Cue-Triggered Addiction and Natural Recovery

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Abstract

We propose a model of natural recovery, a widespread yet unexplained aspect of addictive behavior, starting from the recent theory developed by Bernheim and Rangel (2004). While the Bernheim and Rangel model generates many distinctive patterns of addiction, it does not explicitly consider pathways to natural recovery. Based on insights from the neurosciences, we introduce an "implicit cognitive appraisal" process depending on past experiences as well as on future expected consequences of addictive consumption. Such function affects the individual in two ways: it erodes the payoff from use as the decision maker grows older and it increases the cognitive control competing with the hedonic impulses to use, thus reducing the probability of making mistakes. While we do recognize the importance of allowing for cue-triggered mistakes in individual decision making, our model recovers an important role for cognitive processes, such as subjective cost-benefit evaluations, in explaining natural recovery.

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isn’t it remarkable that the behavior of even reasonably intelligent individuals can be as idiosyncratic, seemingly irrational, and sometimes patently counterproductive as it often appears to be?...

Cohen and Blum (2002)

1 Introduction

Addiction is currently defined as the consequence of repeated use of psychoactive drugs. It is characterized principally by a loss of control over drug seeking behavior with harmful effects on the individual and a high probability of relapse even months or years after cessation of drug taking (Volkow and Fowler, 2000; Kelley, 2004; Weiss, 2005).

The main problem is to understand how this phenomenon “moves”, meaning how the various components of its multifactoriality (individual, substance and environment) can trigger the start, sustain recurrence or generate the frustrating relapse.

Recent economic theories of addiction can be loosely classified as variations or generalizations of the rational addiction model by Becker and Murphy (1988).

Such generalizations allow for the presence of random cues that increase the marginal utility of consumption (Laibson, 2001); “projection bias” (Loewenstein et al., 2003); present-biased preferences and sophisticated or naive expectations (Gruber and Koszegi, 2001); “temptation” (Gul and Pesendorfer, 2001) where well-being depends not only upon the chosen action, but also upon the action not chosen.

The Bernheim and Rangel (B&R) 2004 model instead regards addiction as a progressive susceptibility to stochastic environmental cues that can trigger mistaken usage. The points of departure of this theory from previous ones are: first, its attempt to harmonize economic theory with evidence from psychology, the neurosciences and clinical practice so as to explain the relationship between behavior and the characteristics of the user, substance and the environment mentioned above (B&R, 2004, p.1559); second, the perception of consumption of addictive substances as mistakes. Neuroscience and clinical practice have indeed shown that addictive substances systematically interfere with the proper operation of a process used by the brain to forecast near term hedonic rewards and lead to strong impulses towards consumption that may interfere with higher cognitive control. In this case individual consumption choices are sometimes driven by a rational decision making process, sometimes by strong impulses leading to mistakes, i.e. to a divergence between choices and preferences.

The B&R 2004 model can explain several patterns of addictive behavior, but there is one aspect left unexplained which is spontaneous remittance also known
as natural recovery. Clinical practice shows that this is not an infrequent pattern of behavior in long term addicts, but the reasons for it are still to be recovered. The main purpose of this paper is to extend the B&R 2004 model to explain natural recovery. We introduce in the original framework an “Implicit Cognitive Appraisal” function, depending on past experiences as well as on future consequences of addictive consumption. Such function affects the individual in two ways: it erodes the payoff from use as the decision maker grows older and it increases the cognitive control competing with the hedonic impulses to use, thus reducing the probability of entering the hot mode. Performance analysis of the extended model is then carried out by comparing our results to those of the original model.

The remaining of the paper is structured as follows. Section 2 provides a clinical description of addiction. In section 3 we introduce the phenomenon of natural recovery. Section 4 reports the basic formulation of the B&R model of addiction while section 5 develops the extended model. Policy implications are drawn in Section 6. Concluding remarks and directions for future research are discussed in Section 7.

2 The neuroscience of addictive behavior

In human beings drugs produce an increase of dopamine concentration at target-cells’ receptor levels, as they stimulate the nigrostriatal (controlling motor coordination) and corticolimbic (controlling emotions and cognitive abilities) dopaminergic systems (Wise, 2004).

These cerebral systems have evolved not to entertain addictive substances, but to ensure the survival of the individual by controlling basic functions such as mating, consumption, searching for food and water, etc. Once these systems are engaged by natural rewards (food or sex for example) or by addictive substances (Kelley, 2004; Nestler, 2005) dopamine release in the nucleus accumbens and in other cerebral sites increases, causing specific emotional states (for example, euphoria) that are powerful drivers and reinforce that behavior. Addictive substances have an advantage over natural rewards: they produce a higher dopamine concentration by stimulating the (wanting) system more powerfully and for longer periods (Hyman, 2005). The individual is thus induced to repeat such positive experiences (or avoid them when negative), precisely because he associates the specific function to its hedonic responses (Kelley and Berridge, 2002; Bechara, 2005; Kalivas and Volkow, 2005).

However, there is another important difference between natural rewards and drugs of abuse. In the case of natural rewards, a habit develops after repeated use which reduces the importance of the experiential act. In other words the quality and quantity of the gained pleasure diminishes. On the contrary, addictive substances activate each time a similar hedonic response (liking system, Berridge & Robinson, 2003).

Chronic substance abuse induces profound alterations of the cerebral mechanisms just mentioned which “force”, in a way, the user to make compulsory
choices, i.e. choices that diverge from preferences. In fact, drugs by powerfully activating dopaminergic transmission reinforce excessively the associated learning process, ending up by constraining the individual’s behavioral choices (Berke and Hyman, 2000). In other words, drugs seem to affect the basic forecasting mechanism, a simple system for learning correlations between current conditions, decisions and short term rewards. As Bernheim and Rangel put it (B&R, 2005, p. 109) “this basic forecasting mechanism is very fast and efficient at learning simple action-reward correlations, but it’s inflexible and unsophisticated because it can only learn about a limited range of near-term consequences. Higher cognition is more flexible and sophisticated, but it is comparatively slow.”

They call this process Hedonic Forecasting Mechanism (HFM henceforth). Apparently there is a consensus in the neurosciences according to which addiction results from the impact the addictive substances have on the HFM. With repeated use of a substance, the cues associated with past consumption cause the HFM to forecast exaggerated pleasure responses, creating a disproportionate impulse to use leading to mistakes in decision making (B&R, 2005).

The pleasure following use, the excessive and rapid hedonic expectation induced by the HFM, the progressive failing of the frontal cortex to counterbalance with rational choices the more alluring offer of drugs, all portray a process that invariably regenerates itself and seems to have no end (Kelley and Berridge, 2002; Berridge, 2004). Although drug addiction seems to lead to just one possible result, for still unclear reasons, often the patient stops participating in the ineluctable dynamics of her/his case and ceases to have this insatiable hunger and compulsion for the drug.

