Product Pricing when Demand Follows a Rule of Thumb

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February 16, 2008

In our model, individual consumers follow simple behavioral decision rules governed by imitation and habit as suggested in consumer research. Demand can be viewed as the outcome of a population game in which the revision protocol is determined by the consumers' behavioral rules. The resulting description of consumer dynamics is used to investigate the monopoly market and competition between firms. We derive a Nash-equilibrium in a price-competition model in which demand is determined via such a population game. Our model is consistent with observed market behavior such as product life cycles.

JEL Classification Number: C61, C62, C79, L11, L21, M31
Keywords: bounded rationality, social learning, population game, mean dynamic, product life cycle, monopoly, competition

1. Introduction

On a goods market, rational decisions may in many situations be very complex for consumers, for example when there are large numbers of possible options. Therefore, it is of interest, how markets behave if consumers are not fully rational. However, standard approaches in industrial organization usually do not take bounded rationality into account.

In this paper we explore if it is possible to describe market observations such as product life cycles with just a simple rule of thumb behavior of the consumers.

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Moreover, we investigate the results which we expect from rational pricing decisions of firms if the consumers behave according to such a rule of thumb, i. e. non-rationally.

Bounded rationality has a long history in industrial organization papers. Ellison (2006) has provided an excellent review on the topic, and a very interesting model on this matter is presented in the seminal paper by Smallwood and Conlisk (1979). Although it was already published in 1979, there is still social learning literature building on this model, for example Ellison and Fudenberg (1993). We use an approach quite similar to Smallwood's and Conlisk's paper, extending and refining their ideas.

In order to describe markets, we define a two-level game. On the lower level we use a continuous time consumer model based on behavioral rules which are reasonable according to psychologic and experimental studies (Assael (1984), Venkatesan (1966)). This model is stated in form of a population game as defined in Sandholm (2005) and Sandholm (2006). The upper level describes the strategic behavior of firms, which base their decisions on the results of the lower level model. With such a description at hand we can also transfer the concept of welfare into this framework.

It will be seen that two simple rule of thumb ingredients, imitation and habit, are sufficient to generate typical patterns observed in consumer markets, such as product life cycles (for product life cycles see Kluyver (1977), Brockhoff (1967), Polli and Cook (1969)). Concerning the methodology, we employ a new method to describe interacting consumers in continuous time as well as a new approach to describe supply and demand side of a market via a two-level game.

2. Model and microfoundation of sales

This section proposes a mathematical model for the evolution of sales based on a microfoundation. The microfoundation constitutes a means of macroscopically describing a system of a large number of interacting agents taking part in a so-called modified population game. The agents behave non-rationally and myopically change their activities. The mean over all agents then provides the governing differential equations to describe the overall system behavior.

After briefly introducing the methodology of a microfoundation following Sandholm (2005) and (2006) it is applied to a goods market, yielding a first simple result about the relation between the evolution of product distribution and the sales. Finally, we suggest an exemplary model of consumer behavior and derive the resultant sales evolution via the methods above.

2.1. Methodology

A typical normal form game consists of a population of agents, their actions, and the payoff function which rewards or penalizes the agents' actions depending on the other players' behavior. Since the agents are usually assumed to be rational, their behavior directly follows from maximizing the payoff. However, in the case of non-rational agents the payoff function is insufficient to determine their actions; a revision protocol, for example similar to Sandholm (2006), seems more adequate to describe the agents' behavior.

Definition 1 (Revision protocol)**.** *A* revision protocol ρ *is a map*

$$\rho : X \times \mathbb{R} \to \mathbb{R}_+^{(n+1)\times(n+1)}, \tag{1}$$

where $X = \{x = (x_0, \ldots, x_n) \in [0,1]^{n+1} \,|\sum_{i=0}^{n} x_i = 1\}$ *is called the* state space*. The scalar* $\rho_{ij}(x,t)$ *is called the* conditional switch rate *from activity* i *to activity* j *at time* t *and state* x*. The sum*

$$R_i = \sum_{j=0}^{n} \rho_{ij}(x,t) \tag{2}$$

is called the alarm clock rate *of subpopulation* i*, and the scalar* $p_{ij} = \frac{\rho_{ij}}{R_i}$ *the* switching probability*.*

Before motivating this definition we shall first use the revision protocol to specify the model framework.

Definition 2 (Modified population game)**.** *A* modified population game *is defined by the triple* $\mathcal{G} = (N, \mathcal{A}, \rho)$*, where* N *is the agent* population size*,* $\mathcal{A} = \{0, \ldots, n\}$ *is the* activity set*, and* $\rho \in \mathbb{R}_+^{(n+1)\times(n+1)}$ *is a* revision protocol*.*

The motivation is as follows: In a modified population game a situation with N agents is considered, each of whom plays precisely one activity $i \in \mathcal{A}$ at a time. We use the term 'activity' as opposed to the game theoretic term 'action' in order to emphasize the continuity of the activity in time, whereas actions take place at discrete times (e. g. in repeated games). The subpopulation of those agents playing activity i amounts to the time-varying fraction x_i of the total population so that the set of all possible states is given by $X = \{x \in [0,1]^{n+1} | \sum_{i=0}^{n} x_i = 1\}$. Each agent playing activity i is equipped with an independent *Poisson alarm clock* of rate R_i, i. e. an alarm clock which rings after an exponentially distributed time with expected value R_i^{-1}. Each time the alarm goes off the agent revises her activity and switches to activity $j \in \mathcal{A}$ with probability p_{ij}.

The objective of studying the macroscopic behavior of a system requires averaging over all agents. In order to limit complexity of analysis we shall make use of the continuum hypothesis so that differential equations can be applied. Hence we assume N very large with the effect that the state space X can be approximated as continuous and that the mean agent behavior can be approximated by expected behavior according to the law of large numbers. Under these conditions Sandholm (2006) derives the macroscopic system behavior:

Definition 3 (Mean dynamic)**.** *Let $\mathcal{G}$ be a modified population game, and let ρ be its revision protocol. The* mean dynamic *corresponding to $\mathcal{G}$ is*

$$\dot{x}_i = \sum_{j=0}^{n} x_j \rho_{ji}(x,t) - x_i \sum_{j=0}^{n} \rho_{ij}(x,t), \quad i = 0, \ldots, n, \tag{3}$$

where $\dot{x}_i$ denotes the time derivative of x_i.

2.2. Application to a consumer market

Let us consider a market consisting of N consumers and n different products produced by some firms. As opposed to some standard industrial organization models the focus is here laid on the consumer side of the market, i. e. given the firms' behavior in the sense of product properties[1] (e. g. price and quality) the evolution of the market is governed by the consumers' purchase decisions. Though in principle

[1] The assumption of exogenously given prices are for instance applied in a Bertrand competition for oligopolies.

the application of modified population games allows for time-dependent product properties (i. e. time-varying firm strategies) and consumer characteristics, let us concentrate on a market with static parameters.

