_i + \sigma e_{-i}(\theta) - q_i(\theta) + \rho q_{-i}(\theta))q_i(\theta) - \psi(e_i) \}. $$

By incentive compatibility, we immediately obtain the following local first- and second-order conditions:

(22)  
$$\hat{U}_i(\theta) = (1 + \sigma \hat{e}_{-i}(\theta) + \rho \hat{q}_{-i}(\theta))q_i(\theta),$$

(23)  
$$(1 + \sigma \hat{e}_{-i}(\theta) + \rho \hat{q}_{-i}(\theta))\hat{q}_i(\theta) \geq 0.$$

Incentive feasible allocations must also satisfy the usual participation constraint:

(24)  
$$U_i(\theta) \geq 0.$$

Taking into account that $R_i$’s effort satisfies $e_i(\theta) = \phi(q_i(\theta))$, $S_i$’s problem $(P_i^Q)$ can be rewritten as:

$$(P_i^Q): \max_{\{U_i(\cdot), q_i(\cdot)\}} \int_{\bar{\theta}}^{\theta} \{ (\theta + \phi(q_i(\theta)) + \sigma e_{-i}(\theta) - q_i(\theta) + \rho q_{-i}(\theta))q_i(\theta) - \psi(\phi(q_i(\theta))) - U_i(\theta) \} f(\theta) d\theta,$$

subject to (22), (23) and (24).

Assuming that $(1 + \sigma \hat{e}_{-i}(\theta) + \rho \hat{q}_{-i}(\theta))$ remains positive for all $\theta$ (a condition to be also checked ex post), $U_i(\theta)$ is increasing and thus (24) binds at $\theta$ only. Hence we get:

$$U_i(\theta) = \int_{\bar{\theta}}^{\theta} (1 + \sigma \hat{e}_{-i}(x) + \rho \hat{q}_{-i}(x))q_i(x)dx.$$

Inserting $U_i(\theta)$ into the maximand of $(P_i^Q)$ and integrating by parts yields the new expression of the relaxed program $(P_i^{Qn})$:

$$\max_{\{q_i(\cdot)\}} \int_{\bar{\theta}}^{\theta} \{ p_i(\theta, e_i(\theta), e_{-i}(\theta), q_i(\theta), q_{-i}(\theta))q_i(\theta) - \psi(\phi(q_i(\theta))) - h(\theta)(1 + \sigma \hat{e}_{-i}(\theta) + \rho \hat{q}_{-i}(\theta))q_i(\theta) \} f(\theta) d\theta.$$

Optimizing pointwise yields the following first-order condition:

(25)  
$$\theta + \phi(q_i(\theta)) - 2q_i(\theta) + \sigma e_{-i}(\theta) + \rho q_{-i}(\theta) - h(\theta)(1 + \sigma \hat{e}_{-i}(\theta) + \rho \hat{q}_{-i}(\theta)) = 0,$$

together with the effort optimality condition:

(26)  
$$e_i(\theta) = \phi(q_i(\theta)).$$
Again, effort is chosen with an efficient rule whereas the first-order condition (25) implies that the sales level must be distorted for rent extraction purposes.

6.1 Equilibrium Characterization

In this section we characterize the pure strategy equilibria of our game of contracting modes. As we show, besides the extra agency costs associated to an incomplete contract, two key effects are at play once one moves from a complete to an incomplete contract in a symmetric hierarchies framework. As we have argued above, an incomplete contract makes a retailer willing to exert more effort relative to a complete mechanism since the agent is residual claimant of the full impact of his non-market activities (effort) on demand enhancing when retail price is not monitored. This in turn might affect: (i) the effort-output choice of the competing retailer via the demand-driven externality: a direct strategic effect; and (ii) the own reaction function and, in turn, the output-effort pair of the competing hierarchy: an indirect strategic effect. Depending upon the nature of downstream externalities, i.e., cooperative versus selfish effort and complements versus substitutes goods, these effects might well go in opposite directions. Illustrating the determinants of such a trade-off is the main goal of this section.

Of course, when \( \sigma = \rho = 0 \) we are back to the isolated principal-agent set-up. In this case, retailers do not exert any form of externality on each other and complete contracts are the unique equilibrium of \( G \). For the sake of simplicity, we shall rule out this uninteresting case. The next result extends Proposition 4 to a symmetric hierarchies environment and summarizes our results.

**Proposition 5** Assume that \( \Delta \theta \) is small enough, \(-1 < \sigma + \rho < 1\) and that \( \psi'' \) is large enough so that local and global incentive compatibility constraints are satisfied in each possible outcome of the game of contractual choices. Then the following properties hold:

- If \( \rho \sigma > 0 \), the game \( G \) displays multiple (symmetric) PBE equilibria in the contractual choices.
- If \( \rho \sigma \leq 0 \), only complete contracts are an equilibrium when \( \sigma = 0 \). When \( \sigma \rho < 0 \), the following holds:
  
  (i) incomplete contracts are part of a PBE of \( G \) if and only if:
  \[
  |\rho|\psi'' \leq \sigma \leq 1 \quad \text{and} \quad |\rho|\psi'' < 1,
  \]

  (ii) complete contracts are part of a PBE of \( G \) if and only if:
  \[
  \frac{2|\rho|\psi''}{1 + 2\psi''} \leq \sigma \leq 1,
  \]

  (iii) there are no PBE equilibria in pure strategies otherwise.\(^{23}\)

\(^{23}\)It can be easily checked that the conditions imposed in this equilibrium characterization do not conflict with the lower bound imposed on \( \psi'' \) throughout.
The intuition underlying this result is again driven by the different impact that the two kinds of mechanisms have on retailers’ effort choices in the limit of small uncertainty.

Let us start by explaining why incomplete contracts emerge at a PBE of $G$. Consider an equilibrium candidate where both hierarchies are ruled by incomplete contracts. The crucial issue is understanding whether a supplier, say $S_i$, has an incentive to deviate from this candidate equilibrium by offering a complete RPM contract and, if so, what forces drive this deviation depending on the nature and the magnitude of both market and effort externalities.

To begin with, assume that efforts have a cooperative value and goods are complements. When $S_i$ moves from a complete to an incomplete contract, $R_i$’s effort increases. This has a direct positive effect on $R_i$’s reaction function which is shifted upward, thus making $S_i$ better off as goods are complements. Moreover, a larger $e_i(\theta)$ has also a positive effect on $R_i$’s demand, and so it increases $q_{-i}(\theta)$ via a higher $q_i(\theta)$. Of course, both effects are beneficial to $S_i$ who prefers an incomplete contract to a complete one.

Assume now that efforts have a selfish nature and goods are substitutes. In this case, increasing $R_i$’s effort through the choice of an incomplete contract lowers $R_{-i}$’s reaction function and, as a consequence, it relaxes retail competition. Moreover, it also increases $q_i(\theta)$ so to further reduce $q_{-i}(\theta)$. Both effects again go in the same direction of increasing $S_i$’s profits, and incomplete contracting remains an equilibrium.

Finally, consider the free-riding case. Now, when $S_i$ moves from a complete to an incomplete contract, the increased effort by $R_i$ has a direct positive effect on $R_{-i}$’s reaction function which shifts upward thus making $R_{-i}$ more aggressive at the market stage: a negative competitive effect. However, a larger $q_{-i}(\theta)$ calls for a higher $e_{-i}(\theta)$, which has a positive effect on $R_i$’s demand: a positive free-riding effect. Of course, the latter effect dominates the former one when the (positive) effort spillover is large enough and competition is not very intense, and the converse obtains otherwise. Intuitively, this explains why incomplete contracts remain an equilibrium.

Let us now explain why complete contracts are also an equilibrium of $G$. Consider then an equilibrium candidate where both hierarchies are ruled by complete contracts and assume that $S_i$ deviates, whereas the competing hierarchy sticks with the equilibrium behavior. Again, three relevant cases must be illustrated.

First, assume that efforts have a cooperative value and goods are complements. In this case, moving from an incomplete to a complete contract reduces $R_i$’s incentive to exert effort since the retail price is now downward distorted for screening purposes. This has a direct positive effect on $R_{-i}$’s effort through the RPM condition $e_{-i}(\theta) = 2q_{-i}(\theta) - \rho q_i(\theta) - \sigma e_i(\theta) - \theta$, everything else being kept constant. Hence $R_i$’s reaction function shifts upward. Moreover, as $e_{-i}(\theta)$ increases, $q_{-i}(\theta)$ increases too, thus making $S_i$ better off since goods are complements. Clearly, both effects go in the direction of increasing $S_i$’s profits, and this explains why complete contracts are part of an equilibrium for small demand uncertainty. The same kind of argument drives the intuition whenever $\sigma < 0$ and $\rho < 0$.