This may happen as a consequence of psychological, social, pharmacological and individual interactions as well as all other stimuli found inside and around an individual (deterministic or even stochastic events).

In more general terms, one could say that the multifactoriality sustaining drug addiction sometimes ceases to offer those profits or conveniences considered up till then as indispensable.

3 Natural Recovery

Epidemiological studies, considering pathways out of alcohol abuse without the utilization of professional help (otherwise known as natural recovery), give evidence that the majority of quitting taking place without professional assistance in various countries reveal rates between 66.7% in Germany to 77% in Canada (Bischof et al., 2003). Other studies reveal that although there may be differences in the ways in which it occurs, spontaneous remittance characterizes the whole spectrum of drugs such as alcohol (Cunningham et al. 2006; Bischof et al. 2000; Weisner et al. 2003; Matzger et al. 2005; Bischof et al., 2003), marijuana (Coppersino et al., 2006), multiple drugs, heroin (Waldorf and Biernacki, 1979), binge eating, smoking, sex and gambling (Hanninen et al., 1999) and that the rate of
natural recovery may be much higher than conventional wisdom would suggest.

Despite this striking evidence, natural recovery remains basically an unexplained phenomenon even though it is of interest to different major disciplines, such as economics, psychology and sociology. Natural recovery may occur in at least three different ways: (i) cold turkey quitting due to an exogenous shock; (ii) cold turkey quitting happening without an exogenous shock; (iii) gradual quitting occurring after a period of continuous decrease in consumption. We are particularly interested in case (ii) and argue that quitting is the explicit manifestation of an inner process of "self appraisal" that ultimately brings to quitting consumption of the addictive substance. Such process is also the main determinant of quitting in case (i) where the role played by the exogenous shock is that of a strong incentive accelerating a mechanism that, however, has already begun to develop within the addict. Case (iii) is of minor interest since clinical practice suggests only a minority of natural recoveries happening in this way.

Clinical and experimental research have studied natural recovery from substance abuse since the mid-1970s (Vaillant, 1982; Klingemann, 1991) focusing on triggering mechanisms, maintenance factors and on trying to identify common reasons for change in substance use. To our knowledge, however, there are very few studies describing pathways to natural recovery in an economic model of addiction. Among such studies the Becker & Murphy (1988) model of addictive behavior generates cold turkey quitting through exogenous shocks or stressful events, whereas Suranovic et al. (1999) extend the Becker model to generate cold turkey quitting of cigarettes’ smoking without relying on exogenous shocks or stressful events. The motivation to quit is based instead on changes in the addict’s perspective as he grows older. In addition, this model also shows that some individuals may quit addiction by gradually reducing consumption over time. These key results are obtained by explicitly taking into account the withdrawal effects (quitting costs) experienced when users try to quit and by explicit recognition that the negative health effects of addiction generally appear late in an individual’s life.

Both models are rational models of behavior and presuppose a standard inter temporal decision making implying a complete alignment of choices and time consistent preferences, thereby denying the possibility of mistakes.

However, recent studies in the neurosciences support the view that the consumption of addictive substances can be sometimes rational and sometimes a cue-triggered mistake (B&R, 2005). These insights from neuroscience have led to a new economic theory of addiction that tries to bridge the gap between neuroscience and decision making and depicts addiction as a progressive susceptibility to stochastic environmental cues triggering mistaken usage.

This theory generates many distinctive behavioral patterns of addiction requiring explanation, but it does not explicitly model pathways to natural recovery. The authors, however, hint at possibilities to extend their model including "developing a more complete model of cognitive control in which future consequences may influence the likelihood of overriding HFM-generated impulses (through the threshold \( M^T \))" (see B&R, 2004, p. 1582).
Our main purpose is to explain how natural recovery may occur. To this aim we extend the B&R model introducing a Loss Function that affects the individual in two ways: it erodes the payoff from use as the decision maker (DM) grows older; it rises the threshold $M^f$ so as to increase the cognitive control competing with the hedonic forecasting mechanism (HFM), thus reducing the probability of making mistakes. The key process involves a mechanism that we call “Implicit Cognitive Appraisal” function incorporating future expected losses as well as past experiences from addiction.

Our model may have important policy implications, because it places a high value on measures that increase the likelihood of successful self-regulation without forcing particular choices and eventually leading to natural recovery. It also strengthens the role for “cognitive” policies (B&R, 2005, p. 136), i.e. those increasing cognitive control such as education, creation of counter cues and policies that help the accumulation of social capital.

3.1 Reasons for spontaneous quitting

Matzger et al. (2005), in a study of the reasons for drinking less, assess that triggering mechanisms, the interpersonal and environmental influences that cause a person to move from problematic alcohol use to sustained abstinence or non-problematic use, can be varied and multidimensional and often involve a combination of both short and long term pressures. In this study two groups of problem drinking adults, who reported drinking less at the one year follow up, were identified in Northern California: the first group came from a probability sample in the general population; the second was originated through a survey of consecutive admissions to public and private alcohol and drug problems. A logit model was then used to assess the determinants of sustained remission from problem drinking. Results showed that the two most frequently endorsed reasons for cutting down were the same for both groups: self-evaluation, i.e. weighing the pros and cons of drinking and not drinking and experiencing a major change in lifestyle. Drinking causing health problems was also an important reason for quitting. Self-evaluation implies that recovery is not necessarily triggered by negative or traumatic events, but alternatively comes about through a period of self-reflection. Interventions by medical personnel and family members were either non-significant predictors or significantly negatively related to sustained improvement for both the general population and treated drinkers.

Cunningham et al. (2005) give support to both the “cognitive appraisal” and the “life events” motivations for quitting. They noted that individuals who recovered without treatment went through a process of cognitive appraisal (also known as the motivational explanation for quitting) in which they weighed the pros and cons of drinking and not drinking and decided that the pros outweighed the cons. Anticipated costs and benefits of change is thus one means of measuring the respondents’ motivational explanation for quitting. The “life events” motivation is instead based on past life events. It is hypothesized that addicts’ life events prior
to and after their quit attempt are related to successful quitting attempts. Respondents experiencing the greatest reduction in their negative life events pre to post quit attempts were hypothesized to be most likely to have successfully reduced or quit their addiction.

Reasons for quitting may vary according to the substance of abuse and the addict’s age. Copersino et al. (2006) report that reasons for quitting marijuana by the adults are different from those reported by adolescents. This is important, because individual reasons for quitting may influence the success and duration of the quit attempt. However, the reasons for quitting marijuana reported by the adults are more similar to the reasons given by spontaneous quitters from most licit and illicit substances¹, i.e. concerns about the negative impact on one’s health and on self and social image.