The consumers, who take the role of the agents in the modified population game, own at most one good each, where the different products are substitutes but not perfect substitutes to them. Their activity set comprises the activities "not owning any product", denoted 0, "owning good 1", denoted 1, and so forth. The alarm clock of each consumer announces the time when she can decide to buy a new product and hence to switch her activity. Reasons can be a breakage or a defect when owning a product or a suddenly arising interest of the consumer in a product when currently being without any good. Due to the memorylessness of such incidents, the use of a Poisson alarm clock seems most adequate to model their occurrence. Since the old products are broken, they are disposed and therefore reselling is assumed to be impossible. Finally, let the firms have—as is standard practice in industrial organization—perfect knowledge about the consumer behavior depending on the product properties, and hence they produce exactly as many goods as are demanded at any time.

2.3. The sales equation

We define $S_i(t)$ as the number of units of product $i \in \{1, \ldots, n\}$, sold per time at time t. S_i shall be called the *sales of product* i.

The sales of product i correspond to the rate of consumers switching to or rather buying this good,

$$S_i(t) \quad = \quad N \left[x_i(t)\rho_{ii}(t) + \sum_{\substack{j=0 \\ j \neq i}}^{n} x_j(t)\rho_{ji}(t) \right], \quad i = 0, \ldots, n. \tag{4}$$

Using the mean dynamic (3), equation (4) can be transformed into the *sales equation*, which describes the relationship between sales and consumer subpopulations,

$$\frac{S_i}{N} = \dot{x}_i + x_i R_i, \quad i = 0, \ldots, n. \tag{5}$$

To enhance intuition, the rate of change $\dot{x}_i$ can be interpreted as the difference between consumers adopting and abandoning good i, while the second summand represents all those owners of good i who currently reorientate themselves and either abandon or stick with[2] product type i. As the leaving consumers cancel, the net effect consists of those consumers buying product i.

Let us provide a brief analysis of the sales equation. As indicated above, the equation decomposes into a pair of additive terms, leading to two cases, each corresponding to one term outweighing the other. In the case of small x_i, e. g. during product launch, sales precisely reflect the increasing distribution among the population, e. g. due to growing product awareness. On the other hand, for negligible $\dot{x}_i$ (after reaching a comparatively stable situation) sales of good i solely originate from replacements of both defect units of i and different products. The rate of replacements is equivalent to the rate of consumer reorientation or revision, $x_i R_i$.

2.4. A consumer revision protocol and the resulting mean dynamic

The specific revision protocol which we employ, borrows well-established ideas from social learning (Smallwood and Conlisk 1979) (Ellison and Fudenberg 1993), psychological, experimental and consumer research (Assael 1984) literature. On a goods market there are mainly two factors influencing the buying behavior of a consumer, the goods' properties and their perceived distribution among other consumers. It is for example beyond question that any purchase decision depends on the anticipated product quality, including functionality, reliability, value for money, and many further properties. Also, the life span of a product plays a role as it affects the frequency of purchases. On the other hand, the product distribution among other consumers may have an effect by simply determining the level of product awareness or inducing a fashion or even networking. The revision protocol ought to reflect these influences on the consumer behavior.

A revision protocol $\rho(x, t)$ is uniquely defined by the product-dependent alarm clock rates R_i and the switching probabilities $p_{ij}(x, t)$, for which we shall suggest a specification in the following.

[2]Note that these consumers replace the broken unit they own and buy a new one.

The alarm clock rate represents the average frequency at which consumers think about buying a specific product and hence replacing their old one—unless they do not yet possess any good. Since this frequency depends on the durability of the currently owned product, alarm clock rates in general differ from product to product. For simplicity, we shall assume the rates R_i to be given for each product i and to be invariant with time.

Of those people, who do not own any product, the fraction of consumers deciding to buy product i can be described by the switching probability p_{0i}. When consumers start thinking about buying a specific product they scan the market and become susceptible to various types of information about possible alternatives. In the accompanied decision process passive and active decision mechanisms can be distinguished: When consumers passively encounter a product, its level of familiarity rises, thus increasing the possibility for this product to be bought. On the other hand, consumers may actively imitate others in buying the same good since the popularity of a product might give information about the product's past performance (cf. Ellison and Fudenberg (1993)), so that according to Smallwood and Conlisk (1979) the consumers' choices are sensitive to market shares or popularities of the products. Additionally, studies in social psychology support the individual's conformity to group norms, i. e. that consumers imitate group behavior (Assael (1984), p. 371 and following).

Purely active, purely passive and intermediate decision mechanisms are categorized in the following. They imply a proportionality between the switching probability p_{0i} and the fraction x_i of consumers currently owning product i. The mechanisms are ordered from rather passive to rather active.

- In daily life consumers encounter product i at a frequency proportional to its distribution x_i among the population. Hence, familiarity with the good and proneness to buy it rise accordingly.
- A consumer's idea of an ideal product is partly shaped by the surroundings. Product characteristics often observed are commonly desired. The intensity of influence by good i and thus proneness to buy it may be assumed proportional to its frequency x_i.
- Information about products can be obtained from various media (including the Internet), e. g. experience is exchanged on product evaluation websites.

The media in total roughly reflect the real world including the market; the more widespread a product is, the more frequently the media report about it, thereby increasing product awareness proportionally to x_i.

- The expected quality of a product is often inferred from the number of sales, assuming that superior products always find a ready market. The prevalently perceived indicator is thus the observed distribution x_i.

- Consumers also actively ask around their friends and let their purchase decision be influenced by the found product distribution, which on average matches the x_i. For certain goods, networking may play a role as well, intrinsically implying the positive effect of high x_i on the sales of product i.

- Finally, a fashion might be induced by a high distribution of a specific product, leading to even higher sales of that product.

In summary, we may assume a linear relation between switching probability p_{0i} and x_i as a first approximation, that is $p_{0i} \sim x_i$. The proportionality factor, denoted φ_i, still remains to be determined. Generally, it differs from product to product (Assael (1984), p. 432, 414) and can even be time dependent. We need $\varphi_i \in [0, 1]$, since

$$1 = \sum_{i=0}^{n} p_{0i} = p_{00} + \sum_{i=1}^{n} \varphi_i x_i,$$

must also hold for any state $x_j = 1,\ x_{k \neq j} = 0$. φ_i constitutes the accumulated influence of product frequency on the consumers' purchase decision via all different mechanisms. It can be interpreted as the intensity by which a consumer is convinced during an encounter with the product and is therefore termed *convincement factor* in the following. Of course, φ_i does depend on the good properties as there are the price, the (expected) quality, the strength of networking and fashion effects for that product etc. To summarize,

$$p_{0i} = \varphi_i x_i,\ i \neq 0. \tag{6}$$

Let us now turn to those people owning product i. Someone who is content with that good, tends to buy a new unit of that good when the alarm clock rings, even though a better product might exist. Surveying the market is time-consuming,

and furthermore, consumers usually act conservatively and avoid changes, so that the same product i is bought. Assael (1984) (p. 53) summarizes several studies on the topic and comes to the conclusion that a form of habit evolves, leading to repeat purchases of a product without further information search or evaluating brand alternatives.