Finally, in the free-riding case the lower incentives to exert effort for $R_i$ under a complete contract have a direct positive effect on $R_{-i}$’s effort through the RPM condition $e_{-i}(\theta) = 2q_{-i}(\theta) - \rho q_i(\theta) - \sigma e_i(\theta) - \theta$. This shifts upward $R_i$’s reaction function: a positive free-riding effect. However, a higher $e_{-i}(\theta)$ also raises $q_{-i}(\theta)$. 
This makes $R_{-1}$ more aggressive at the market stage and, in turn, $S_i$ worse off when competition is strong enough: a negative competitive effect. When $\sigma$ is large relative to $\rho$ the former effect dominates the latter one. Hence, complete contracts remain part of an equilibrium in the limit of small uncertainty.

Since the game $G$ might admit multiple equilibria, in the next Proposition we order these equilibria according to a Pareto criterion from the suppliers’ viewpoint. As we will see, the key point will be the nature of effort externalities. This provides a rough of equilibrium refinement which will be useful in the welfare analysis performed in the next section.

**Proposition 6** Assume that $\Delta \theta$ is small enough, $-1 < \sigma + \rho < 1$ and that $\psi''$ is large enough. Then, when $G$ has multiple equilibria, suppliers prefer to coordinate on incomplete contracts when effort has a cooperative value; and the converse obtains otherwise.

The result shows that incomplete contracts are mutually beneficial to suppliers relative to complete ones only when effort spillovers are positive or effort has a cooperative value. Intuitively, in the limit of small uncertainty, as retailers are residual claimants for the full impact of their effort choices under incomplete contracts, consumers’ willingness to pay increases under incomplete contracting relative to complete contracts, thus improving upon productive efficiency.

### 6.2 Welfare Analysis

This section investigates the welfare effects of contracting incompleteness on welfare. In fact, our results can also be used to address relevant policy issues regarding the welfare desirability of vertical price control. This question has been intensely debated over the last decades and dates back to the Chicago school arguments. Stemming from the seminal work by Spengler (1950) and Telser (1960), a major challenge in the IO research agenda has been to understand what are the determinants of firms’ incentives to impose vertical restraints. Many theoretical and empirical papers have addressed the question of whether, and under what conditions, such private incentives are aligned with social incentives. The theoretical and political debate on this ground has been developed around two opposing views. On the one hand, scholars belonging to the so called Chicago school have proposed arguments in favor of vertical restraints, arguing that manufacturers are able to internalize vertical and horizontal externalities, potentially detrimental to social welfare, only through a careful use of vertical control. On the other hand, a more recent view (Rey and Tirole, 1988, and Jullien and Rey, 2000, among many others) has pointed out that the Chicago conjecture may fail under some significant circumstances, such as asymmetric information and uncertainty, where vertical control might generate undesirable monopoly power. Yet, as suggested by Rey and Tirole (1988), the theoretical work on this ground has often failed to recognize that informational issues, a major source of the delegation problem between vertical related firms, are at the root of double marginalization in such environment. Moving in this direction, we devote the rest of this section to assess the impact of retail price restrictions on third parties such as consumers.\footnote{Much scholars have indeed advocated that the sole role of Antitrust policies should be to promote consumers' surplus. See Bork (1978, Chapter 2, pp. 51) for instance.}

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Proposition 7 Assume that \( \Delta \theta \) is small enough, \(-1 < \sigma + \rho < 1\) and \( \psi'' \) is large enough. Then, RPM is detrimental to consumers relative to QF if effort has a cooperative value; and the opposite obtains if effort has a selfish value.

This result\(^{25}\) unveils that the welfare impact of vertical price restrictions on consumers depends upon the sign of the non-market externalities that retailers impose on each other through promotional, marketing and R&D activities. Intuitively, since incomplete contracts force firms to behave more aggressively at the market stage when effort has a cooperative value, contracts based only upon sales must be beneficial to consumers relative to more complex arrangements also controlling the retail prices. Exactly the same kind of argument allows to conclude that the converse obtains when effort has a selfish nature.

Since in Proposition 6 we have also showed that suppliers are jointly better off (resp. worse) under QF relative to RPM when effort has a cooperative value (resp. selfish), an interesting corollary of this result is that suppliers’ and consumers’ preferences are aligned with respect to the preferred contractual mode. Moreover, since information rents have only a distributive effect on welfare, it also follows that the best contract for the suppliers’ viewpoint is also that which maximizes total welfare. In this regard, an important contribution of this paper is to provide simple testable implications making a step forward towards a careful classification of circumstances under which certain kinds of vertical restraints are socially undesirable relative to others.

7 Concluding Remarks

Our analysis provides a new rationale for incomplete contracting in games of competing hierarchies. Focusing on a simple manufacturer-retailers economy, we have sowed that the equilibrium determinants of incompleteness rest upon three somewhat natural aspects of information asymmetries: (i) the way different screening instruments shape agency costs; (ii) the type of externalities that bilateral negotiations between principal-agent pairs impose on competing organizations; (iii) principals’ ability to precommit to certain contractual rules. The main result is that principals dealing with (exclusive) competing agents, may prefer to leave contracts silent on some (potentially) verifiable performance measures whenever certain other aspects of agents’ activity remain non-contractible. The key idea is that by allowing agents to respond more efficiently to competition, an incomplete contract has a strategic value in that it may induce a more friendly behavior by rivals at the market stage. This result is fairly general and, although we have developed our formal arguments in a familiar IO setting, the scope of our conclusions is much broader. They can be applied basically to any competing hierarchies model involving vertical and horizontal contractual externalities, be it procurement contracting, executive compensations, patent licensing, insurance or credit relationships, to name only a few. They have the surprising feature that, in contrast with the common wisdom, equilibrium contracts might display some forms of incompleteness whenever agency costs are small enough.

\(^{25}\)Proposition 7 extends the results provided in Martimort and Piccolo (2006), dealing with an isolated manufacturer-retailer relationship, to games of competing hierarchies.
Furthermore, besides shading new light on the incompleteness puzzle, the paper also delivers a theoretical framework for understanding what are the sources of inefficiency associated to vertical control in games of competing hierarchies with asymmetric information and secret contracting. Our welfare analysis provides simple testable conditions under which vertical price restrictions exacerbate the double marginalization effect driven by retailers’ information superiority. Accordingly, the analysis reveals that the concerns about social desirability of RPM, typically addressed by antitrust authorities, are justified only under specific conditions related to the underlying economic environment. On a more applied ground, our characterization makes a step forward towards a careful classification of circumstances under which certain kinds of vertical restraints are socially undesirable relative to others.

Finally, we do believe that our notion of strategic value of incompletely written contracts extends to many other contracting and organizational environments. Provided that precommitment on the type of screening variables is viable and that contractual offers are secret, our conjecture is that similar conclusions obtain in multiagent settings where a single principal contracts with competing agents. Also, since here we have assumed that agents hold a Nash behavior at the revelation stage, it would be worth studying whether the strategic value of incomplete contracting survives once the possibility of collusion between agents is introduced. We leave these issues for future research.

8 Appendix

8.1 Proof of Lemma 1

To prove the claim it suffices to verify that vertical price control allows $S_1$ to learn (ex post) the realized demand state. To this end, suppose that $R_1$ lies at the revelation stage and announces a message $\tilde{\theta} \neq \theta$, $S_1$ then supplies $q_1(\tilde{\theta})$ units of inputs, receives the fixed fee $t(\tilde{\theta})$ and expects to observe (ex post) a retail price level $p_1(\tilde{\theta}) = (2 + \rho)(\tilde{\theta} + \rho q_1(\tilde{\theta}))$. However, since $S_2-R_2$ produces at $q_2(\theta) = (\theta + \rho q_1(\tilde{\theta}))/2$, ex post $S_1$ will observe $(2 + \rho)(\theta + \rho q_1(\tilde{\theta})) \neq p_1(\tilde{\theta})$ for each $\tilde{\theta} \neq \theta$. $R_1$’s deviation will then be immediately detected.

8.2 Proof of Proposition 2

As a preliminary step we show that in a properly defined neighborhood of $\tilde{\theta}$, throughout denoted $B(\tilde{\theta})$, it must be $q_1^Q(\theta) \leq q_1^P(\theta)$ with equality holding only at $\tilde{\theta}$. That $q_1^Q(\tilde{\theta}) = q_1^P(\tilde{\theta}) = q^*(\tilde{\theta})$ is obvious. Notice then, that for all $\theta \in B(\tilde{\theta})$, a first-order Taylor expansion of $q_1^Q(\theta)$ around $\tilde{\theta}$ yields:

$$q_1^Q(\theta) \approx q_1^P(\tilde{\theta}) - q_1^Q(\tilde{\theta})(\tilde{\theta} - \theta),$$

where $q_1^Q(\tilde{\theta})$ can be obtained by using l’Hopital’s rule to the differential equation (5):