4 The Bernheim & Rangel Model of Addiction

The following analysis is related to the behavioral addiction model developed by Bernheim and Rangel (2004)². The B&R theory is based on the following premises: a) consumption among addicts is frequently a mistake; b) previous experience with an addictive good sensitizes an individual to environmental cues that trigger mistaken usage; c) awareness of sensitivity to cue-triggered mistakes produces attempts to manage the process with some degree of sophistication. While the second and third premises are also present in other recent models of addictive consumption, such as Laibson’s Cue-Theory of Consumption (2001) for instance, the perception of consumption of addictive substances as mistakes is an original feature of the B&R model. This stems from recent advances in neurosciences stressing that addictive substances are different from others in the way they interfere with the normal operation of the brain. The combination of these two factors, i.e. a rather precise understanding of the alterations of the cerebral mechanisms produced by addictive substances and the perception of consumption as a mistake which follows represents an advancement with respect to other economic models of addiction.

The model involves a decision maker (DM) living for an infinite number of discrete periods who can operate either in a cold (involving rationality) or hot mode (where decisions and preferences may diverge). Time is discrete, indexed by the nonnegative integers, \( t \in \{0, 1, 2, \ldots\} \). In each time period, \( t \), the DM makes two decisions in succession. First, he selects a lifestyle \( a \) from the set \{E, A, R\} (e.g. going to a bar or staying at home watching TV or reading a book). If lifestyle \( E \), "exposure", is chosen there is a high likelihood that the DM will encounter a large number of substance-related cues. Activity \( A \), "avoidance", entails fewer

¹Except for cigarettes smokers who report quitting more because of feelings of disgust and the desire and will to quit.
²The model described in this Section is based on the theory developed by Bernheim and Rangel, while functional forms and parameters’ values are chosen by the authors.
substance-related cues and may also reduce sensitivity to environmental cues. Activity $R$, "rehabilitation", implies a commitment to clinical treatment, the cost of which is $r_s$, and it may further reduce exposure and sensitivity to substance-related cues.

Second, he allocates resources between a potentially addictive good/substance, $x \in \{0, 1\}$, the price of which is $q$, and a non addictive good ($e_s \geq 0$). By assumption the DM can not borrow or save. Each period is entered in cold mode and the DM chooses his lifestyle rationally. This choice, along with the addictive state, $s$, determines the probability $p_s^E$ with which he encounters cues that trigger the hot mode (see below). With some transition probability $p_T$, consumption of the addictive substance in state $s$ at time $t$ moves the individual to a higher addictive state, $s + 1$ at time $t + 1$, and abstention moves him to a lower addictive state $s - 1$ at time $t + 1$. There are $S + 1$ addictive states labeled $s = 0, 1, ..., S$. The system dynamics is described by the evolution of state $s_t$ according to the following equation:

$$s_{t+1} = \begin{cases} \min[p_T (s_t + 1) + (1 - p_T)s_t, S] & \text{if } x_t = 1, a_t \in \{E, A\} \\ \max[1, p_T (s_t - 1) + (1 - p_T)s_t] & \text{if } x_t = 0, a_t \in \{E, A, R\} \end{cases}$$

Equation (1) implies that usage in state $s$ leads to state $\min[S, s_t + 1]$ in the next period with probability $p_T$. No use leads to state $\max[1, s_t - 1]$ with probability $p_T$ from state $s > 1$ and to state $s = 0$ from state $s = 0$.

The volume of substance related environmental cues encountered, $c(a, \omega, t)$, depends on the lifestyle and on an exogenous state of nature $\omega$ drawn randomly from a state space $\Omega$ according to some probability measure $\mu$. The impulses $c(a, \omega, t)$ place the DM in hot mode when their intensity, measured by the function $M(c, s, a, \omega, t)$ which represents the DM’s sensitivity to the cues, exceeds some exogenously given threshold $M^T$. These functions summarize the working of the HFM.

Since people become sensitized to cues through repeated use, another assumption of the model is $M(c, s', a, \omega, t) < M(c, s'', a, \omega, t)$ for $s'' > s'$ and $M(c, 0, a, \omega, t) < M^T$.

Let $T(s, a) = \{\omega \in \Omega : M(c, s, a, \omega, t) \geq M^T\}$. The DM enters the hot mode if and only if $\omega \in T(s, a)$. Moreover, let $p_t^\omega = \mu(T(s, a))$ denote the probability of entering the hot mode at time $t$ in addictive state $s$ and lifestyle $a$.

An increase in the addictive state $s$ raises the likelihood of entering the hot mode at any moment, because the sensitivity to random environmental cues has increased. So, by assumption, at each time instant $p_{s+1}^\omega \geq p_s^\omega$, $p_0^\omega = 0$ and $p_s^E \geq p_s^R$.

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3We re-write the B&R model by emphasizing that non addicted goods $e$, depend on the addictive state $s$. The reason for this choice will become clear in the next Sections, where an extension of the model will be presented. Taking the composite good $e$ as the numeraire implies that $e_s$ is the amount of income $y_s$ spent on goods other than the addictive one.
In state $s$ the DM receives an immediate hedonic payoff $w_s(e_s, x, a) = u(e_s) + v_s(x, a)$ where utility derived from non addictive goods, $u(e_s)$, is assumed to be separable from utility derived from addictive consumption. According to assumption 2 in the B&R model, $w_s$ is increasing, unbounded, strictly concave and twice differentiable with bounded second derivative in the variable $e_s$. Moreover $v_s(x, a) \equiv u^a_s + b^a_s$, where $u^a_s$ represents the baseline payoff associated with successful abstinence in state $s$ and activity $a$ and $b^a_s$ represents the marginal instantaneous benefit from use the individual receives in state $s$ after taking activity $a$. By the same assumption, at any instant $u^E_s > u^A_s \geq u^R_s$ and $u^E_s + b^E_s > u^A_s + b^A_s$. The DM discounts future hedonic payoffs using an exponential discount function with discount factor $\delta$. His choices in the cold mode correspond to the solution of a dynamic stochastic programming problem with a value function $V_s(\theta)$ and Bellman equation equal to:

$$V_h(\theta) = \max_{(a, x) \in C} u^a_h + \sigma^{a,x}_h b^a_h + \delta \left[ (1 - \sigma^{a,x}_h) V_{h-1}(\theta) + \sigma^{a,x}_h V_{h+1}(\theta) \right], \quad (2)$$

s.t.