We may deduce that the fraction p_{ii} of consumers sticking to product i only depends on the habit induced by this good and is independent of any other factors. Also, we will assume this habit level to represent a characteristic of the product, as justified by the fact that a consumer usually gets the more discontent with a product the more often it breaks down or the less satisfactory its functionality seems. In our model, a fixed, product-specific percentage of consumers will develop a buying habit, so that finally

$$p_{ii} = s_i \in [0,1],\ i \neq 0. \tag{7}$$

Obviously, the fraction of switching consumers $(1 - p_{ii})$ divides up into the fractions p_{ij} of people switching to product $j \neq i$. Naturally, they behave just like those consumers not yet owning any good, except that they do not purchase product i again. Therefore,

$$\begin{aligned} p_{ij} &= (1 - p_{ii})p_{0j} \\ &= (1 - s_i)\varphi_j x_j,\ i \neq 0 \wedge j \neq 0, i. \end{aligned} \tag{8}$$

The switching probabilities p_{i0} and p_{00} are now uniquely determined by the constraints $\sum_{j=0}^{n} p_{ij} = 1$ and $\sum_{j=0}^{n} p_{0j} = 1$,

$$p_{00} = 1 - \sum_{j=1}^{n} p_{0j} = 1 - \sum_{j=1}^{n} \varphi_j x_j, \tag{9}$$

$$\begin{aligned} p_{i0} &= 1 - \sum_{j=1}^{n} p_{ij} = (1 - p_{ii})\,(p_{00} + p_{0i}) \\ &= (1 - s_i)\left(1 - \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_j x_j\right). \end{aligned} \tag{10}$$

For $i = 1, \ldots, n$, the mean dynamic (3) eventually takes the form

$$\begin{aligned} \dot{x}_i &= \rho_{0i} + \sum_{j=1}^{n} x_j(\rho_{ji} - \rho_{0i}) - x_i R_i \\ &= x_i \left(\varphi_i R_0 - (1 - s_i) R_i - \varphi_i \sum_{\substack{j=1 \\ j \neq i}}^{n} \left[R_0 - (1 - s_j) R_j \right] x_j - \varphi_i R_0 x_i \right). \end{aligned} \tag{11}$$

This specific mean dynamic has Lotka-Volterra form $\dot{x}_i = x_i \left(a_i - \sum_{j=1}^{n} b_{ij} x_j \right)$ of competitive species with coefficients $a_i = \varphi_i R_0 - (1 - s_i) R_i$, $b_{ii} = \varphi_i R_0$, and for $j \neq i$, $b_{ij} = \varphi_i \left[R_0 - (1 - s_j) R_j \right]$.

All constants R_i, φ_i and s_i may in principle be time-dependent so that product modifications or fashion trends can be modeled. However, it is also possible to examine the long-term evolution of competing product classes and hence focus on the more fundamental consumer decision to buy an entity of one specific product class and not another. For a theoretical analysis, we shall first assume the coefficients to be constant.

3. Model analysis

3.1. Single good market

For the single good case the mean dynamic has the following form

$$\dot{x}_1 = x_1 \varphi_1 R_0 \left(\Psi - x_1 \right) \tag{12}$$

with $\Psi = 1 - \frac{R_1}{R_0} \frac{1 - s_1}{\varphi_1}$. This ordinary differential equation can be solved analytically. Using partial fraction and separation of variables the solution reads

$$x_1(t) = \frac{\Psi}{1 + \left(\frac{\Psi}{x_1(0)} - 1 \right) \exp\left[-\tau \Psi \varphi_1 \right]}, \tag{13}$$

with dimensionless time $\tau = t R_0$. The evolution of the population x_1 has a sigmoidal shape (differentiable, non-negative first derivative, exactly one inflection point) with initial exponential growth and saturation values Ψ or zero. A sketch of the curve can be found in figure 1.

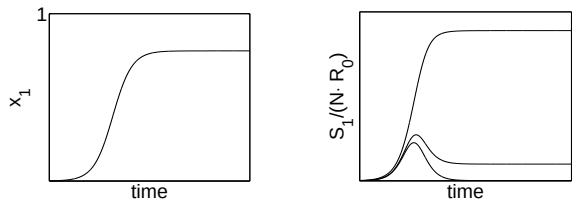

Figure 1: Sketch of a typical behavior of x_1 (left) and $\frac{S_1}{N \cdot R_0}$ (right) in time.

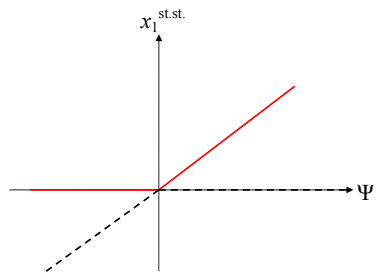

Figure 2: Transcritical bifurcation with stable (solid line) and unstable steady states (dashed line).

Definition 4 (Feasibility)**.** *A good is* feasible *if a positive number of consumers owns the product in the steady state.*

In this case, the number of sales is positive at least within a certain time period. Mathematically, feasibility of a good is determined by the following lemma.

Lemma 1. *The single product on the market is feasible iff* $\Psi > 0$.

The proof can be obtained by algebraically investigating the stability of the (unique) steady states $x_1 = 0$ and $x_1 = \Psi$ respectively.

Diagram 2 shows the stability of the steady states and a transcritical bifurcation:

1. $x_1 = 0$ is stable steady state if $\Psi < 0$
2. $x_1 = \Psi$ is stable steady state if $\Psi > 0$.

Hence, for a feasible good the steady state $x_1 = \Psi$ is stable.

We may assume that the initial condition $x_1(0) > 0$ is near zero. The sales equation for the single good market with a feasible good then takes the explicit

form

$$\frac{S_1(t)}{N \cdot R_0} = \frac{\Psi}{1 + \left(\frac{\Psi}{x_1(0)} - 1\right) \exp\left[-\tau \Psi \varphi_1\right]} \left[\frac{R_1}{R_0} + \varphi_1 \Psi - \frac{\varphi_1 \Psi}{1 + \left(\frac{\Psi}{x_1(0)} - 1\right) \exp\left[-\tau \Psi \varphi_1\right]}\right] \quad (14)$$

with saturation value

$$\lim_{t \to \infty} \frac{S_1(t)}{N \cdot R_0} = \Psi \frac{R_1}{R_0}. \quad (15)$$

Depending on $\frac{R_1}{R_0} \cdot \frac{2-s_1}{\varphi_1}$, the sales can take qualitatively different shapes corresponding to different patterns of monopoly product life cycles. For $\frac{R_1}{R_0} \cdot \frac{2-s_1}{\varphi_1} < 1$ we find an initial exponential growth, a maximum and then a decay with saturation. Intuitively, the stronger the effect of habituation or imitation (i. e. the more convincing the product is), the stronger is the increase of the subpopulation. After a maximum is reached however, the effect of product durability gains in importance, since the most purchases are replacements of broken products. Hence, the sales decrease and approach a saturation value. In the special case $\frac{R_1}{R_0} \cdot \frac{2-s_1}{\varphi_1} = 0$ of an infinitely durable good ($R_1 = 0$), the saturation value is zero since once the good is bought, the consumer owns it forever and does not purchase another unit of this product. For $\frac{R_1}{R_0} \cdot \frac{2-s_1}{\varphi_1} \geq 1$ the curve is sigmoidal. In this case habit and imitation cause an initial increase, then the good has to be replaced quite often and a saturation value is approached. The possible shapes are sketched in the right graph of figure 1.