$$\lim_{\theta \to \tilde{\theta}} q_1^Q(\theta) = \frac{2 + \rho}{\rho^2} \lim_{\theta \to \tilde{\theta}} \frac{1 - (2 - \rho)q_1^Q(\theta) - h(\theta)}{h(\theta)}.$$
Using the fact that $\dot{h}(\bar{\theta}) = -1$ and rearranging terms in the above equation we have $q_{11}^{Q}(\bar{\theta}) = (2 + \rho)(2 - \rho^2)^{-1} \geq q_{11}^{P}(\bar{\theta}) = (2 - \rho)^{-1}$. Therefore, we have $q_{11}^{Q}(\theta) \leq q_{11}^{P}(\theta)$ for all $\theta \in B(\bar{\theta})$ since $q_{11}^{Q}(\bar{\theta}) = q_{11}^{P}(\bar{\theta})$. Now, let $B^c(\bar{\theta}) = \{ \theta \in \Theta | \theta \notin B(\bar{\theta}) \}$, we show that the result also holds globally for all $\theta \in B^c(\bar{\theta})$. Suppose, indeed, that there exists a $\theta^* \in B^c(\bar{\theta})$ such that $q_{11}^{Q}(\theta^*) = q_{11}^{P}(\theta^*)$, and without loss of generality, consider that $\theta^*$ is the lowest of such values. By using equation (5) one can easily verify that $q_{11}^{Q}(\theta^*) = -(2 + \rho)\rho^{-2} < 0 < q_{11}^{P}(\theta^*) = (2 - \rho)^{-1}$. Hence, in a properly defined neighborhood of $\theta^*$, say $B(\theta^*)$, one must have $q_{11}^{Q}(\theta) \geq q_{11}^{P}(\theta)$ if and only if $\theta \leq \theta^*$, and the converse obtains otherwise. This yields a contradiction with the definition of $\theta^*$.

We shall now prove the following results: Information rents are increasing in $\theta$, the local second-order condition (IC$_2$) holds, program (P') has a unique solution and global incentive compatibility holds. As a preliminary step we prove that $q_{11}^{Q}(\theta) \leq q_{11}^{m}(\theta)$ for all $\theta$, where $q_{11}^{m}(\theta)$ satisfies:

$$ q_{11}^{Q}(\theta) = q_{11}^{Q}(\theta) - h(\theta) = 0. $$

To begin with, we show that the result holds in a properly defined neighborhood of $\bar{\theta}$, say $B^m(\bar{\theta})$. Differentiating equation (27) with respect to $\theta$ we have $q_{11}^{m}(\bar{\theta}) = 2(2 - \rho)^{-1} \leq (2 - \rho)(2 - \rho^2)^{-1} = q_{11}^{Q}(\bar{\theta})$. Therefore, since $q_{11}^{m}(\bar{\theta}) = q_{11}^{Q}(\bar{\theta}) = q_{11}^{P}(\bar{\theta})$, it must be $q_{11}^{m}(\theta) \geq q_{11}^{Q}(\theta)$ for all $\theta \in B^m(\bar{\theta})$. Moreover, one can also show that the result remains true for all $\theta \in \{ \theta \in \Theta | \theta \notin B^m(\bar{\theta}) \}$. Indeed, suppose that there exists a $\theta^{**} \in \Theta$ such that $q_{11}^{m}(\theta^{**}) = q_{11}^{Q}(\theta^{**})$, and without loss of generality assume that $\theta^{**}$ is the lowest of such values. In this case, from equation (5) one immediately gets $q_{11}^{Q}(\theta^{**}) = 0$. Since $q_{11}^{m}(\theta) = (2 - \rho)^{-1}(1 - h(\theta)) > 0$, we have $q_{11}^{m}(\theta^{**}) = q_{11}^{Q}(\theta^{**})$ and $q_{11}^{m}(\theta) < q_{11}^{Q}(\theta)$ for $\theta \leq \theta^{**}$, a contradiction.

By using the same logic one can immediately show that (IC$_2$) holds too. Also, uniqueness follows directly by linearity of $p_i(.)$ for $i = 1, 2$ and strict concavity of (P').

Let now $U_1^{Q}(\theta, \hat{\theta})$ define $R_1$’s profits when the true demand state is $\theta$ but he announces a message $\hat{\theta} \neq \theta$ to $S_1$. To show that global incentive compatibility constraints hold, it must be that $\Gamma(\theta, \hat{\theta}) = U_1^{Q}(\theta, \theta) - U_1^{Q}(\theta, \hat{\theta}) > 0$ for each pair $(\theta, \hat{\theta}) \in \Theta^2$. Assume then $\theta > \hat{\theta}$ without loss of generality, simple algebraic manipulations allow to rewrite $\Gamma(.)$ as:

$$ \Gamma(\theta, \hat{\theta}) = \int_{\theta}^{\hat{\theta}} \left\{ -i_{11}^{Q}(s) + (\theta - q_{11}^{Q}(s) + \rho q_{22}^{Q}(\theta))q_{11}^{Q}(s) - q_{11}^{Q}(s) q_{11}^{Q}(s) \right\} ds. $$

By using (IC$_1$) and substituting for $i_{11}^{Q}(s) = (s - q_{11}^{Q}(s) + \rho q_{22}^{Q}(s))q_{11}^{Q}(s) - q_{11}^{Q}(s) q_{11}^{Q}(s)$ into the above equation one obtains:

$$ \Gamma(\theta, \hat{\theta}) = \int_{\theta}^{\hat{\theta}} 2q_{11}^{Q}(s) \left\{ \int_{s}^{\theta} (1 + \rho q_{22}^{Q}(x))dx \right\} ds. $$

Since we have already proved that $q_{11}^{Q}(\theta) > 0$ and $1 + \rho q_{22}^{Q}(\theta) > 0$ for all $\theta$, it follows that $\Gamma(\theta, \hat{\theta}) > 0$, which concludes the proof.

Finally, let $\Pi_1^{P}(\theta)$ and $\Pi_1^{Q}(\theta)$ be $S_1$’s state contingent profits under a complete and an incomplete contract, respectively. Formally we have:

$$ \Pi_1^{Q}(\theta) = q_{11}^{Q}(\theta)(\theta + \rho q_{22}^{Q}(\theta) - q_{11}^{Q}(\theta) - h(\theta)(1 + \rho q_{22}^{Q}(\theta))). $$
and

\[ \Pi^P(\theta) = q^P_1(\theta)(\theta + \rho q^P_2(\theta) - q^P_1(\theta)). \]

from the first-order conditions (3) and (4) one has \( \Pi^P_1 = E_{\tilde{\theta}}[q^P_1(\theta)]^2 \) and \( \Pi^Q_1 = E_{\tilde{\theta}}[q^Q_1(\theta)]^2 \). Then \( \Pi^P_1 > \Pi^Q_1 \) follows immediately from \( q^Q_1(\theta) \leq q^P_1(\theta) \) for all \( \theta \in \Theta \).

### 8.3 Market Equilibrium under Complete Contracting

In this section we provide a formal characterization of the market equilibrium when \( S_1 \) offers a complete contract. For future references remember that, in the asymmetric environment at hand, the complete information allocation is implicitly defined by the following equation at each \( \theta \):

\[(2 + \rho)\theta + (2 + \rho \sigma)\psi(q^*_1(\theta)) - (4 - \rho^2)q^*_1(\theta) = 0,
\]

together with the effort optimality condition \( q^*_1(\theta) = \psi'(e^*_1(\theta)) \). Next lemma summarizes the results.

**Lemma 8** Assume that \( \Delta \theta \) is small enough and \( \psi''(e) > \max\left\{ \frac{1}{2}, \frac{4+2\sigma \rho}{1-2(\sigma+\rho)} \right\} \) for all \( e \in \mathbb{R}_+ \). The following properties hold:

- \( e^P_1(\theta) \leq e^*_1(\theta) \) and \( q^P_1(\theta) \leq q^*_1(\theta) \) for all \( \theta \) (with equality holding only at \( \bar{\theta} \))
- Information rents are increasing in \( \theta \) and IC\(_2\) holds.

**Proof.** Since we are considering \( \Delta \theta \) small, we can use the following first-order Taylor expansions in describing output and effort:

\[ q^P_1(\theta) \approx q^*_1(\bar{\theta}) - q^P_1(\bar{\theta})(\bar{\theta} - \theta) \quad \text{and} \quad e^P_1(\theta) \approx e^*_1(\bar{\theta}) - e^P_1(\bar{\theta})(\bar{\theta} - \theta) \]

for all \( \theta \).

To show that \( \dot{q}_1^P(\theta) \geq 0 \), simple application of l'Hopital’s rule yields:

\[
\lim_{\theta \to \bar{\theta}} \dot{q}_1^P(\theta) = \lim_{\theta \to \bar{\theta}} \frac{2(\dot{q}_1^P(\theta) - \psi''(e_1^P(\theta))\dot{e}_1^P(\theta)) - (2 + \rho(1 + \sigma \dot{e}_1^P(\theta)))(\dot{h}(\theta)\psi''(e_1^P(\theta)) - h(\theta)\psi'''(e_1^P(\theta))\dot{e}_1^P(\theta))}{\rho^2 (\dot{h}(\theta)\psi''(e_1^P(\theta)) + h(\theta)\psi'''(e_1^P(\theta))\dot{e}_1^P(\theta))},
\]

which, from (11) together with \( \dot{h}(\bar{\theta}) = -1 \) and \( h(\bar{\theta}) = 0 \), yields:

\[
\dot{q}_1^P(\bar{\theta}) = \frac{2\psi''(2 + \rho)}{\psi''(4 - 2\rho(\sigma + \rho)) - \sigma \rho - 2}.
\]

Therefore, monotonicity is guaranteed if the following restriction holds:

\[
\psi''(e) > \frac{2 + \sigma \rho}{4 - 2(\sigma + \rho)} \quad \forall \ e \in \mathbb{R}_+.
\]

By using the above definition of \( q^*_1(\theta) \) we get:

\[
\dot{q}_1^*(\bar{\theta}) = \frac{\psi''(2 + \rho)}{\psi''(4 - \rho^2) - \sigma \rho - 2},
\]

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which is clearly positive as we have assumed \( \psi''(e) > \max \left\{ \frac{1}{2}, \frac{2+\sigma\rho}{4-2\rho(\sigma+\rho)} \right\} \).