$0 \leq h \leq S,$

$h - 1 = \max \{1, s - 1\},$

$h + 1 = \min \{S, s + 1\}$

In equation (2) $C$ is the set of decision states $\{(E, 1), (E, 0), (A, 0), (R, 0)\}$, while $\sigma^{a,x}_h$ represents the probability of consuming the substance in state $x$ with contingent plan $(a, x)$; $\theta$ is a vector specifying the model parameters. The stationarity of equation (2) follows from the assumption that the DM takes his decision at the beginning of each period. This model generates the following patterns of addiction:

a) **Unsuccessful attempts to quit** occur when there is an unanticipated or anticipated and sufficiently slow shift in parameters $\theta_s = (p_s, u_s, b_s)$, from $\theta'$ to $\theta''$. 

b) **Cue-triggered recidivism** are associated with high exposure to relatively intense cues, e.g. high realizations of $c(a, \omega)$. 

c) **Self-described mistakes** occur when the DM chooses $(E, 0)$ or $(A, 0)$ in cold mode, but then he enters the hot mode. 

d) **Self-control through precommitment** is given by the choice $(R, 0)$ which implies a costly pre-commitment. 

e) **Self-control through behavioral and cognitive therapy** is modeled through choice $(A, 0)$ which implies costly cue avoidance.$^4$

$^4$Each of these patterns have been produced via numerical simulations.
We are particularly interested in the choice set \((E, 0)\). In this case impulses to use are not forcibly controlled through rehabilitation, but abstinence occurs for high enough \(M^T\), the threshold level impulses required to defeat cognitive control.

Our purpose is to find mechanisms that decrease the probability of entering the hot mode and the convenience to use when in cold mode so that the DM is inclined to abstain from consumption for a reasonably long period of time \(^5\). This implies building a more complete model of cognitive control where such mechanisms influence the likelihood of overriding the HFM-generated impulses by increasing the threshold \(M^T\).

### 4.1 Model specification

Taking a quadratic approximation in all the arguments except \(e_s\) the instantaneous payoff function \(w_s\) is\(^6\):

\[
    w_s(e_s, x, a) = b_s^a + w(s) + u(e_s) = b_s^a + u_s^a, \tag{3}
\]

where

\[
    b_s^a = a_s^a x + \frac{a_{sx}^a}{2} x^2 + \frac{a_{xs}^a}{2} x s,
\]

\[
    w(s) = a_s^s s + \frac{a_{ss}^s}{2} s^2 + \frac{a_{xs}^s}{2} x s,
\]

\[
    u(e_s) = \alpha_e \log(e_s) + \alpha_{ee} \log(e_s) + \alpha_{xe} x e + \alpha_{se} s e,
\]

\[
    u_s^a = w(s) + u(e_s).
\]

\(w(x)\) and \(u(e_s)\) are increasing and concave in \(x\) and \(e\), \(w(s)\) is decreasing in \(s\) and the interaction terms \(\alpha_{se}\) and \(\alpha_{xe}\) are zero by the separability assumption. Monotonicity and concavity of \(w(x)\) and \(u(e_s)\) follow from standard arguments, whereas the properties of \(w(s)\) incorporate the effect of past usage on current well being, i.e. tolerance, deterioration of health, depression, illness, etc. (see figure 1).

We then consider a function \(M\) (which denotes the "power" assigned to the drug by the HFM-generated impulses) that, on average, has a logistic shape, in line with the literature in neurosciences and pharmacology:

\[
    M(c(a, \omega_a), s, a, \omega_a) = c(a, \omega_a) + \frac{M_0 e^{\lambda s}}{1 + M_0 (e^{\lambda s} - 1)} \tag{4}
\]

where \(a \in \{R, E, A\}\) and \(M_0 = M(s = 0)\) and \(\lambda\) is the growth rate of the HFM generated impulses.

Di Chiara (2002), for instance, defines four different phases of addiction, delimited in Figure 2 by vertical dotted lines: controlled drug use, drug abuse, drug addiction, post-addiction stage.

\(^5\)Clinical practice suggests that we can speak of recovery after about at least two years of sustained abstinence from use.

\(^6\)To simplify the notation we omit, henceforth, the time index from variables and equations.
In the first stage, as a result of curiosity, peer pressure, social factors, personality traits, etc. (lifestyles $a$ and environmental cues $c$) the DM comes into contact with a drug. Sensitization facilitates further experimentation and increases the power of the HFM ($M$ weakly increasing in $s$). At this stage the subject responds to the drug-related stimuli in a controlled manner.

With repeated drug exposure the DM progressively enters the stage of drug abuse. In this stage sensitization becomes very powerful and drug-related stimuli are associated to craving ($M$ strongly increasing in $s$).

The stage of drug addiction is characterized by the preceding stage to which is added that of tolerance and physical dependence (the slope of the $M$ function starts decreasing).

In the post-addiction stage abstinence as well as sensitization progressively disappear but the HFM-generated impulses remain active (saturating $M$ function).

This $M$ function satisfies the assumptions of the B&R model (see B&R, 2004, p. 1565):

- $M(c, s, R, \omega) \leq M(c, s, A, \omega) \leq M(c, s, E, \omega)$, i.e. the lifestyle affects the DM sensitization to environmental cues;

- $M(c, s', a, \omega) < M(c, s'', a, \omega)$ for $s' < s''$ and $M(c, 0, a, \omega) < M^T$, where $M^T$ is the HFM activating threshold. $M(c, s, a, \omega) > M^T$ allows the DM to enter the HOT mode.

Moreover we introduce the following ASSUMPTION 1: The power function $M$ is strictly increasing and twice continuously differentiable in $s$. 

![Figure 1: Payoff functions.](image-url)
The function $c(a, \omega)$ included in $M$ is specified by $c(a, \omega_a) = k_1^a + k_2^a \omega_a$, where $\omega_a$ is a normally distributed random process with variance $\sigma^2 = 1$ and mean depending on the life style $a$. The parameters $k_1^a$ and $k_2^a$ depend on the lifestyle $a$.

5 The extended model

We assume that consumption of addictive substances has negative effects as the addictive state $s$ increases. This is consistent with assumptions on the payoff function $w$ stated in B&R. However this model doesn’t refer to the role of time and age of the DM in the payoff. Our model conceives a loss function which accounts for past experiences with addictive goods and for the future negative effects of current addiction. We assume that, due to increasing awareness of both, the DM may experience a change of perspective as he grows older sufficient to induce quitting even without an exogenous shock or a stressful event occurring to generate this outcome.

As regards as future expected losses the DM evaluates future losses from current addictive consumption by calculating the present discounted value of expected reductions in the length of life. These future expected losses may affect behavior in three distinct ways:

(i) they increase the threshold $M^T$ thus reducing the probability of entering the hot mode. In our model the Mesolimbic Dopamine System plays an important role in determining the choice to consume an addictive good at each point in time. Even though with age the wanting system declines inducing also a reduction in drugs use (Badanich, Adlich and Kirstein, 2006) this is not, in itself, sufficient to generate Natural Recovery, as empirical evidence suggests. Moreover, for policy
purposes studying the factors leading to Natural Recovery of young consumers seems more relevant. Therefore other factors must be involved, besides Dopamine decline with age, in explaining spontaneous remittance. In our model, structures in the frontal cortex may activate competing "cognitive incentives" by identifying alternative courses of action or projecting the future consequences of choices (B&R, 2004, p. 1563). Higher cognitive incentives triggered by future expected losses from addiction could even override HFM-generated impulses.