S_1 is uniquely determined by three parameters, s_1, φ_1, and $\frac{R_1}{R_0}$, which together with $\frac{R_1}{R_0} \cdot \frac{2-s_1}{\varphi_1}$ lend themselves for a comparative statics analysis (figure 3). Generally, the time scale inherent in the sales evolution, decreases for rising φ_1, s_1, $\frac{R_1}{R_0}$, $\frac{R_1}{R_0} \cdot \frac{2-s_1}{\varphi_1}$, (i. e. the system becomes faster), and the saturation value increases, which can also be directly inferred from equation (15) and the mean dynamic (12). The effect is especially sensitive to changes in φ_1. The results are intuitive, since rising habit, imitation and replacement rates are indeed expected to speed up the system and to cause more frequent purchases.

A single product market can be a model of different firm constellations: Either a monopoly produces the good or an oligopoly, where all firms produce exactly the same good and the brands do not influence the consumers' decision; the consumers do not even differentiate between them by any means.

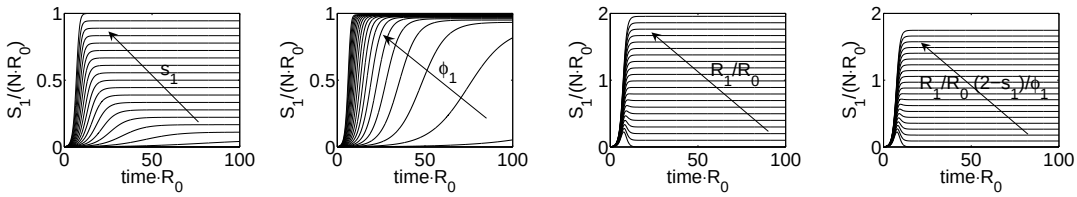

Figure 3: Comparative statics of $\frac{S_1}{N \cdot R_0}$ with respect to the parameters s_1, φ_1, $\frac{R_1}{R_0}$ and $\frac{R_1}{R_0} \cdot \frac{2-s_1}{\varphi_1}$. In each graph, the fixed parameters have the values $R_0 = R_1 = 1$, $x_1(0) = 0.001$, $\varphi_1 = 0.9$, $s_1 = 0.99$. The arrows indicate the variation of the curve for an increasing parameter value, where the parameter values of s_1 and φ_1 are varied from zero to one in steps of 0.05 and the values of $\frac{R_1}{R_0}$ and $\frac{R_1}{R_0} \cdot \frac{2-s_1}{\varphi_1}$ are varied from zero to two in steps of 0.1.

3.2. Two goods market

In the two good market the mean dynamic has the form

$$\dot{x}_i \quad = \quad x_i \varphi_i R_0 \left[\Psi_i - x_i - \Phi_j x_j \right], \; i = 1, 2, \; j \neq i, \tag{16}$$

where the $\Phi_j = 1 - (1 - s_j)\frac{R_j}{R_0}$ are the slopes of the nullclines (i. e. the lines along which $\dot{x}_i = 0$) in the corresponding phase plane (figure 4) and the $\Psi_i = 1 - \frac{R_i}{R_0}\frac{1-s_i}{\varphi_i}$ are the intersections of nullclines with the axes and thus represent the respective saturation values of the single good case. Obviously, the Ψ_i have to be positive for feasible goods. Phase planes are a very elegant representation of the dynamic system, in which the nullclines divide the state space into regions with $\dot{x}_i > 0$ and $\dot{x}_i < 0$. Steady states are given by the intersection points $x = (x_1, x_2)$ of nullclines $\dot{x}_1 = 0$ and $\dot{x}_2 = 0$. Obviously, in the two goods case there are four distinct steady states,

$$\mathbf{x}^A \quad = \quad \left(\frac{\Psi_1 - \Phi_2 \Psi_2}{1 - \Phi_1 \Phi_2}, \frac{\Psi_2 - \Phi_1 \Psi_1}{1 - \Phi_1 \Phi_2} \right), \;\; \mathbf{x}^B = (0, \Psi_2), \;\; \mathbf{x}^C = (\Psi_1, 0), \;\; \mathbf{x}^D = (0, 0).$$

$\mathbf{x}^D$, the trivial steady state, is instable. For a stability analysis of the other steady states two cases have to be distinguished, $\mathbf{x}^A \in \mathbb{R}^2_{\geq 0}$ and $\mathbf{x}^A \notin \mathbb{R}^2_{\geq 0}$ respectively (cf. figure 4). For the description of the time evolution of the populations x_1 and x_2 and the according product life cycles we may again assume $x_1(0), x_2(0) > 0$ to be near zero.

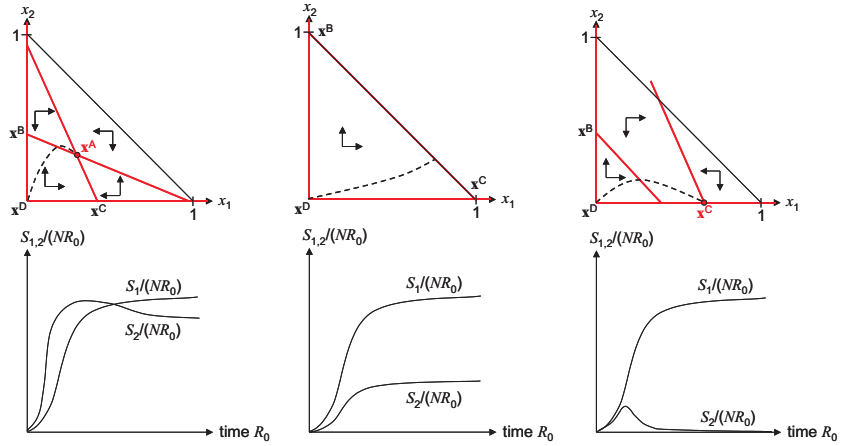

Figure 4: Phase planes for each of the three cases 1. a), 1. b), 2. (from left to right), with isoclines (solid lines) and one trajectory (dotted line) starting near zero each. The (asymptotically) stable steady states are indicated by filled circles. Resulting product life cycles are displayed beneath each diagram.