Now, since \( q_1^P(\theta) - q_1^s(\theta) \approx (q_1^P(\bar{\theta}) - q_1^s(\bar{\theta}))(\bar{\theta} - \theta) \) for all \( \theta \), we get:

\[
q_1^P(\bar{\theta}) - q_1^s(\bar{\theta}) = \frac{(2 + \sigma\rho)(2\psi'' - 1)(2 + \rho)\psi''}{(\psi''(4 - 2\rho(\sigma + \rho)) - \sigma\rho - 2)(\psi''(4 - \rho^2) - \sigma\rho - 2)}
\]

which immediately implies \( q_1^s(\bar{\theta}) < q_1^P(\bar{\theta}) \) since \( \psi''(e) > \max \left\{ \frac{1}{2}, \frac{2+\sigma\rho}{4-2\rho(\sigma+\rho)} \right\} \) for all \( e \in \mathbb{R}_+ \). Hence \( q_1^P(\theta) \leq q_1^s(\theta) \) for all \( \theta \) for \( \Delta \theta \) small enough with equality only at \( \bar{\theta} \).

By using the same kind of argument, we have:

\[
\frac{\dot{e}_1^P(\bar{\theta})}{\psi''(4 - 2\rho(\sigma + \rho)) - \sigma\rho - 2} = \frac{(2\psi'' + 1)(2 + \rho)}{(2 + \rho)\psi''}.
\]

Then, since \( \psi''q_1^s(\bar{\theta}) = \dot{e}_1^s(\bar{\theta}) \), one gets:

\[
\frac{\dot{e}_1^P(\bar{\theta}) - \dot{e}_1^s(\bar{\theta})}{\psi''(4 - 2\rho(\sigma + \rho)) - \sigma\rho - 2} = \frac{2(2 + \rho)\psi''}{(\psi''(4 - 2\rho(\sigma + \rho)) - \sigma\rho - 2)(\psi''(4 - \rho^2) - \sigma\rho - 2)};
\]

implying \( \dot{e}_1^s(\bar{\theta}) < \dot{e}_1^P(\bar{\theta}) \) since \( \psi''(e) > \max \left\{ \frac{1}{2}, \frac{2+\sigma\rho}{4-2\rho(\sigma+\rho)} \right\} \) for all \( e \in \mathbb{R}_+ \) and, as a consequence, \( e_1^P(\theta) \leq e_1^s(\theta) \) for all \( \theta \).

Notice also that program \((P^0)\) displays interior solutions whenever \( \Delta \theta \) is sufficiently small. Moreover, from the above results one can show that incentive rents are increasing in \( \theta \). In fact, simple algebra yields:

\[
0 < \dot{U}_1^P(\theta) = \frac{(2 + \rho)(2\psi'' - 1)\psi'(e_1^P(\theta))}{\psi''(4 - 2\rho(\sigma + \rho)) - \sigma\rho - 2}.
\]

The same kind of argument allows to show that the monotonicity condition holds, i.e., \( 2\dot{q}_1^P(\theta)(1 + \rho\dot{q}_2^P(\theta)) > 0 \) for all \( \theta \). Moreover, uniqueness simply follows from linearity of \( p_i(.) \) for \( i = 1, 2 \) and strictly concavity of \((P^0)\).

Finally, let \( U_1^P(\theta, \hat{\theta}) \) be \( R_1 \)'s profits when the true retailer's type is \( \theta \) but he announces a message \( \hat{\theta} \neq \theta \) to \( S_1 \). To show that global incentive compatibility constraints hold we must have \( \Gamma^P(\theta, \hat{\theta}) = U_1^P(\theta, \theta) - U_1^P(\theta, \hat{\theta}) > 0 \) for each pair \((\theta, \hat{\theta}) \in \Theta^2\). Assume then \( \theta > \hat{\theta} \) without loss of generality, simple algebraic manipulations allow to rewrite \( \Gamma^P(\cdot, \hat{\theta}) \) as \( \Gamma^P(\theta, \hat{\theta}) = \int_{\hat{\theta}}^{\theta} \left\{ \dot{I}_1^P(s) - 2\psi'(e_1^P(s, \theta))q_1^P(s) \right\} ds \), where \( e_1^P(s, \theta) = \int_{\hat{\theta}}^{\theta} (1 + \rho\dot{q}_2^P(\theta))^{-1} ds \). By using \((IC_1)\) and substituting for \( \dot{I}_1^P(s) = 2\psi'(e_1^P(s))q_1^P(s) \) into \( \Gamma^P(\cdot, \hat{\theta}) \), one obtains:

\[
\Gamma^P(\theta, \hat{\theta}) = 2 \int_{\hat{\theta}}^{\theta} \dot{q}_1^P(s) \left\{ \int_{\hat{\theta}}^{\theta} \psi''(e(s, x))(1 + \rho\dot{q}_2^P(x))dx \right\} ds.
\]

Then, since \( \dot{q}_1^P(\bar{\theta}) > 0 \) whenever \( \psi''(e) > \max \left\{ \frac{1}{2}, \frac{2+\sigma\rho}{4-2\rho(\sigma+\rho)} \right\} \) for all \( e \in \mathbb{R}_+ \), it follows that \( \Gamma^P(\theta, \hat{\theta}) > 0 \) for all \( \theta \) and \( \hat{\theta} \), which concludes the proof. \( \blacksquare \)
8.4 Market Equilibrium under Incomplete Contracting

We provide now a formal characterization of the market equilibrium when $S_1$ offers an incomplete contract. Next lemma summarizes the results.

**Lemma 9** Assume that $\Delta \theta$ is small enough and $\psi''(e) > \max \left\{ \frac{1}{2}, \frac{1+\sigma \rho}{2-\rho^2} \right\}$ for all $e \in \mathbb{R}_+$. The following properties hold:

- $e_1^Q(\theta) \leq e_1(\theta)$ and $q_1^Q(\theta) \leq q_1(\theta)$ for all $\theta$ (with equality only at $\bar{\theta}$);
- Information rents are increasing in $\theta$ and $IC_2$ holds.

**Proof.** As before, we consider the approximation $q_1^Q(\theta) \approx q_1^*(\theta) - q_1^Q(\theta)(\bar{\theta} - \theta)$ for all $\theta$. From (16), l'Hopital’s rule yields:

$$
\lim_{\theta \to \bar{\theta}} \hat{q}_1^Q(\theta) = \lim_{\theta \to \bar{\theta}} \frac{(2 + \rho)(1 - \hat{h}(\theta)) + (2 + \rho \sigma)\psi'(q_1^Q(\theta))q_1^Q(\theta) - q_1^Q(\theta)(4 - \rho^2)}{\rho \left( \hat{h}(\theta)(\psi'(q_1^Q(\theta)) + \rho) + \sigma \hat{h}(\theta)\psi''(q_1^Q(\theta))q_1^Q(\theta) \right)},
$$

hence, using $\hat{h}(\bar{\theta}) = -1$ and $\hat{h}(\bar{\theta}) = 0$, we get:

$$
\hat{q}_1^Q(\bar{\theta}) = \frac{(2 + \rho)\psi''}{\psi''(2 - \rho^2) - \sigma \rho - 1},
$$

where monotonicity follows, $\hat{q}_1^Q(\bar{\theta}) > 0$, if the following restriction holds:

$$
\psi''(e) > \frac{1 + \sigma \rho}{2 - \rho^2} \forall e \in \mathbb{R}_+.
$$

In the limit of small uncertainty then we have $q_1^Q(\theta) - q_1^Q(\theta) \approx (\hat{q}_1^*(\theta) - q_1^Q(\theta))(\bar{\theta} - \theta)$ for all $\theta$ where:

$$
\hat{q}_1^Q(\bar{\theta}) - \hat{q}_1^*(\bar{\theta}) = \frac{(2 + \rho)(2\psi'' - 1)\psi''}{(\psi''(2 - \rho^2) - \sigma \rho - 1)(\psi''(4 - \rho^2) - \sigma \rho - 2)},
$$

which immediately implies $\hat{q}_1^*(\bar{\theta}) < \hat{q}_1^Q(\bar{\theta})$ whenever $\psi''(e) > \max \left\{ \frac{1}{2}, \frac{1+\sigma \rho}{2-\rho^2} \right\}$ for all $e \in \mathbb{R}_+$. Hence $q_1^Q(\theta) \leq q_1^*(\theta)$ for all $\theta$ with equality only at $\bar{\theta}$. By using the same kind of argument one also has $e_1^Q(\theta) \approx e_1^*(\bar{\theta}) - e_1^Q(\bar{\theta})(\bar{\theta} - \theta)$. Hence:

$$
e_1^Q(\bar{\theta}) = \phi'(q_1^*(\bar{\theta}))q_1^Q(\bar{\theta}) > \phi'(q_1^*(\bar{\theta}))\hat{q}_1^*(\bar{\theta}) = e_1^*(\bar{\theta}),$$

which directly implies $e_1^Q(\theta) \leq e_1^*(\theta)$ for all $\theta$.