(ii) They affect the dynamic programming decision process through the decreased probability of use $\sigma$.

(iii) They erode the marginal instantaneous benefit from use.

The effect of past experiences, instead, is accounted for introducing the variable $H = \text{Max}\{s_i\}, i = 0, 1, ..., t - 1$ which is the DM’s maximum addictive state reached up to the current time period, $t$.

5.1 Future expected losses

Drawing from Suranovic et al. (1999) we assume that the DM is currently $Y$ years old and let $T(Y)$ be the number of years remaining representing a non addict’s life expectancy at age $Y$. $T(Y)$ is linear in $Y$ with $T'(Y) < 0$. An addict’s life expectancy at age $Y$ can be represented as $T(Y) - \beta H$ with $\beta$ being a parameter weighting the reduction in life expectancy caused by $H$.

The present value of an addict’s expected future utility stream ($V$) from consumption at age $Y$ can be defined as:

$$V_{Y,H}(s) = \int_{t=Y}^{T(Y)+Y-\beta(H+s)} e^{-r(t-Y)} b_s dt$$

where $r$ is the fixed discount rate, $e^{-r(t-Y)}$ is the discount factor at time $t$ and $b_s$ is the individual’s expected utility of consuming the addictive good at time $t$. $\beta(H+s)$ is the average lost life caused by the maximum addictive state reached in the past and by the current addictive state $s$.

For a DM aged $Y$ and maximum addictive state $H$ the present value of the expected future losses at time $t$ is given by:

$$L_{Y,H}(s) = V_{Y,H}(s) - V_{Y,H}(s + 1) = \int_{T(Y)+Y-\beta(H+s)}^{T(Y)+Y-\beta(H+s+1)} e^{-r(t-Y)} b_s dt$$

In writing equation (6) we do not account for transition probabilities affecting the evolution of addictive state $s$, because the DM evaluates future losses independently from the speed of transition between addictive states.

Differentiation of equation (6) with respect to $s$ leads to:

$$L'_{Y,H}(s) = -\beta e^{-r[T(Y)-\beta(H+s)]} b_s T(Y)+Y-\beta(H+s) + \frac{\beta}{2} e^{-r[T(Y)-\beta(H+s+1)]} b_s T(Y)+Y-\beta(H+s+1)$$

(7)
This is weakly positive because
\[ e^{-\frac{r}{2} [T(Y) - \beta]} < e^{-\frac{r}{2} [T(Y) - \beta + 1]} \]

and
\[ b_s, T(Y) + Y - \frac{\beta}{2} \leq b_s, T(Y) + Y - \frac{\beta + 1}{2}. \]

Future losses increase with the addictive state. Higher addictive states eliminate the expected benefits of the final moments of life.

As the DM gets older, the loss function \( L_{Y,H}(s) \), which is a function of age, rises. Differentiating equation (6) with respect to age \( Y \) brings to:

\[ \frac{\delta L_Y}{\delta Y} = (T'(Y) + 1) b_s, T(Y) + Y - \beta s e^{-rT(Y) - \beta} - (T'(Y) + 1) b_s, T(Y) + Y - \beta (s + 1) e^{-rT(Y) - \beta (s + 1)} \]

\[ + \int_{T(Y) + Y - \beta (s + 1)}^{T(Y) + Y - \beta s} e^{-r(Y - t)} b_dt \geq 0 \]  

(8)

Future losses rise with age as one gets older, because the discount factor used to weight end-of-life utility rises as aging draws one closer to the terminal date. Stated differently, due to discounting, end-of-life utilities are given more weight as one gets closer to the terminal date, because they are closer to the present. On the other hand, at a younger age, end-of-life utilities are given much less weight because they are far away in the future\(^7\).

In our model this loss function enters the cold mode of operating, in the value function \( V_s \) (see equation (11)), as \( b_s^0 - L_{Y,H} \).

5.2 The process leading to natural recovery

We conceive the process leading to natural recovery as a competition between HFM-generated impulses to use and cognitive control impulses. To this end we need to develop a more complete model of cognitive control in which future consequences may influence the likelihood of overriding HFM-generated impulses, as suggested by B&R (2004, p. 1582).

Berridge and Robinson (2003, p. 508) explain that the motivational component of reward can be parsed into two different psychological components: an implicit and an explicit one. Explicit processes are consciously experienced whereas implicit psychological processes may not operate at a conscious level. They also stress that additional psychological processes of cognitive awareness can transform the products of implicit processes into explicit representations. This is also

\(^7\)Intuitively we can think of future losses as irrelevant when life expectancy is known to approach zero, thus eventually leading the DM to raise, rather than reduce, her/his consumption. This is taken into account by observing that future losses, described by equation (6), tend to zero as \( \lim_{Y \to \infty} T(Y) = 0 \). Stated differently, in this case future addiction costs are ineffective given the extremely short life time remaining.
consistent with recent advances in the neurosciences that strive to bridge the gap between moral and biological lines and allow the addiction treatment “to reduce the rewarding properties of drugs while enhancing those of alternative reinforcers, inhibit conditioned memories and strengthen cognitive control.” (see Baler et al., 2006).

We therefore introduce an “Implicit Cognitive Appraisal” process representing cognitive incentives competing with the HFM’s generated impulses to use. Such process incorporates future expected losses from addiction representing an additional psychological drive that may transform the implicit cognitive mechanism into the dominant one thus overriding the HFM generated impulses.

The Implicit Cognitive Appraisal process ($I$) is modeled as a variation of the deterministic part of the $M$ function with initial condition $I_0 = I(s = 0)$ representing the a priori level of cognitive control. Following Orphanides and Zervos (1995) we let the population of DMs consist of two distinct groups: non addicts and potential addicts. For non addicts $I_0 \geq M_0$ and for potential addicts $I_0 < M_0$. A non addict DM may never become an addict, because its level of competing cognitive incentives is high enough to decrease the probability of entering the hot mode. On the other hand $I_0 < M_0$ represents the case of a DM who has not yet gained experience with the addictive good and is thus less aware of its potential consequences. We focus on this class of DMs.

The $I$ function for potential addicts is related to the loss function $L_{Y,H}(s)$ as follows:

$$I(s, Y) = \frac{I_0 e^{\lambda s}}{1 + I_0 (e^{\lambda s} - 1)},$$

(9)

where $\lambda$ is the same as in equation (4) and the initial condition is now defined as

$$I_0 = I_0 + \gamma L_{Y,H}.$$  

(10)

$I$ satisfies the following properties:

- $I(s', Y) < I(s'', Y)$ for $s' < s''$;
- $I(s, Y') < I(s, Y'')$ for $Y' < Y''$.