1. $\mathbf{x}^A \in \mathbb{R}^2_{\geq 0}$. This is possible if either a) the (equal) denominator of x_1^A and x_2^A is positive and the numerators are positive or b) the denominator and the numerators are all zero, i. e. the isoclines lie on top of each other.

   a) $1 > \Phi_1\Phi_2,\ \Psi_1 > \Phi_2\Psi_2,\ \Psi_2 > \Phi_1\Psi_1$ (figure 4 left). The only stable steady state is $\mathbf{x}^A$. Hence the products coexist lastingly on the market. Depending on the initial condition, x_1 or x_2 or none of them can have a maximum before approaching the saturation value; the form of the trajectory strongly depends on its starting point. For the product life cycles—similar to the single good case—a maximum may or may not exist depending on the parameter values. The number of products sold initially and overall strongly depends on the initial condition.

   b) $1 = \Phi_1\Phi_2,\ \Psi_1 = \Phi_2\Psi_2,\ \Psi_2 = \Phi_1\Psi_1$ (figure 4 middle). This case is possible if either $R_i = 0$ or $s_i = 1$ for $i = 1, 2$, i. e. for infinitely durable goods or perfect habituation. $\mathbf{x}^A$ does not exist (since the isoclines lie on top of each other); instead there is a continuous line of (not asymptotically) stable steady states. For infinitely durable (and feasible) goods $R_i = 0$, $i = 1, 2$, the population evolutions $x_1(t)$ and $x_2(t)$ have no maxima, while the product life cycles do reach a maximum

and then approach the saturation value zero. For completely satisfying goods, maxima may exist for the product life cycle depending on R_i $(i = 1, 2)$ but not for the $x_i(t)$.

2. $\mathbf{x}^A \notin \mathbb{R}^2_{\geq 0}$ (figure 4 right). Without loss of generality let $\Psi_1 > \Phi_2\Psi_2$ (otherwise simply renumber the species; $\Psi_1 < \Phi_2\Psi_2$ and $\Psi_2 < \Phi_1\Psi_1$ is not possible for $\Phi_i \leq 1$). Since either x_1^A or x_2^A has to be negative to prevent an intersection of nullclines in $\mathbb{R}^2_{\geq 0}$, this implies $1 = \Phi_1\Phi_2$ or $\Psi_2 < \Phi_1\Psi_1$. $\mathbf{x}^C$ is stable and $\mathbf{x}^B$ is instable (reverse for $\Psi_1 < \Phi_2\Psi_2$ and $\Psi_2 > \Phi_1\Psi_1$). Hence, one product (here product 2) dies out and the other product survives on the market. The qualitative form of the evolution of the population owning product 2 and the product life cycle for this product reaches a maximum and then approaches its saturation value zero. The shapes depend strongly on the initial condition. The population evolution of the first product has a sigmoidal form whereas the product life cycle can take different forms (sigmoidal or maximum and then saturation)—similar to the single good case.

The bifurcation diagram in figure 5 illustrates the parameter regions for the different cases. For the white region, $\Phi_1\Phi_2 > 1$ which is impossible. Region (1a) represents the area of coexistence of both products and corresponds to the case 1. a) above. The case 1. b) is illustrated by the point denoted (1b). The region (2a) corresponds to the 2. case where good two dies out and product one survives on the market. The reverse situation is represented by the area denoted (2b).

The previous analysis immediately implies the following lemma:

Lemma 2. *Product i is feasible on the two-goods market, iff $\Psi_i \geq \Phi_j\Psi_j$ $(i \neq j)$.*

From figure 4 it follows directly

$$x_i^{stable} \leq \Psi_i \leq x_1^{stable} + x_2^{stable}, \quad i = 1, 2, \tag{17}$$

where x_i^{stable} is the (or a) stable steady state of the system and Ψ_i is the saturation value of the single good case. Intuitively, this means that when a competing product appears on the market, the total distribution of both products together increases, while, of course, the distribution of the old product shrinks.

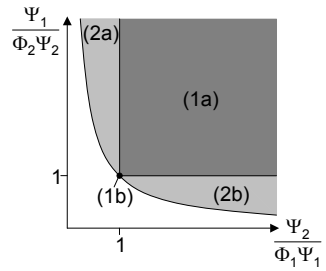

Figure 5: Bifurcation diagram. (1a) represents the coexistence case 1. a) of both products, (1b) is its special case 1. b) and (2a) is the 2. case where good 2 dies out and product 1 survives on the market and in (2b) the situation is reversed.

As in the single good case, a two product market can model different firm constellations: There might be a monopoly producing both goods, or the products are offered by an duopoly or even oligopoly, but with only two distinguishable goods. Equation (17) suggests that it might be beneficial for a monopoly to offer a variety of goods instead of just one, even for completely homogeneous consumers.

The extension to multi-goods markets, where the mean dynamic takes the form

$$\dot{x}_i = x_i \left(\varphi_i R_0 - (1 - s_i) R_i - \varphi_i \sum_{\substack{j=1 \\ j \neq i}}^{n} \left[R_0 - (1 - s_j) R_j \right] x_j - \varphi_i R_0 x_i \right) . \quad (18)$$

is canonic and will therefore be omitted in this paper.

3.3. Price competition

Sofar, we modeled the base framework of the consumers' behavior and the revision of their decisions. It is now interesting to examine the situation in which firms anticipate the consumers' behavior and set their prices accordingly. Hence, it remains to introduce prices in order to analyze these firms' price competition. Naturally, the imitation factor φ_i and habit coefficient s_i depend on the good prices, i. e. $\varphi_i = \varphi_i(\xi_1, \ldots, \xi_n)$, $s_i = s_i(\xi_1, \ldots, \xi_n)$, $i = 1, \ldots, n$, where prices are denoted by ξ_j. To keep things simple by minimizing the number of parameters while at the same time being quite realistic, we shall first assume φ_i and s_i to depend on ξ_i only.

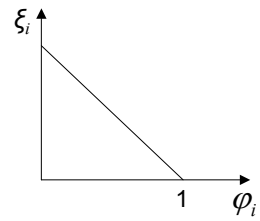

Figure 6: Linear form of the imitation function $\varphi_i(\xi_i)$.

Recall that the imitation function has the following interpretation: a consumer owning no good or switching product imitates the population of consumers owning good i with probability $\varphi_i(\xi_i)$ if the price of this product i is ξ_i. Equivalently, the fraction $\varphi_i(\xi_i)$ of the whole population would purchase good i at a price of ξ_i. Obviously, $\varphi_i(\xi_i)$ represents the normalized demand function of product i, or termed in probabilistic terms, $\varphi_i(\xi_i)$ is the demand distribution of product i. Hence, it is realistic to assume $\varphi_i(\xi_i)$ to decrease with rising ξ_i, i. e. $\varphi'(\xi_i) < 0$.