That program $(P'^Q)$ displays interior solutions whenever $\Delta \theta$ is small enough is obvious. Hence, for $q_1^Q(\theta)$ being interior, information rents are increasing in $\theta$ since $\psi''(e) > \max \left\{ \frac{1}{2}, \frac{1+\sigma \rho}{2-\rho^2} \right\}$ for all $e \in \mathbb{R}_+$ implies:

$$
0 < \hat{U}_1^Q(\theta) = \frac{q_1^Q(\theta)(2\psi'' - 1)(2 + \rho)}{2(\psi''(2 - \rho^2) - \sigma \rho - 1)}.
$$

30
Since $q^Q_1(\bar{\theta}) > 0$, one can check that the monotonicity condition also holds, i.e., $(1 + \rho q^Q_2(\theta))q^Q_1(\theta) > 0$. Moreover, uniqueness simply follows from linearity of $p_1(.)$ and strictly concavity of $(P^Q)$.

Finally, let $U^Q_1(\theta, \hat{\theta})$ define the retailer’s profits when the true demand realization is $\theta$ but he announces a message $\hat{\theta} \neq \theta$ to $S_1$. To show that global incentive compatibility constraints hold we must have $\Gamma^Q(\theta, \hat{\theta}) \equiv U^Q_1(\theta, \theta) - U^Q_1(\theta, \hat{\theta}) > 0$ for each pair $(\theta, \hat{\theta}) \in \Theta^2$. Assume then $\theta > \hat{\theta}$ without loss of generality, simple algebraic manipulations allow to rewrite $\Gamma^Q(\cdot)$ as:

$$\Gamma^Q(\theta, \hat{\theta}) = \int_{\theta}^{\hat{\theta}} \left\{ -\eta^Q_1(s) + q^Q_1(s)p_1(\theta, q^Q_1(s), q^Q_1(s), q^Q_1(s)) - q^Q_1(s)q^Q_1(s) \right\} ds.$$

By using $IC_1$ and substituting for $\eta^Q(s) \equiv q^Q_1(s)p_1(\theta, q^Q_1(s), q^Q_1(s), q^Q_1(s)) - q^Q_1(s)q^Q_1(s)$ into the above equation, we get:

$$\Gamma^Q(\theta, \hat{\theta}) = \int_{\theta}^{\hat{\theta}} q^Q_1(s) \left\{ \int_s^{\theta} (1 + \rho q^Q_2(x)) dx \right\} ds.$$

Then, since we have proved above that $1 + \rho q^Q_2(\theta) > 0$ when $\psi''(e) > \max \left\{ 1, \frac{1+\sigma \rho}{2-\rho^2} \right\}$ for all $e \in \mathbb{R}_+$, one has $\Gamma^Q(\theta, \hat{\theta}) > 0$, which concludes the proof. ■

### 8.5 Proof of Proposition 3

Observe that $e^Q_1(\theta) - e^P_1(\theta) \approx (\dot{e}^P_1(\bar{\theta}) - \dot{e}^Q_1(\bar{\theta}))(\bar{\theta} - \theta)$ for all $\theta$. Using the definition of $\dot{e}^P_1(\bar{\theta})$ and $\dot{e}^Q_1(\bar{\theta})$ we get:

$$\dot{e}^P_1(\bar{\theta}) - \dot{e}^Q_1(\bar{\theta}) = \frac{(2\psi'' - 1)(2 + \rho)(\psi''(2 - \rho^2) - 1)}{(2\psi''(2 - \rho(\sigma + \rho)) - \sigma \rho - 2)(\psi''(2 - \rho^2) - \sigma \rho - 1)},$$

which immediately implies $\dot{e}^P_1(\bar{\theta}) > \dot{e}^Q_1(\bar{\theta})$ as we have assumed $\psi''(e) > \max \left\{ 1, \frac{2+\sigma \rho}{2(2-\rho(\sigma + \rho))}, \frac{1+\sigma \rho}{2-\rho^2} \right\}$ for all $e \in \mathbb{R}_+$.

Hence, $e^Q(\theta) \geq e^P(\theta)$ for all $\theta$ with equality only at $\bar{\theta}$.

By using the same kind of argument we have $q^Q_2(\theta) - q^P_2(\theta) \approx (\dot{q}^P_2(\bar{\theta}) - \dot{q}^Q_2(\bar{\theta}))(\bar{\theta} - \theta)$ for all $\theta$. Hence:

$$\dot{q}^P_2(\bar{\theta}) - \dot{q}^Q_2(\bar{\theta}) = \frac{\sigma(2 + \rho)(2\psi'' - 1)^2}{2(2\psi''(2 - \rho(\sigma + \rho)) - \sigma \rho - 2)(\psi''(2 - \rho^2) - \sigma \rho - 1)},$$

which immediately yields the result since $\psi''(e) > \max \left\{ \frac{1}{2}, \frac{2+\rho}{2(2-\rho(\sigma + \rho))}, \frac{1+\rho}{2-\rho^2} \right\}$ for all $e \in \mathbb{R}_+$.

Similarly, we have:

$$\dot{q}^P_1(\bar{\theta}) - \dot{q}^Q_1(\bar{\theta}) = \frac{\sigma \rho(2\psi'' - 1)(2 + \rho) \psi''}{(2\psi''(2 - \rho(\sigma + \rho)) - \sigma \rho - 2)(\psi''(2 - \rho^2) - \sigma \rho - 1)},$$

yielding $\text{sign}(q^Q_1(\theta) - q^P_1(\theta)) = \text{sign}(\sigma \rho)$ since $\psi''(e) > \max \left\{ \frac{1}{2}, \frac{2+\rho}{2(2-\rho(\sigma + \rho))}, \frac{1+\rho}{2-\rho^2} \right\}$ for all $e \in \mathbb{R}_+$.
8.6 Proof of Proposition 4

First, observe that in the limit of small uncertainty the type-contingent profit of the upstream supplier under both contractual regimes can be obtained by using a second-order Taylor expansion of (virtual) profits around \( \hat{\theta} \):

\[
\Pi'_1 = E_{\hat{\theta}}[\Pi'_1(\theta)] \approx \Pi'_1(\hat{\theta}) - \Pi'_1(\hat{\theta})E_{\hat{\theta}}[\theta - \hat{\theta}] + \frac{1}{2} \Pi''_1(\hat{\theta})E_{\hat{\theta}}[\theta - \hat{\theta}]^2 \quad \text{for each } \omega \in \{QF, RPM\}.
\]

Observe that when \( \Delta \theta \) is sufficiently small one can also approximate the distribution of \( F(.) \) with a uniform, i.e., \( \hat{\theta} \sim U[\theta] \), yielding immediately \( E_{\hat{\theta}}[\theta - \hat{\theta}] \approx \Delta \theta / 2 \) and \( E_{\hat{\theta}}[\theta - \hat{\theta}]^2 \approx \Delta \theta^2 / 3 \). Taking expectations on both sides of (30) we get up to terms of order at least 3:

\[
\Pi'_1(\theta) - \Pi'_p \approx \left( \Pi'_1(\hat{\theta}) - \Pi'_p(\hat{\theta}) \right) \frac{\Delta \theta}{2} + \left( \Pi''_1(\hat{\theta}) - \Pi''_p(\hat{\theta}) \right) \frac{\Delta \theta^2}{6},
\]

We need then to compute each term appearing in (31). First, consider program \( (P^P) \), differentiating with respect to \( \theta \) and using the Envelope Theorem we have:

\[
\Pi''_1(\hat{\theta}) = \psi'(e'_1(\theta))(1 + \rho q^P_1(\theta))(1 - \hat{h}(\theta)) + h(\theta)\psi''(e'_1(\theta))(1 + \rho q^P_1(\theta))^2.
\]

From \( h(\hat{\theta}) = 0 \) and \( \hat{h}(\hat{\theta}) = -1 \), taking limits for \( \theta \to \hat{\theta} \) on both sides it follows \( \Pi''_1(\hat{\theta}) = 2q^1_1(1 + \rho q^P_1(\hat{\theta})) \). Using the first-order condition (9) in \( \Pi''_1(\hat{\theta}) \), differentiating again with respect to \( \theta \), we have \( \Pi''_1(\hat{\theta}) = (\hat{q}^P_1(\hat{\theta}) + \psi'' e'_1(\hat{\theta}))(1 + \rho q^P_1(\hat{\theta})) \).