**ASSUMPTION 2:** $I$ is strictly increasing in $L_{Y,H}(s)$ and twice continuously differentiable in the variable $s$.

We let $\gamma$ in equation (10) indicate the presence of learning processes related to past history of consumption, age and awareness of future expected losses. We assume $0 \leq \gamma \leq 1$, where $\gamma = 1$ implies perfect learning and $\gamma = 0$ signals absence of learning. Given $I_0$, the presence of learning may drive the implicit cognitive incentives to override the HFM impulses to use for sufficiently high $Y$ and $H$. Since different individuals have different learning capacities and histories $I_0$ and $\gamma$ account for DMs heterogeneity.
In Figure 3 we plot the \( I \) function corresponding to different values of the initial condition \( \bar{I}_0 \).

![Graph showing different functions corresponding to different initial conditions.](image)

Figure 3: \( M \) and \( I \) functions corresponding to different assumptions on \( \bar{I}_0 \). Dashed line: \( \bar{I}_0 < M_0 \) (for \( \gamma L_YH < M_0 - I_0 \)), solid line: \( \bar{I}_0 = M_0 \) (for \( \gamma L_YH = M_0 - I_0 \)), dashdot line: \( \bar{I}_0 > M_0 \) (for \( \gamma L_YH > M_0 - I_0 \)).

For a given \( \gamma \), the \( I \) function shifts upwards as time \( t \) and the addictive state \( s \) increase, so that different values of \( I \) may be associated with the same \( s \) reached at different time periods. Such process may continue until the \( I \) function overrides the HFM and the probability of entering the hot mode may even decline to zero. An analogous process arises when the a priori level of cognitive control \( \bar{I}_0 \) increases.

We then study the conditions under which \( I(s, Y) > M(s, a, \omega_a) \). To this aim we prove the following propositions.

PROPOSITION 1. (i) Higher values of \( \bar{I}_0 \) decrease the probability \( p_s \) and thus the probabilities \( \sigma_{E}^{s,0} \) and \( \sigma_{A}^{s,0} \); (ii) higher values of \( \gamma \) decrease the probability \( p_s \) and thus the probabilities \( \sigma_{E}^{s,0} \) and \( \sigma_{A}^{s,0} \).

PROOF. (i) Let \( I'_0 \) and \( I''_0 \) be two distinct initial conditions of the \( I \) function, such that \( I'_0 < I''_0 \). From equation (9) it follows that \( I(s, Y, I'_0) < I(s, Y, I''_0) \) \( \forall s = 0, 1, \ldots, S \) and \( T(s, a, I'_0) \subset T(s, a, I''_0) \). It follows that \( \mu(T(s, a, I'_0)) > \mu(T(s, a, I''_0)) \). (ii) Analogously, \( I(s, Y, \gamma') < I(s, Y, \gamma'') \) for \( \gamma' < \gamma'' \) and \( \mu(T(s, a, \gamma')) > \mu(T(s, a, \gamma'')) \). Thus spontaneous remittance may be a result of the extended model.

We next show that the equilibrium solution \( s^{eq} = 1 \) is globally asymptotically stable for the dynamic system described by equation (1). We start by noting that equation (1) is a hybrid dynamic systems as it evolves according to different dy-
ynamics depending on the specific point in the state-input space under consideration. In general, for example, Piecewise Affine Systems allow to consider fundamental hybrid features such as linear-threshold events and mode switching. In our case the regime shifts depend on the DM’s choices at each time period and the resulting dynamic systems are the following:

\[ s_{t+1} = \begin{cases} 
    s_t + 1, & \text{if } x_t = 1, \\
    s_t - 1, & \text{if } x_t = 0, \\
    s_t, & \text{if } (x_t = 1 \land s_t = S) \lor (x_t = 0 \land s_t = 1).
\end{cases} \]

In the first two regimes, no equilibrium solutions exist and the dynamics is always increasing or decreasing; in the third dynamics there are two equilibria: \( s^{eq_1} = 1 \) and \( s^{eq_2} = S \). Asymptotic stability of one of them corresponds to either addictive behavior leading to chronic addiction (\( s = S \)) or to natural recovery (\( s = 1 \)). Any oscillating dynamics is due to shifts or to transient behavior. In order to study the stability properties of the dynamic system we focus only on the third regime and on the two single-point sets \( M = \{ s^{eq_1} \} \) and \( N = \{ s^{eq_2} \} \).

**PROPOSITION 2.** The equilibrium solution \( s = s^{eq_1} \) is globally asymptotically stable.

**PROOF.** Let \( L(s_t) = V_{max} - V(s_t) \) be a function defined in the open set \( G = \{0, 1, 2, \ldots, S - 1\} \) of the values reached by the state variable \( s \). \( L \) is a Liapunov function on \( G \); in fact it is continuous on \( G \) and \( L(s_t) = L(s_{t+1}) - L(s_t) \leq 0 \) for all \( s_t \in G \) (see Figure 4). Furthermore, \( M \) is the largest invariant set in \( G \) and \( G \) is a bounded open positively invariant set. Then, the theorem on asymptotic stability of the set \( M \) (La Salle, 1997) shows that the equilibrium \( s = s^{eq_1} \) is asymptotically stable on \( G \). This completes the proof.

![Figure 4: The function L on the set G.](image-url)
Since the loss function decreases the instantaneous marginal benefit from use and from proposition 2 just proved we expect this self evaluation process to lead\(^8\) the DM eventually to choose \((E,0)\) when in cold mode and for a number of time periods sufficient to generate natural recovery. Taking into account clinical evidence reported in Section 3, spontaneous remittance may be a result of the model depicted in this Section.

We have made several numerical simulations of the model. Results of such simulations, such as user’s behavior and the specific case of natural recovery, are shown in the appendix.

Now let \(\phi\) be the parameters’ vector, \(\phi = (\delta, r, q, y, I_0, M_0, \gamma)\) such that natural recovery may occur.

**PROPOSITION 3.** Assume fixed all the parameters in \(\phi\) except for \(I_0\):

1. (i) on average an increase in \(I_0\) lengthens the interval between the initial use and the maximum addictive state \(H\) and shortens the interval between \(H\) and natural recovery.
2. (ii) On average an increase in \(I_0\) lowers the maximum addictive state \(H\).

**PROOF.** (i) Given \(\bar{I}_0 = I_0 + \gamma L_{Y,H}\), an increase in \(\bar{I}_0\) is determined by a change in the a priori level of cognitive control \(I_0\). For a given stochastic process \(\omega\) and lifestyle \(\rho\), this causes \(p_s^\alpha\) to decrease (see Proposition 1) at each \(t\) thus reducing consumption in hot mode and reducing the velocity with which \(s\) increases.