The imitation function can for instance take a linear form according to figure 6. We are now able to describe a competition normal form game of the firms, technically similar to Bertrand's model.

Definition 5 (Competition normal form game)**.** *The* competition normal form game *is a normal form game $G = (n, \mathfrak{S}, \Pi)$ with*

- *the number of agents n, being the number of firms, where the firms are understood to be characterized by $s_i(\xi_1, \ldots, \xi_n)$ and $\varphi_i(\xi_1, \ldots, \xi_n)$,*
- *the set $\mathfrak{S}$ of all possible strategy combinations with $\mathfrak{S} \subseteq [\mathbb{R}_+^{\mathbb{R}_+}]^n = \mathbb{R}_+^{\mathbb{R}_+} \times \ldots \times \mathbb{R}_+^{\mathbb{R}_+}$ (a subset of the space of n-tuples over maps $\xi_i : \mathbb{R}_+ \to \mathbb{R}_+$, $t \mapsto \xi_i(t)$, where $\xi_i(t)$ denotes the price of good i at time t),*
- *the utility function $\Pi : \mathfrak{S} \to \mathbb{R}_+^n$, $\Pi_i(\xi_1, \ldots, \xi_n) = F\left[(\xi_i(t) - c_i)S_i(\vec{s}(\vec{\xi}(t)), \vec{\varphi}(\vec{\xi}(t)), t)\right]$, being the firm's profit, where c_i denotes the marginal cost of production for good i and $\vec{S}(\vec{s}, \vec{\varphi}(\vec{\xi}(t)), t)$ is the sales vector according to the modified population game $\mathcal{G} = (N, \mathcal{A}, \rho)$ with ρ defined according to equations (6), (7), (8), (9) and (10). The operator F may assume different forms.*

The imitation and habit function $\vec{\varphi}(\vec{\xi})$ and $\vec{s}(\vec{\xi})$ may in general be time dependent. The operator F can e. g. yield the *steady state profit* for a time-invariant price,

that if for some function $\pi(t)$

$$F\left[\pi(t)\right] = \lim_{t\to\infty} \pi(t),$$

where $\pi(t)$ is meant to be the firm's profit at time t, or the *cumulated discounted profit* over a certain time period

$$F_T\left[\pi(t)\right] = \int_0^T \exp[-rt]\pi(t)dt.$$

The steady state profit is for example of interest, when the firms would like to validate their prices in an equilibrated market.

Note, that in definition 5 we allow for a time dependent price. The firms set their price paths initially and then follow this path. This strategy contains no disadvantage, since the consumer model is deterministic anyway, but as a next step, the competition normal form game could be extended to a dynamic game, which allows the firms to revise their price paths.

As an example, let us assume the case of fixed prices (e. g. an equilibrated market or other price restrictions). Mathematically, this means that $\mathfrak{S} = \mathbb{R}_+^n$ is the set of all n-tuples (representing constant functions $\xi_i(t) = \xi_i$). A *monopoly price* can now be obtained by calculating the Nash equilibrium for given $\varphi_1(\xi_1)$, $s_1(\xi_1)$ using the steady state profit (cf. example 1). Analogously, the Nash equilibrium for a system of n firms yields *oligopoly prices* (cf. example 2).

Example 1 (Steady state monopoly price)**.** *Consider the following normal form game:*

$$G = \left(1, \mathbb{R}_+, \lim_{t\to\infty}(\xi_1 - c_1)S_1(s_1, \varphi_1(\xi_1), t)\right),$$

with $\mathfrak{S} = \mathbb{R}_+$ here representing the set of constant functions in $\mathbb{R}_+$, i. e. we consider constant prices ξ_1 only. Moreover, take the imitation function $\varphi_1(\xi_1) = 1 - \frac{1}{\bar{\xi}_1}\xi_1$, where $\bar{\xi}_1 \geq c_1$ is the maximum reservation price, and a constant habit function s_1. The strategy of the unique Nash-equilibrium can be calculated by solving

$$\max_{\xi_1}\left[\lim_{t\to\infty}(\xi_1 - c_1)S_1(s_1, \varphi_1(\xi_1), t)\right]$$

and is given by $\xi_1^\infty = \bar{\xi}_1 - \sqrt{\frac{R_1}{R_0}(1-s_1)\bar{\xi}_1(\bar{\xi}_1 - c_1)}$. For $\xi_1^\infty < c_1$ the firm is recommended not to produce. The product therefore is feasible, iff $c_1 \leq \bar{\xi}_1\left[1 - \frac{R_1}{R_0}(1-s_1)\right]$, which is just a different representation of Lemma 1.

Of course, the constant s_1 in the above example is a very crude approximation (though reasonable, if a certain range of prices or not exceeded), since then, obviously, the monopolist could gain infinite profit by suddenly charging infinite prices (which the consumers will pay due to their habit). Hence, when looking at time dependent prices, one should for example set $s_1(\xi_1) = \varphi_1(\xi_1)$. If the price is allowed to vary, one might think that over the long-term for instance an oscillating price (e. g. a high and a low level price, alternating with fixed time periods) yields more profit than a constant price, however, we indeed find the special case of a fixed price leading to the maximum profit for the monopolist (for $s(\xi) = \varphi(\xi)$).

Example 2 (Steady state oligopoly prices)**.** *Consider the same situation as in example 1, only for the general oligopoly case and this time with* $\varphi_i = \frac{1}{1+\frac{\xi_i}{\bar{\xi}_i}}$ *(this choice renders the system analytically solvable and is a sufficient approximation to a linear* $\varphi_i(\xi_i)$ *for low prices). Hence, the following normal form game is considered:*

$$G = \left(n, \mathbb{R}^n_+, \lim_{t\to\infty} \left((\xi_i - c_i)S_i(\vec{s}, \vec{\varphi}(\vec{\xi}), t)\right)_{i=1,\dots,n}\right).$$

For the case that all n *products are feasible appendix A.1 shows the optimal steady state oligopoly price to be given by*

$$\begin{aligned}\vec{\xi}^{\infty} &= \begin{pmatrix} \frac{\bar{\xi}_1}{a_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{\bar{\xi}_n}{a_n} \end{pmatrix} \left[A^{-1} + \begin{pmatrix} \frac{\det(B_{11})}{\det(A)} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{\det(B_{nn})}{\det(A)} \end{pmatrix} \right]^{-1} \\ &\quad \cdot \left[\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} + \begin{pmatrix} a_1 \frac{\det(B_{11})}{\det(A)}(1 + \frac{c_1}{\xi_1}) \\ \vdots \\ a_n \frac{\det(B_{nn})}{\det(A)}(1 + \frac{c_n}{\xi_n}) \end{pmatrix} \right], \end{aligned} \tag{19}$$

with $a_i := \frac{(1-s_i)R_i}{R_0}$, $i = 1, \dots, n$,

$$A_{ij} := \begin{cases} 1, & j = i \\ 1 - a_j, & j \neq i \end{cases},$$

and B_{ij} *being matrix* A *with the* j*th column replaced by the* i*th Cartesian unit vector.*