Consider now program \( (P^Q) \), differentiating with respect to \( \theta \) and using the Envelope Theorem we have:

\[
\Pi''_1(\hat{\theta}) = (1 + \rho q^Q_2(\hat{\theta}))q^Q_1(\hat{\theta}) - \hat{h}(\theta)(1 + \rho q^Q_2(\hat{\theta}))q^Q_1(\theta),
\]

taking limits for \( \theta \to \hat{\theta} \) on both sides and using \( \hat{h}(\hat{\theta}) = -1 \) we have \( \Pi''_1(\hat{\theta}) = 2q^1_1(1 + \rho q^Q_2(\hat{\theta})) \). By using the same kind of argument, we get \( \Pi''_1(\hat{\theta}) = (q^Q_1(\hat{\theta}) + \psi'' e'_1(\hat{\theta}))(1 + \rho q^Q_2(\hat{\theta})) \).

Substituting \( \Pi''_1(\hat{\theta}) \) and \( \Pi''_1(\hat{\theta}) \), for \( \omega \in \{QF, RPM\} \), into \( \Pi'_1 - \Pi'_p \), it follows:

\[
\Pi'_1 - \Pi'_p \approx \rho q^*(\hat{\theta}) \left( \hat{q}^P_2(\hat{\theta}) - \hat{q}^Q_2(\hat{\theta}) \right) \Delta \theta - \left( \Pi''_1(\hat{\theta}) - \Pi''_p(\hat{\theta}) \right) \frac{\Delta \theta^2}{6}.
\]

Taking \( \Delta \theta \) small enough and substituting (29) into the above equation we have:

\[
\Pi'_1 - \Pi'_p \approx \frac{\sigma \rho q^*(\hat{\theta})(2\psi'' - 1) (2 + \rho) \psi'' \Delta \theta}{(2\psi''(2 - \rho)(\sigma + \rho) - \sigma \rho - 2)(\psi''(2 - \rho)^2 - \sigma \rho - 1)},
\]

which immediately yields the result since \( \Pi'_1 - \Pi'_p > 0 \) whenever \( \rho \sigma > 0 \), and the converse obtains otherwise.

To conclude the proof we must consider the case where \( \sigma \rho = 0 \). A first-order approximation is now not
enough to sign $\Pi^Q_1 - \Pi^P_1$, hence we will use the second-order term of the Taylor approximation (31):

$$
\Pi^Q_1 - \Pi^P_1 \approx \left( (\dot{q}^Q_1(\bar{\theta}) + \psi'' \dot{e}^Q_1(\bar{\theta}))(1 + \rho q^Q_2(\bar{\theta})) - (\dot{q}^P_1(\bar{\theta}) + \psi'' \dot{e}^P_1(\bar{\theta}))(1 + \rho q^P_2(\bar{\theta})) \right) \frac{\Delta \theta^2}{6}.
$$

First assume $\sigma = 0$, in this case $\dot{q}^P_1(\bar{\theta}) = \dot{q}^Q_1(\bar{\theta})$ and $\dot{q}^P_2(\bar{\theta}) = \dot{q}^Q_2(\bar{\theta})$, then we have:

$$
\lim_{\rho \to 0} \left( \Pi^Q_1 - \Pi^P_1 \right) \approx -(1 + \rho q^Q_2(\bar{\theta}))(\dot{e}^P_1(\bar{\theta}) - \dot{e}^Q_1(\bar{\theta})) \frac{\Delta \theta^2}{6},
$$

which yields the result since $\dot{e}^P_1(\bar{\theta}) > \dot{e}^Q_1(\bar{\theta})$. The same kind of argument allows to show that

$$
\lim_{\rho \to 0} \left( \Pi^Q_1 - \Pi^P_1 \right) \approx -\psi''(\dot{e}^P_1(\bar{\theta}) - \dot{e}^Q_1(\bar{\theta})) \frac{\Delta \theta^2}{6} < 0,
$$

which concludes the proof. ■

8.7 Incentive Feasible Allocations under Complete Contracting

We show that when both suppliers announce a complete contract the market allocation satisfies standard properties of adverse selection models in the limit of small uncertainty. The next lemma summarizes the result.

**Lemma 10** Assume that $\Delta \theta$ is small enough, $\sigma + \rho < 1$ and $\psi''(e) > \max \left\{ \frac{1}{2}, \frac{1+\sigma}{2-\rho}, \frac{1+\sigma}{2(1-\sigma-\rho)} \right\}$ for all $e \in \mathbb{R}_+$. Then the following properties hold:

- $e^P(\theta) \leq e^*(\theta)$ and $q^P(\theta) \leq q^*(\theta)$ for all $\theta$ (with equality only at $\bar{\theta}$);
- Information rents are increasing in $\theta$ and $IC_2$ holds.

**Proof.** We follow the same approximation techniques used before. First, notice that the allocation $\{q^P(\theta), e^P(\theta)\}_{\theta \in \Theta}$ solves the following system of differential equations:

$$
\begin{align*}
\dot{q}^P(\theta)(2 - \rho) - 1 - \dot{e}^P(\theta)(1 + \sigma) &= 0, \\
q^P(\theta) - \psi'(e^P(\theta)) - h(\theta)(1 + \sigma e^P(\theta) + \rho q^P(\theta))\psi''(e^P(\theta)) &= 0,
\end{align*}
$$

with boundary conditions $q^P(\bar{\theta}) = q^*(\bar{\theta})$ and $e^P(\bar{\theta}) = e^*(\bar{\theta})$. As we have assumed $\Delta \theta$ small enough, from a first-order Taylor expansion of $q^P(\cdot)$ around $\bar{\theta}$ we have $q^P(\theta) \approx q^*(\bar{\theta}) - q^*(\bar{\theta})(\bar{\theta} - \theta)$. From a simple application of l’Hôpital’s rule we have:

$$
q^P(\bar{\theta}) = \frac{2\psi''}{2\psi''(1 - \rho - \sigma) - (1 + \sigma)} \quad \text{and} \quad \dot{e}^P(\theta) = \frac{2\psi'' - 1}{2\psi''(1 - \rho - \sigma) - (1 + \sigma)}.
$$

\[27\] Again these assumptions are needed to insure that the problem is well behaved and, in particular, that the equilibrium allocation satisfies monotonicity.
Having assumed $\sigma + \rho < 1$, monotonicity is insured under the following restriction:

$$\psi''(e) > \frac{1 + \sigma}{2(1 - \rho - \sigma)} \forall e \in \mathbb{R}_+,$$

Now, it is easy to show that in the complete information benchmark we have:

$$\tilde{q}^*(\bar{\theta}) = \frac{\psi''}{\psi''(2 - \rho) - (1 + \sigma)},$$

with $\tilde{q}^*(\bar{\theta}) > 0$ since $\psi''(e) > (1 + \sigma)/(2 - \rho)$ for all $e \in \mathbb{R}_+$.

Hence, as $q^P(\bar{\theta}) = q^*(\bar{\theta})$, it follows that $q^P(\theta) > q^*(\theta)$ must imply $q^P(\theta) < q^*(\theta)$ for all $\theta < \bar{\theta}$. Simple algebra then yields:

$$\dot{q}^P(\bar{\theta}) - \dot{q}^*(\bar{\theta}) = \frac{(2\psi'' - 1)(1 + \sigma)\psi''}{(\psi''(2 - \rho) - (1 + \sigma))(2\psi''(1 - \rho - \sigma) - (1 + \sigma))},$$

which implies the result since $\psi''(e) > 1/2$ for all $e \in \mathbb{R}_+$. By using the same kind of argument one shows that $e^P(\theta) \leq e^*(\theta)$ for all $\theta \leq \bar{\theta}$. Finally, notice that for $\Delta \theta$ small enough, efforts and outputs will be positive. The rest of the proof is omitted as it follows Martimort (1996).

### 8.8 Incentive Feasible Allocations under Incomplete Contracting

We now show that when both suppliers announce an incomplete contract the market allocation satisfies no distortion at the top and underproduction at the bottom. The next lemma summarizes the result.

**Lemma 11** Assume that $\Delta \theta$ is small enough and $\psi''(e) > \max \left\{ \frac{1}{2}, \frac{1 + \sigma}{2 - \rho}, \frac{1 + 2\sigma}{2(1 - \rho)} \right\}$ for all $e \in \mathbb{R}_+$. Then the following properties hold:

- $e^Q(\theta) \leq e^*(\theta)$ and $q^Q(\theta) \leq q^*(\theta)$ for all $\theta$ (with equality only at $\bar{\theta}$);
- Information rents are increasing in $\theta$ and $IC_2$ holds.