(ii) Let \(I_0'\) and \(I_0''\) be two distinct initial conditions of the \(I\) function, such that \(I_0' < I_0''\). The maximum levels of \(s \bar{H}(I_0')\) and \(H'(I_0'')\) are reached at two different time instants \(t'\) and \(t''\). From (i) it follows that \(t' \leq t''\). Since by definition \(L(H, Y)\) is increasing in time, \(H''(I_0') \leq H''(I_0'')\).

**PROPOSITION 4.** Assume fixed all the parameters in \(\phi\) except for \(\gamma\). A decrease in \(\gamma\) lengthens the drug addiction stage and delays natural recovery.

**PROOF.** A decrease in \(\gamma\) shifts the \(I\) function downwards. From Proposition 1 this implies an increase in \(p_s^\alpha\) which causes a delay in the effects of the loss function.

### 6 Policy Implications

Public policies towards addictive substances are targeted at reducing negative externalities (e.g. second hand smoke, social or familial discomfort connected to the addictive state) and social costs (e.g. alcohol related violence and crime, road accidents and extra costs to the health and social security system) of addiction and at reducing direct personal or health costs that may occur as a consequence of addiction.

---

\(^8\)Stated differently, there exist a subset of the relevant parameters satisfying the conditions leading to natural recovery.
Bernheim and Rangel (2005) argue that if consumers are sometimes rational and sometimes driven by cue-triggered mistakes, traditional public policy approaches, i.e. regulation versus incentives, may produce undesirable results. While strict regulation or prohibition may be mostly effective and discourage consumption in rational individuals by raising the monetary and non-monetary costs of consumption, they are not effective if addiction is induced by cue-triggered mistakes. In this case such measures only raise the costs of consumption without reaching the target of reducing it. Similar considerations apply to tax policies. While price increases of legal addictive substances (such as alcohol and tobacco) may induce a reduction in demand in rational people, they may only raise the costs of consumption if this is driven by compulsive choices. Thus both criminalization and taxation may be socially counterproductive and ineffective at reaching their goals, because those who become addicted incur higher monetary costs.

Such conclusions, however, are modified if spontaneous remittance, i.e. spontaneous cessation of consumption, occurs through increased awareness of future expected costs and through learning from past experiences. In this case cognitive policies, education and information campaigns or a combination of them may be best suited to activate cognitive control mechanisms, but more traditional approaches such as regulation and taxation still play an important role.

Cognitive therapies may help consumers to activate a process of self evaluation that raises the value of the future negative consequences of addiction thus reinforcing the motivation to change habits. Education may also help identifying the social, health and psychological consequences of substance abuse increasing the present value of uncertain and remote future costs. As Trosclair et al. (2002) have emphasized, more educated individuals are far more likely to quit smoking successfully, for instance, as education helps activating the competing "cognitive incentives" necessary to override the HFM. Therefore, contrary to what Bernheim and Rangel have suggested (2005, p.131), we believe that education campaigns may be effective, to some extent, in reducing consumption even among those already addicted and not only as a prevention policy to discourage initial experimentation. Even though education and information campaigns may not alter the mechanism through which individuals engage in compulsive use (i.e. the HFM), they may help activating the competing cognitive incentive mechanisms which trigger a process of self evaluation.

Furthermore, the same type of role in triggering the competing cognitive incentive mechanism may arise from regulation and/or taxation of addictive substances, because they increase the monetary and non monetary costs of future consumption as well as of current one. Once a process of self evaluation has been activated, even more traditional policy measures may be of help in carrying out the mechanisms of self appraisal which may lead to spontaneous quitting.

Finally, our model suggests that policy strategies should be differentiated according to the age profile of the addicts. If natural recovery is the result of two mechanisms: a physical maturation process combined with increased awareness of the negative consequences of past, current and future substance abuse, then strict
regulation and taxation are potentially more effective at discouraging substance use among the youngest groups of consumers, because such measures help them shift the cost-benefit payoff away from drug using and its associated lifestyle, whereas such shift in behavior cannot yet be associated with ageing and a phenomenon of maturing out.

7 Conclusion

In this paper we explain how natural recovery from addiction may arise from increasing awareness of future and uncertain consequences of consumption and from learning from past experiences. These processes may be critical for understanding spontaneous remittance, a widespread yet unexplained pattern of addictive behavior.

Our model explains how even strongly addicted persons may find their way out of substance abuse without the utilization of professional help and it also highlights the main factors driving such process. Drawing from clinical and experimental research we introduce an "Implicit Cognitive Appraisal" function depending on past experiences as well as on future expected consequences of addictive consumption. Such function affects the DM in two ways: it erodes the payoff from use as the decision maker grows older and it increases the cognitive control competing with the hedonic impulses to use, thus reducing the probability of entering the hot mode.

Some of the policy implications of the B&R model are modified. Strict regulation or taxation of addictive substances may still play a role in activating the cognitive control process, because they raise the monetary and non monetary costs of current and future addiction. Moreover, education and information campaigns can be effective policy measures not only to discourage initial experimentation, but also to trigger a process of self evaluation. This process may eventually lead to natural recovery, because it helps the DM identifying the social, health and psychological consequences of substance abuse.

We believe that the possibility of making cue-triggered mistakes greatly improves our understanding of the phenomenon of addiction and also leads to counter intuitive policy implications (see Section 6). However, our explanation of spontaneous remittance recovers an important role for cognitive processes and rational decision making, because when the DM becomes increasingly aware of the costs and benefits of addiction he can successfully overcome its hedonic impulses to use.
Appendix

This appendix contains some simulation results of the model. The simulations are obtained by assigning appropriate values to the model parameters and maximizing the value function reported in equation (2).

In the model the DM can operate in either of two modes: the hot mode and the cold mode. The hot mode can be activated by environmental cues as well as the DM’s history of use and lifestyle. In the cold mode the DM selects his most preferred alternatives by imposing cognitive control.

The parameters of the functions $M$ and $I$, specified by equation (4) and (9) respectively, are the following: $\lambda = 0.1$, $M_0 = 0.09$, $I_0 = 0.07$ and $\gamma = 1$. The function $c(a, \omega)$ included in $M$ is specified by $c(a, \omega_a) = k_1^a a + k_2^a \omega_a$, where $\omega_a$ is a normally distributed random process with variance $\sigma_a^2 = 1$ and mean depending on the lifestyle $a$. The parameters $k_1^a$ and $k_2^a$ depend on the lifestyle $a$.

The payoff function $w_s$, $t$ is specified by equation (3), where $\alpha_x = 10$, $\alpha_{xx} = -0.5$, $\alpha_s = -1.0$, $\alpha_{ss} = 0.9$, $\alpha_{e} = 30$, $\alpha_{ee} = -1$, $e_s = y_s$.