Lemma 3. *Let the assumptions of example 2 hold. In the case of a symmetric oligopoly with identical firms, where each firm optimally chooses the same price $\xi^{\infty,n}$, for an increasing number of firms the price decreases. For a perfect competition ($n \to \infty$) the price converges to the marginal cost c.*

Proof. *Define $a = \frac{(1-s)R}{R_0}$. Then (19) yields*

$$\xi^{\infty,n} = \frac{\bar{\xi} - a\bar{\xi} + (n-1)c - (n-2)ca}{(n-1) - (n-3)a},$$

from which we directly imply

$$\lim_{n\to\infty} \xi^{\infty,n} = c.$$

Furthermore,

$$\begin{aligned}
\frac{\xi^{\infty,n+1}}{\xi^{\infty,n}} &= \frac{n + \left(\frac{\bar{\xi}}{c} + \frac{a}{1-a}\right)}{n - 1 + \underbrace{\left(\frac{\bar{\xi}}{c} + \frac{a}{1-a}\right)}_{=:C_1>0}} \cdot \frac{n + \frac{3a-1}{1-a}}{n + 1 + \underbrace{\frac{3a-1}{1-a}}_{=:C_2\geq -1}} \\
&= \frac{(n+C_1)(n+C_2)}{\underbrace{(n+C_1)}_{\geq 1}\underbrace{(n+C_2)}_{\geq 0} + C_1 - C_2 - 1} < 1 \\
&\Leftrightarrow \quad C_1 - C_2 - 1 > 0 \\
&\Leftrightarrow \quad \Psi(\xi = c) > 0,
\end{aligned}$$

which is true for feasible products. □

For both examples considered so far it was possible to get analytical insight—at least for a certain form of $\varphi_i(\xi_i)$. For F being the cumulated discounted profit, the optimality system cannot be solved analytically. Nevertheless, for the monopoly case, a qualitative insight into the problem can be gained. Let us turn to this problem in example 3.

Example 3 (Cumulated discounted profit in a monopoly setting)**.** *For simplicity, let us only allow for linear price functions $\xi_1(t) \in \mathfrak{L}([0,T]) := \{f : [0,T] \to \mathbb{R} | \exists a, b : f(t) = a + bt\}$. Consider the competition game*

$$G = \left(1, \mathfrak{L}([0,T]), \int_0^T \exp[-rt](\xi_1(t) - c_1) S_1\left(s_1(\xi_1(t)), \varphi_1(\xi_1(t)), t\right) dt\right),$$

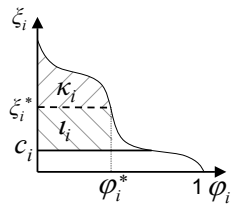

Figure 7: The consumer surplus equals $CS_i = \frac{\kappa_i}{\varphi_i^*} S_i$ and the producer surplus is given by $PS_i = \frac{\iota_i}{\varphi_i^*} S_i$.

then the optimal price $\xi_1(t)$ can be found numerically. For example, we obtain for certain parameter values that the product is initially sold below marginal costs, and then the price rises.

Note, that for the monopoly case the situation is deterministic, i. e. the single firm has no incentive to change the price path at any time. Hence, even if the monopolist would be able to change the price within the period, the monopolist would not do so, unless an exterior event changes the market. Of course the monopolist could then change the price path for the next period starting from that moment on.

Let us finally define a welfare within our setting to allow for theories of social implications. The producer surplus can be calculated as usual for the Marshallian definition, whereas the consumer surplus has to be obtained differently as a consequence of the special consumer behavior.

Definition 6 (Welfare)**.** *Let the imitation function for good i be $\varphi_i(\xi_i)$ with $\varphi_i'(\xi_i) \leq 0$. Consumer and producer surplus at time t can then be calculated as*

$$PS_i(t) = (\xi_i(t) - c_i) S_i(t) \tag{20}$$

$$CS_i(t) = \frac{S_i}{\varphi_i(\xi_i(t))} \left(\int_0^{\varphi_i(\xi_i(t))} \left[\varphi_i^{-1}(\varphi) - \xi_i(t) \right] d\varphi \right). \tag{21}$$

Hence, the welfare takes the form

$$W(t) = \sum_{i=1}^{n} PS_i(t) + CS_i(t) \tag{22}$$

For an arbitrary imitation function, consumer and producer surplus can be illustrated as in figure 7. A motivation is as follows:

The imitation function $\varphi_i(\xi_i)$ is the demand distribution for good i. In other words, for a consumer the probability to have a reservation price RP for good i larger than or equal to some price ξ_i^* is given by the demand distribution, i. e.

$$\varphi_i(\xi_i^*) = P[RP \geq \xi_i^*].$$

Phrased differently, the imitation function $\varphi_i(\xi_i)$ constitutes the probability distribution of the reservation price ξ_i of a set of heterogenous consumers and the reservation price ξ_i is distributed according to the density $-\varphi_i'(\xi_i)$. Hence, the expected value of the utility per consumer U_i is given by

$$E[U_i] \quad = \quad \int_{\xi_i^*}^{\infty} -\varphi_i(\xi_i)(\xi_i - \xi_i^*)d\xi_i = \int_{\xi_i^*}^{\infty} \varphi_i(\xi_i)d\xi_i.$$

The expected value of the utility per purchased product u_i is thus equal to

$$E[u_i] = \frac{E[U_i]}{\varphi_i^*}.$$

Finally, the expected consumer surplus is then given by

$$CS_i = E[u_i]S_i = \frac{E[U_i]}{\varphi_i^*}S_i.$$

We therefore have chosen a definition similar to the standard approach: CS_i is equal to the difference between average reservation price among all buyers and the actual price.