**Proof.** By definition the allocation $\{q^Q(\theta), e^Q(\theta)\}_{\theta \in \Theta}$ solves the following system of differential equations:

$$\theta + (1 + \sigma)e^Q(\theta) - q^Q(\theta)(2 + \rho) - h(\theta)(1 + \sigma e^Q(\theta) + \rho q^Q(\theta)) = 0,$$

$$\dot{e}^Q(\theta) = \phi'(q^Q(\theta))\dot{q}^Q(\theta),$$

with boundary conditions $q^R(\bar{\theta}) = q^*(\bar{\theta})$ and $e^R(\bar{\theta}) = e^*(\bar{\theta})$. Again, as we have assumed $\Delta \theta$ small, l’Hopital’s rule yields:

$$\dot{q}^Q(\bar{\theta}) = \frac{2\psi''}{2\psi''(1 - \rho) - (1 + 2\sigma)} \quad \text{and} \quad \dot{e}^Q(\bar{\theta}) = \frac{2}{2\psi''(1 - \rho) - (1 + 2\sigma)}.$$
which directly implies monotonicity if:

\[ \psi''(\epsilon) > \frac{1 + 2\sigma}{2(1 - \rho)} \quad \forall \ \epsilon \in \mathbb{R}_+. \]

Simple algebra then yields:

\[ q^Q(\theta) - q^*(\theta) = \frac{(2\psi'' - 1) \psi''}{(2\psi''(1 - \rho) - (1 + 2\sigma)(\psi''(2 - \rho) - (1 + \sigma))}, \]

that immediately implies \( q^Q(\theta) \leq q^*(\theta) \) for all \( \theta \) with equality holding only at \( \bar{\theta} \). The same kind of argument allows to show that \( e^Q(\theta) \leq e^*(\theta) \) for all \( \theta \) with equality holding only at \( \bar{\theta} \). Finally, \( (P^Q') \) has interior solutions whenever \( \Delta \theta \) is small enough. The rest of the proof is omitted as it follows Martimort (1996).

\[ \square \]

### 8.9 Proof of Proposition 5

To begin with, we need to characterize allocations in an asymmetric play of \( G \). So, suppose that \( S_i \) chooses a complete contract while \( S_{-i} \) plays an incomplete one. By definition the allocations \( \{q^P_i, e^P_i\}_\theta \) and \( \{q^P_{-i}, e^P_{-i}\}_\theta \) solve the following system of differential equations:

\[ q^P_i(\theta) - \psi'\left( e^P_i(\theta) \right) - h(\theta)(1 + \sigma e^P_{-i}(\theta) + \rho q^P_{-i}(\theta)) \psi''(e^P_i(\theta)) = 0, \]

(34)

\[ e^P_i(\theta) = 2q^P_i(\theta) - \sigma e^P_{-i}(\theta) - \rho q^P_{-i}(\theta) - \theta, \]

(35)

\[ \theta + e^P_{-i}(\theta) - 2q^P_{-i}(\theta) + \sigma e^P_i(\theta) + \rho q^P_i(\theta) - h(\theta)(1 + \sigma e^P_i(\theta) + \rho q^P_i(\theta)) = 0, \]

(36)

\[ e^P_{-i}(\theta) = \phi(q^P_{-i}(\theta)). \]

(37)

with boundary conditions \( q^P_i(\bar{\theta}) = q^P_{-i}(\bar{\theta}) = q^*(\theta) \) and \( e^P_i(\theta) = e^P_{-i}(\theta) = e^*(\theta) \).

Differentiating (37) it follows \( \dot{e}^P_{-i}(\theta) = \phi'(q^P_{-i}(\theta)) \dot{q}^P_{-i}(\theta) \), then plugging \( \dot{e}^P_{-i}(\theta) \) into (34)-(36) and linearizing the reduced system around the point \( (\bar{\theta}, q^*(\bar{\theta}), e^*(\bar{\theta})) \) we have:

\[ 2q^P_i(\bar{\theta}) - 1 - e^P_i(\bar{\theta}) - \frac{\sigma}{\psi'} q^P_{-i}(\bar{\theta}) - \rho q^P_{-i}(\bar{\theta}) = 0, \]

(38)

\[ q^P_i(\bar{\theta}) - \psi'' e^P_i(\bar{\theta}) + \left( 1 + \frac{\sigma}{\psi'} q^P_{-i}(\bar{\theta}) + \rho q^P_{-i}(\bar{\theta}) \right) \psi'' = 0, \]

(39)
\[(40)\quad 1 + \frac{1}{\psi''} \bar{q}^{Q,P}_i(\bar{\theta}) - 2 \bar{q}^{Q,P}_i(\bar{\theta}) + \sigma \dot{e}^{P,Q}_i(\bar{\theta}) + \rho \dot{e}^{P,Q}_i(\bar{\theta}) + (1 + \sigma \dot{e}^{P,Q}_i(\bar{\theta}) + \rho \dot{e}^{P,Q}_i(\bar{\theta})) = 0.\]

One can check that the solution of the linearized system (38)-(40) yields:

\[
\dot{e}^{P,Q}_i(\bar{\theta}) = \frac{-2(1 - 2\sigma) + 2\psi''(2\sigma + \rho) + 4(\psi''')^2(1 + \rho)}{a + b\psi'' + c(\psi''')^2},
\]

\[
\bar{q}^{P,Q}_i(\bar{\theta}) = \frac{-2(1 - 2\sigma) + 4(\psi''')^2(1 + \rho)}{a + b\psi'' + c(\psi''')^2},
\]

and

\[
\bar{q}^{Q,P}_i(\bar{\theta}) = \frac{-2\psi''(1 - \sigma) + (\psi''')^2(1 + \sigma + \rho)}{a + b\psi'' + c(\psi''')^2}.
\]

Moreover, \(\dot{e}^{Q,P}_i(\bar{\theta}) = \phi'(q^*(\bar{\theta}))\dot{q}^{Q,P}_i(\bar{\theta})\) implies:

\[
\dot{e}^{Q,P}_i(\bar{\theta}) = \frac{1}{\psi''} \left( -2\psi''(1 - \sigma) + (\psi''')^2(1 + \sigma + \rho) \right) \left( a + b\psi'' + c(\psi''')^2 \right),
\]

where we have defined \(a = (1 - 2\sigma),\ b = -2(3\sigma\rho + 2\sigma^2 + 2)\) and \(c = 4(1 - \rho^2 - \sigma\rho)\). With \(c > 0\) since we have assumed \(1 > \sigma + \rho\). Notice when \(\psi''\) is sufficiently large and \(-1 < \sigma + \rho < 1\), monotonicity follows under all possible contractual regimes. That is, \(\ddot{q}^{\omega,\omega'}(\theta) \geq 0\) for all pairs \((\omega, \omega') \in \{QF, RPM\}\) and \(\theta \in \Theta\) if the following inequality holds:

\[
\psi''(e) > \max \left\{ \frac{1 - 2\sigma}{1 + \rho}, \frac{1 - \sigma}{1 + \sigma + \rho}, \frac{1 + 2\sigma}{2(1 - \rho)}, \frac{1 + \sigma}{2(1 - \rho - \sigma)}, \psi''_1, \psi''_2 \right\}, \forall e \in \mathbb{R}_+
\]

where we have defined:

\[
\psi''_1 = \max \left\{ \psi'' \in \mathbb{R}_+ \middle| - (1 - 2\sigma) + 2\psi''(2\sigma + \rho) + 4(\psi''')^2(1 + \rho) = 0 \right\},
\]

and,

\[
\psi''_2 = \max \left\{ \psi'' \in \mathbb{R}_+ \middle| a + b\psi'' + c(\psi''')^2 = 0 \right\}.
\]

Equipped with this characterization we can conclude the proof. Again, since \(\Delta\theta\) is small enough, from a second-order Taylor approximation around \(\bar{\theta}\) the \(S_i\)’s type-contingent profit when he chooses a mechanism \(\omega\) while \(S_{-i}\) chooses \(\omega'\) can be written as:

\[
\Pi_i^{\omega,\omega'} = E_{\bar{\theta}}[\Pi_i^{\omega,\omega'}(\theta)] \approx \Pi_i^{\omega,\omega'}(\bar{\theta}) - \bar{\Pi}_i^{\omega,\omega'}(\bar{\theta})E_{\bar{\theta}}[\bar{\theta} - \theta] + \frac{1}{2} \bar{\Pi}_i^{\omega,\omega'}(\bar{\theta})E_{\bar{\theta}}[\bar{\theta} - \theta]^2.
\]
Then, using $E_{\bar{\theta}}[\bar{\theta} - \theta] \approx \Delta \theta/2$ and $E_{\bar{\theta}}[(\bar{\theta} - \theta)^2] \approx \Delta \theta^2/3$ it follows that $\Pi_i^{v,\omega} \geq \Pi_i^{v',\omega'}$ (resp. $\leq$) if:

$$-(\Pi_i^{v,\omega'}(\bar{\theta}) - \Pi_i^{v',\omega'}(\bar{\theta})) \frac{\Delta \theta}{2} + (\Pi_i^{v,\omega'}(\bar{\theta}) - \Pi_i^{v',\omega'}(\bar{\theta})) \frac{\Delta \theta^2}{6} \geq 0 \quad (41)$$