**Stochastic dynamic programming problem**

The DM discounts future hedonic payoffs using an exponential discount function with discount factor $\delta$. His choices in the cold mode correspond to the solution of a dynamic stochastic programming problem with a value function $V_s$ and Bellman equation equal to:

$$V_h = \max_{(a, x) \in C} \left[ u_h^a + \sigma_h^{a,x} b_h^a + \delta \left( 1 - \sigma_h^{a,x} \right) V_{h-1} + \sigma_h^{a,x} V_{h+1} \right], \quad (11)$$

s.t. $0 \leq h \leq S$,

$h - 1 = \max \{1, s - 1\}$,

$h + 1 = \min \{S, s + 1\}$

In equation (11) $C$ is the set of decision states $\{(E, 1), (E, 0), (A, 0), (R, 0)\}$, while $\sigma_h^{a,x}$ represents the probability of consuming the substance in state $x$ with contingent plan $(a, x)$. The stationarity of equation (11) follows from the assumption that the DM takes his decision at the beginning of each period.

At each time period $t$ and in cold mode, the DM takes two decisions in sequence: he chooses the lifestyle $a$ and whether consuming $(x = 1)$ or not consuming $(x = 0)$ the addictive good. His choices in the cold mode correspond to the solution of the dynamic stochastic programming problem of equation 11 in the set of decision states $C = \{(E, 1), (E, 0), (A, 0), (R, 0)\}$, where $\sigma_s^{a,x}$ represents the probability of consuming the substance in state $s$ with contingent plan $(a, x)$. The DM discounts future hedonic payoffs at a constant rate $\delta$. $S$ is the highest addictive state.

The problem can be solved recursively as follows:
Step 1: initialization. By equation (11), for \( s = S \) the function \( V \) is

\[
V_s = \max_{(a, x) \in C} a^S + \sigma^a S h^a - \delta \left( \left( 1 - \sigma^a S \right) V_{s-1} + \sigma^a S V_s \right).
\] (12)

Equation 12 implicitly defines \( V_s \) as a function of \( V_{s-1} \) by

\[
V_s = h^S_s (V_{s-1}).
\] (13)

We search \( V_{s-1} \) within an interval \([V_{s-1}, V_{s-1}]\).

Step 2: backward induction For each \( k = S - 1, \ldots, 2 \), by equation (11) we find

\[
V_k = g^k_k (V_{k-1}, V_{k+1}) = g^k_k (V_{k-1}, h^{k+1}_{k+1} (V_k)),
\] (14)

where the function \( h^{k+1}_{k+1} \) is defined implicitly by the previous steps. Hence, equation (14) implicitly defines a sequence of functions \( h^k \) such that

\[
V_k = h^k (V_{k-1}).
\] (15)

Step 3: evaluation of terminal condition We find

\[
V_1 : g_1^1 (V_1, V_2) = g_1^1 (V_1, h^2_1 (V_1))
\] (16)

by solving equation

\[
V_1 = \max_{(a, x) \in C} a^1 + \sigma^a 1 b^1_1 + \delta \left( \left( 1 - \sigma^a 1 \right) V_1 + \sigma^a 1 h^2_1 (V_1) \right).
\]

Equation (16) is nonlinear in the variable \( V_1 \) and can be solved numerically. Let \( \hat{V}_1 \) be the optimal solution. We also evaluate \( V_0 \) by solving equation

\[
V_0 = \max_{(a, x) \in C} a^0 + \sigma^a 0 b^0_0 + \delta \left( \left( 1 - \sigma^a 0 \right) V_0 + \sigma^a 0 \hat{V}_1 \right)
\]

Step 4: computation of optimal values the optimal sequence \( \hat{V}_0, \hat{V}_1, \hat{V}_2, \ldots, \hat{V}_S \) is backward recovered by applying the functions \( h^k \) defined by equation (15).

Simulation results

In this section we show some simulations\(^9\) obtained by initializing the model as follows:

- \( S = 50 \);
- \( y_s = 800 \) $;
- time period \( t \): 1 week;

\(^9\)Numerical simulations and dynamic programming are run on MATLAB 7.0.4.
Figure 5: Frequency of decisions for each addictive state $s$.

- simulation length: 1000 periods ($\approx$ 20 years);
- cost of addictive substance: 200 $;
- costs for rehabilitation: 250 $.
- decisions set: $(E, 1), (E, 0), (A, 0), (R, 0)$.

Figure 5 shows the probability that in the whole simulation time and in addictive state $s$ the DM chooses one of the four decisions.

In Table 1 and 2 we report some statistics of simulations performed by varying income $y$ and initial cognitive control $I_0$ with respect to a baseline value ($y^*, I_0^*$) of the same parameters. Means (1st column), Standard Deviations (2nd column), Absolute Maxima (3rd column) and time periods at which Natural Recovery occurs (5th column) are shown. STD of Max (4th column) is the standard deviation of the absolute maximum corresponding to each run. All statistics are evaluated on 10 runs of the evolution of the addictive state $s$. The simulation length is 1000 periods.

**Table 1: Summary statistics on income $y$.**

<table>
<thead>
<tr>
<th>$y$</th>
<th>Mean</th>
<th>STD</th>
<th>Max</th>
<th>STD of Max</th>
<th>Nat. Rec. (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>7.88</td>
<td>7.69</td>
<td>30</td>
<td>1.94</td>
<td>808</td>
</tr>
<tr>
<td>800*</td>
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<td>7.39</td>
<td>28</td>
<td>1.88</td>
<td>737</td>
</tr>
<tr>
<td>900</td>
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<td>7.29</td>
<td>27.6</td>
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</tbody>
</table>
Table 2: Summary statistics on initial cognitive control $I_0$.

<table>
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<th>$I_0$</th>
<th>Mean</th>
<th>STD</th>
<th>Max</th>
<th>STD of Max</th>
<th>Nat. Rec. (t)</th>
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</thead>
<tbody>
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<td>8.97</td>
<td>33.1</td>
<td>2.18</td>
<td>733</td>
</tr>
<tr>
<td>0.07*</td>
<td>7.42</td>
<td>7.39</td>
<td>28</td>
<td>1.88</td>
<td>737</td>
</tr>
<tr>
<td>0.09</td>
<td>6.8</td>
<td>6.39</td>
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<td>1.61</td>
<td>741</td>
</tr>
</tbody>
</table>

Figure 6: Choices over time: cold mode (top) and hot mode (bottom) decision making.

Figure 6 shows the cold mode decision making process and the actual decision making (Hedonic Forecasting Mechanism active) and, in particular, how a DM who starts a process of self evaluation (as modelled in this paper) may eventually stop entering the hot mode and always choose $(E, 0)$ when in cold mode (see also Figure 7).
Figure 7: Evolution of the addictive state $s$ when a self-evaluation process is introduced in decision making leading to natural recovery.

References