3.4. Product life cycles and results from the model

In this section we briefly illustrate how a realistic product life cycle may emerge from our model. As a simple example consider a consecutive introduction of many products, all competing with each other. Consider the market of mobile phones, where new (innovative) phones frequently enter the market. The maximum reservation price for a product may be assumed highest at its introduction on the market when it still represents the state of the art and then it decreases in time, as innovation goes on. Hence, also habit and imitation function are highest at the time of product introduction. The most simple setting is to assume fixed prices ξ_i, simple imitation and habit functions, e. g. $\varphi_i(\xi) = 1 - \frac{\xi}{\xi_i}$, $s_i(\xi) = 1 - \frac{\xi}{\xi_i}$,

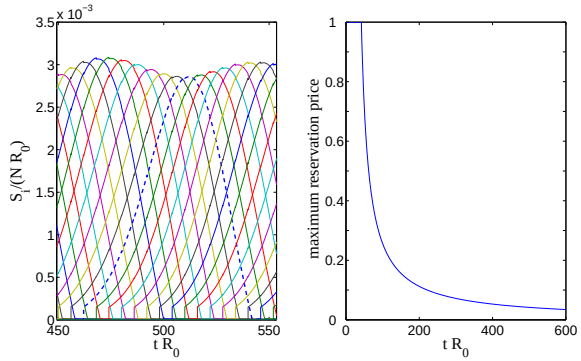

Figure 8: Product life cycles when many products enter the market successively (left) and the maximum reservation price $\bar{\xi}_i$ is decreasing in time (right). The habit and imitation functions depend on the price according to $s(\xi_i) = \varphi(\xi_i) = 1 - \frac{2}{\bar{\xi}_i}\xi_i$. The following parameter values were used: time interval between introduction of products $T = \frac{6}{R_0}$, alarm clock rate of the goods $R_i = 0.025R_0$, initial subpopulation $x_0 = 10^{-3}$, product prices $\xi_i = 0.1\bar{\xi}$, maximum reservation price $\bar{\xi}_i = \frac{\bar{\xi}}{1+\alpha(t-t_i)}$.

new product introductions equally distributed over time, and a simple evolution of the maximum reservation price $\bar{\xi}_i$ with time, e. g. $\bar{\xi}_i = \frac{\bar{\xi}}{1+\alpha(t-t_i)}$, where t_i is the time of introduction of product i. Figure 8 shows the resulting product life cycles of the successively introduced products.

These obtained form of product life cycles have a very similar form to the empirically examined product life cycles for industrial component products in two major markets as presented in Kluyver (1977) and other empirical work.

4. Conclusion

We examined the evolution of a consumer market, where non-rational consumers follow a rule of thumb, basing their decisions on imitation and habit. To achieve this goal we set up a population game with a corresponding revision protocol as framework, we then examined strategic pricing for a monopoly and an oligopoly when demand follows these rules of thumb and investigated product feasibility under these conditions. The assumed non-rational consumer behavior lead to observed market patterns such as product life cycles, and the introduced method-

ology allows for broad applications and qualitative theoretical analysis. Moreover and finally, we proposed a definition of an adequate welfare within the model.

Acknowledgement

For many valuable comments we are grateful to Frank Riedel, Paul Heidhues, Timon Tatur, Tymofiy Mylovanov, Johannes Münster and all seminar participants of the BGSE Micro Workshop at the University of Bonn.

A. Appendix

A.1. Solution of example 2

We use the same notation a_i, A and B_{ij} for $i, j = 1, \ldots, n$. Under the assumption of coexistence of all n goods in the steady state equation (18) can be transformed into

$$\underbrace{\begin{pmatrix} 1 - \frac{a_1}{\varphi_1} \\ \vdots \\ 1 - \frac{a_n}{\varphi_n} \end{pmatrix}}_{=:\vec{b}} = A \cdot \underbrace{\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}}_{=:\vec{x}}$$

The sales $\vec{S}$ and the profit $\vec{\pi}$ then assume the following form:

$$\vec{S} = \underbrace{\begin{pmatrix} R_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & R_n \end{pmatrix}}_{=:R} A^{-1}\vec{b} \text{ and } \vec{\pi} = \left[\underbrace{\begin{pmatrix} \xi_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \xi_n \end{pmatrix}}_{=:\Xi} - \underbrace{\begin{pmatrix} c_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & c_n \end{pmatrix}}_{=:C} \right] RA^{-1}\vec{b}.$$

Letting M_i denote the ith row of Matrix M, the Nash-equilibrium is found by

solving

$$\frac{d\pi_i}{d\xi_i} = R_i \left(A^{-1}\right)_i \left[\vec{b} + (\xi_i - c_i) \underbrace{\begin{pmatrix} 0 \\ \vdots \\ \frac{a_i}{\varphi_i^2} \\ 0 \\ \vdots \end{pmatrix}}_{=:\frac{d\vec{b}}{d\xi_i}} \frac{d\varphi_i}{d\xi_i} \right] = 0 \quad i = 1, \ldots, n.$$

For the system to be analytically solvable, assume $\varphi_i = \frac{1}{1+\frac{\xi_i}{\xi_i}}$ and hence $\frac{d\varphi_i}{d\xi_i} = -\frac{\varphi_i^2}{\xi_i}$. Then, for $i = 1, \ldots, n$,

$$R_i \left(A^{-1}\right)_i \left[\begin{pmatrix} 1 - a_1(1 + \frac{\xi_1}{\xi_1}) \\ \vdots \\ 1 - a_n(1 + \frac{\xi_n}{\xi_n}) \end{pmatrix} - (\xi_i - c_i) \begin{pmatrix} 0 \\ \vdots \\ \frac{a_i}{\xi_i} \\ 0 \\ \vdots \end{pmatrix} \right] = 0$$

$$\Leftrightarrow \det(A)(A^{-1})_{i:} \begin{pmatrix} 1 - \frac{a_1}{\xi_1}(\bar{\xi}_1 + \xi_1) \\ \vdots \\ 1 - \frac{a_i}{\xi_i}(\bar{\xi}_i - c_i + 2\xi_i) \\ 1 - \frac{a_{i+1}}{\xi_{i+1}}(\bar{\xi}_{i+1} + \xi_{i+1}) \\ \vdots \end{pmatrix} = 0.$$

Since $(A^{-1})_{ij} = \frac{\det(B_{ij})}{\det(A)}$, this is equivalent to

$$\sum_{j=1}^{n} \det(B_{ij}) \left(1 - a_j - \frac{a_j}{\bar{\xi}_j}\xi_j\right) - \det(B_{ii}) \left(\frac{a_i}{\bar{\xi}_i}\xi_i - \frac{a_i c_i}{\bar{\xi}_i}\right) = 0, \quad i = 1, \ldots, n.$$

According to Laplace, $\det(A) = \sum_{j=1}^{n} (A)_{ij} \det(B_{ij})$, and the equation becomes

$$\det(A) \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} + \begin{pmatrix} a_1 \det(B_{11}) \left(1 + \frac{c_1}{\xi_1}\right) \\ \vdots \\ a_n \det(B_{nn}) \left(1 + \frac{c_n}{\xi_n}\right) \end{pmatrix}$$

$$= \left[\underbrace{\begin{pmatrix} \det(B_{11}) & \cdots & \det(B_{1n}) \\ \vdots & & \vdots \\ \det(B_{n1}) & \cdots & \det(B_{nn}) \end{pmatrix}}_{=\det(A)A^{-1}} + \begin{pmatrix} \det(B_{11}) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \det(B_{nn}) \end{pmatrix} \right] \begin{pmatrix} \frac{a_1}{\xi_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{a_n}{\xi_n} \end{pmatrix} \cdot \vec{\xi}$$

Finally, the price solution reads as given in example 2.

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