We first show that incomplete contracts are part of a PBE of $\mathcal{G}$ if one of the following conditions holds: (i) $\rho \sigma > 0$, or (ii) $\sigma > |\rho|\psi''$ when $\sigma > 0$ and $\rho \leq 0$. Since players are symmetric, we only need to show that $\Pi_i^{Q,Q} \geq \Pi_i^{P,Q}$. Consider then program $(P_i^{Q,Q})$, differentiating with respect to $\theta$ and using the Envelope Theorem one can easily show that $\Pi_i^{Q,Q}(\bar{\theta}) = 2q^*(\bar{\theta})(1 + \sigma \epsilon_{P,Q}(\bar{\theta}) + \eta q^Q(\bar{\theta}))$ and $\Pi_i^{Q,Q}(\bar{\theta}) = (q^Q(\bar{\theta}) + \psi'' \epsilon^Q(\bar{\theta}))(1 + \sigma \epsilon_{P,Q}(\bar{\theta}) + \eta q^Q(\bar{\theta}))$. By using the same kind of arguments, differentiating program $(P_i^{P,Q})$ with respect to $\theta$ and taking limits for $\theta \to \bar{\theta}$ one also gets $\Pi_i^{P,Q}(\bar{\theta}) = 2q^*(\bar{\theta})(1 + \sigma \epsilon_{P,Q}(\bar{\theta}) + \eta q^Q(\bar{\theta}))$ and $\Pi_i^{P,Q}(\bar{\theta}) = (q^P(\bar{\theta}) + \psi'' \epsilon^P(\bar{\theta}))(1 + \sigma \epsilon_{P,Q}(\bar{\theta}) + \eta q^Q(\bar{\theta}))$. Substituting in (41) and taking $\Delta \theta$ small we have:

$$\Pi_i^{Q,Q} - \Pi_i^{P,Q} \approx q^*(\bar{\theta}) \left( \sigma (\epsilon_{P,i}^{Q,Q}(\bar{\theta}) - \epsilon^Q(\bar{\theta})) + \rho (q_{P,i}^{Q,Q}(\bar{\theta}) - q^Q(\bar{\theta})) \right) \Delta \theta,$$

using the solutions of the linearized system of equations (38)-(40) together with $q^Q(\bar{\theta})$ and $\epsilon^Q(\bar{\theta})$, we have:

$$\Pi_i^{Q,Q} - \Pi_i^{P,Q} \approx \frac{2 \sigma (\sigma + \rho \psi'') q^*(\bar{\theta}) (2 \psi'' - 1)^2 \Delta \theta}{(a + b \psi' + c (\psi'')^2)(2 \psi'' (1 - \rho - \sigma) - (1 + \sigma))},$$

which yields immediately the result when $\sigma + \rho \psi'' \neq 0$.

Assume now $\sigma + \rho \psi'' = 0$, for $\Delta \theta$ small the sign of $\Pi_i^{Q,Q} - \Pi_i^{P,Q}$ is given by the second-order terms of equation (41):

$$\lim_{\rho \to -\frac{\psi''}{\psi'}} \left( \Pi_i^{Q,Q} - \Pi_i^{P,Q} \right) = \frac{\Delta \theta^2}{6} \lim_{\rho \to -\frac{\psi''}{\psi'}} \left( \Pi_i^{Q,Q}(\bar{\theta}) - \Pi_i^{P,Q}(\bar{\theta}) \right) = -\psi'' \frac{\Delta \theta^2}{6} < 0,$$

it is easy to check that the same result holds when $\sigma = 0$ and $\sigma + \rho \psi'' \neq 0$.

We now show that complete contracts are part of a PBE of $\mathcal{G}$ if one of the following conditions holds: (i) $\rho \sigma > 0$, or (ii) $\sigma \geq 2 |\rho|\psi''/(1 + 2 \psi'')$ when $\sigma \geq 0$ and $\rho \leq 0$. Consider now the following difference $\Pi_i^{P,P} - \Pi_i^{Q,P}$, for $\Delta \theta$ small, the same argument used above allows to obtain:

$$\Pi_i^{P,P} - \Pi_i^{Q,P} \approx q^*(\bar{\theta}) \left( \sigma (\epsilon_{P,i}^{P,P}(\bar{\theta}) - \epsilon^P(\bar{\theta})) + \rho (q_{P,i}^{P,Q}(\bar{\theta}) - q^P(\bar{\theta})) \right) \Delta \theta.$$

After simple algebra we have:

$$\Pi_i^{P,P} - \Pi_i^{Q,P} \approx \frac{\sigma (\sigma + 2 \psi''(\sigma + \rho)) q^*(\bar{\theta}) (2 \psi'' - 1)^2 \Delta \theta}{(a + b \psi' + c (\psi'')^2)(2 \psi'' (1 - \rho - \sigma) - (1 + \sigma))},$$

yielding the result for $\sigma + 2 \psi''(\sigma + \rho) \neq 0$.

Consider now the case $\sigma + 2 \psi''(\sigma + \rho) = 0$, for $\Delta \theta$ small the sign of $\Pi_i^{P,P} - \Pi_i^{Q,P}$ is given by the second-order
terms of equation (41):
\[
\lim_{\rho \to 0} \frac{\Delta \theta^2}{2} \left( \Pi_i^{P,P} - \Pi_i^{Q,P} \right) \approx \frac{\Delta \theta^2}{6} \left( \bar{\Pi}_i^{P,P}(\bar{\theta}) - \bar{\Pi}_i^{Q,P}(\bar{\theta}) \right) = \psi' \frac{\Delta \theta^2}{6} > 0,
\]
it is easy to check that the same result holds when \( \sigma = 0 \) and \( \sigma + 2\psi'(\sigma + \rho) \neq 0 \). This concludes the proof.

8.10 Proof of Proposition 6

Again, as we have assumed \( \Delta \theta \) small:

\[
\Pi^{Q,Q} - \Pi^{P,P} \approx q^*(\bar{\theta}) \left( \sigma(\epsilon^P(\bar{\theta}) - \epsilon^Q(\bar{\theta})) + \rho(q^P(\bar{\theta}) - q^Q(\bar{\theta})) \right) \Delta \theta.
\]

From equations (32) and (33) we have:

\[
\Pi^{Q,Q} - \Pi^{P,P} \approx \frac{\sigma q^*(\bar{\theta})(2\psi'' - 1)^2 \Delta \theta}{(2\psi''(1 - \rho - \sigma) - (1 + \sigma))(2\psi''(1 - \rho) - (1 + 2\sigma))},
\]

which directly proves that \( \Pi^{Q,Q} > \Pi^{P,P} \) if \( \sigma > 0 \), and that the converse obtains otherwise.

When \( \sigma = 0 \), the sign of \( \Pi^{Q,Q} - \Pi^{P,P} \) is given by the second-order terms of (41):

\[
\lim_{\sigma \to 0} \left( \Pi^{Q,Q} - \Pi^{P,P} \right) \approx \frac{\Delta \theta^2}{6} \lim_{\sigma \to 0} \left( \bar{\Pi}^{Q,Q}(\bar{\theta}) - \bar{\Pi}^{P,P}(\bar{\theta}) \right) = -\psi'' \frac{\Delta \theta^2}{6} \left( \frac{(2\psi'' - 1)^2}{(1 - 2\psi''(1 - \rho))^2} \right),
\]

which directly implies the claim.

8.11 Proof of Proposition 7

Since we have considered a simple representative consumer economy where, for any given wealth level \( w \), demands are derived as a solution of the program,

\[
\max_{(q_1, q_2, I)} \left\{ V(q_1, q_2, I, \theta) : \sum_{i=1,2} p_i q_i + I \leq w \right\},
\]

with:

\[
V(q_1, q_2, I, \theta) = \sum_{i,j=1,2; i \neq j} e_i(q_i + \sigma q_j) + \theta \sum_{i=1,2} q_i - \frac{1}{2} \left( \sum_{i=1,2} q_i^2 - 2\rho q_1 q_2 \right) + I.
\]

Hence, as we focus only on what happens to consumers in the symmetric equilibria of contracting choices we have:

\[
\Delta V(\theta) = V(q^Q(\theta), q^Q(\theta), I, \theta) - V(q^P(\theta), q^P(\theta), I, \theta) = (q^Q(\theta) - q^P(\theta))(q^Q(\theta) + q^P(\theta)).
\]
Therefore one can check that \( \text{sign}(\Delta V(\theta)) = -\text{sign}(q^Q(\theta) - q^P(\theta)) \) for all \( \theta \). As we have assumed \( \Delta \theta \) small enough, the result follows from:

\[
q^Q(\overline{\theta}) - q^P(\overline{\theta}) = -\frac{2\sigma \psi''(2\psi'' - 1)}{(2\psi'(1 - \rho) - (1 + 2\sigma))(2\psi'(1 - \rho - \sigma) - (1 + \sigma))},
\]

which directly yields \( \text{sign}(q^Q(\theta) - q^P(\theta)) = \text{sign}(\sigma) \) for all \( \theta \). ■

References


