RATIONALIZABILITY OF CHOICE BY SEQUENTIAL PROCEDURES

JOSE APESTEGUIA† AND MIGUEL A. BALLESTER‡

ABSTRACT. In a sequential rationalization of choice, the successive application of rationales reduces the set of alternatives to a unique element, which is the one actually chosen. We offer a property, Independence of One Irrelevant Alternative, that consists on a relaxation of the classical property Independence of Irrelevant Alternatives, that completely characterizes the set of choice functions that are sequentially rationalizable, no matter the number of rationales required. Our property provides a novel tool with which to study how other behavioral concepts are related to sequential rationalizability, and establish a priori unexpected implications. In particular, we show that the concept of rationalizability by game trees, which, in principle, had little to do with sequential rationalizability, is contained in the latter. We also show that some prominent voting mechanisms and choice procedures based on the ordered elimination of alternatives are sequentially rationalizable. Finally, we prove that choice functions exhibiting a status-quo bias are also sequentially rationalizable.

Keywords: Individual rationality, Bounded rationality, Behavioral economics.
JEL classification numbers: B41, D01.

1. Introduction

The classic choice model in economics encompasses choice behaviors resulting from the maximization of a single preference relation. When this is the case, it is typically said that behavior is rationalizable. Over the last decades, however, the research has produced increasing amounts of evidence documenting systematic and predictable deviations from the notion of rationality implied in the above definition. Not surprisingly, these inconsistencies between theoretical models and applications have triggered a number of alternative rationalizability models to flourish in the literature (see, e.g., Kalai, Rubinstein and Spiegler 2002, Manzini and Mariotti 2007, and Xu and Zhou 2007).

In this paper we study the behavioral structure of sequential models of choice. Sequential choice is very appealing from a behavioral perspective. It considers a decision-maker (DM) that, when faced with a choice problem, applies a number of

Date: First version: May, 2008. This version: February, 2009.
†Universitat Pompeu Fabra. E-mail: jose.apesteguia@upf.edu.
‡Universitat Autonoma de Barcelona. E-mail: miguelangel.ballester@uab.es.

criteria in a fixed order of priority, gradually narrowing down the set of alternatives until one is identified as the choice. Clearly, this sequential procedure does not necessarily satisfy the standard axioms of choice, and hence it rationalizes behavior typically regarded as irrational. For example, cyclic choice patterns, often observed in empirical studies, can be easily accommodated as the result of choice by a sequential procedure. However, not every choice pattern can be accommodated by the sequential application of criteria, even when applying an arbitrarily large number of them. The question then arises on what kinds of observed choices are sequentially rationalizable. In this paper we offer a property, Independence of One Irrelevant Alternative (IOIA), and show that it completely characterizes the set of choice functions that are sequentially rationalizable, no matter the number of rationales required.

According to IOIA, the DM simplifies the evaluation of a choice problem A by focusing on the pairs of alternatives that compose it. She identifies a single pair as the most relevant one, and imposes that the choice from A should not depend on the dominated alternative from this pair of alternatives. The DM selects the most relevant pair by following a rule that is fully consistent across choice problems. This rule may represent, for example, the familiarity the DM may have with the different pairs of alternatives. That is, the DM may be more used to compare certain pairs of alternatives than others, and for every choice problem she decides to focus on the most familiar pair. Also, the alternatives may be perceived by the DM in a particular physical order, and then the DM when evaluating a choice problem A she may select the first two alternatives in A according to the physical order.

Clearly, IOIA is a simple relaxation of the classical property of Independence of Irrelevant Alternatives (IIA). While IIA requires that the choice from the problem A should be consistent with the choice from $A \setminus \{x\}$, for every single x other than the chosen element from A, IOIA requires such consistency but *for only one such element*. The contrast between IIA and IOIA shows immediately that IOIA constitutes a strict weakening of IIA that allows for certain menu effects.

Our property allows the possibility of linking sequential rationalizability with other choice models. In the second part of the paper we give three such applications. We first show that the notion of rationalizability by game trees, due to Xu and Zhou (2007), which, in principle, had little to do with sequential rationalizability, is a strict refinement of the latter. We then show that agenda rationalizability, a rationalizability notion that is introduced here and is rooted in certain models of choice by ordered elimination as well as in voting mechanisms based on successive elimination, is also sequentially rationalizable. Finally, we show that choice functions exhibiting the well-known status-quo bias also are sequentially rationalizable. The proofs of these links with sequential rationalizability constitute one of the direct implications of our characterization result.

The closest paper to ours is Manzini and Mariotti (2007). We adopt their model of sequential choice for the most part.[1] Notably, Manzini and Mariotti provide characterizations for those choice functions that are sequentially rationalizable by two criteria (called Rational Shortlist Methods) or three criteria.[2]

The paper is organized as follows. Section 2 introduces the notation and the main definitions to be used thereafter. Section 3 contains the characterization result. Section 4 presents three applications of our characterization. Finally, Section 5 concludes.

2. Basic Notation and Definitions

Let X be a finite set of $n \geq 2$ objects. We denote by $\mathcal{P}(X)$ the set of all non-empty subsets of X, by $\mathcal{P}_2(X)$ the set of all subsets of X with at least two alternatives, and by $\mathcal{B}(X)$ the collection of 2-alternative sets (binary problems) in X. A choice function c on $\mathcal{P}(X)$ assigns to every $A \in \mathcal{P}(X)$ a unique element $c(A) \in A$.

A binary selector f is a single-valued function $f : \mathcal{P}_2(X) \to \mathcal{B}(X)$ that for every choice problem A with at least two alternatives gives a binary problem in A, i.e. $f(A) \subseteq A$. Given a binary selector f the direct revealed relation S on $\mathcal{B}(X)$ is defined by $(B_1, B_2) \in S$ if and only if there is a choice problem $A \in \mathcal{P}_2(X)$ such that $B_1, B_2 \subseteq A$ and $B_1 = f(A)$. Denote the transitive closure of S by $\overline{S}$. We say that the binary selector f is rational if and only if for all $A \in \mathcal{P}_2(X)$, for all $B_1, B_2 \subseteq A$, $B_1 \overline{S} B_2$ implies that $B_2 \neq f(A)$. The latter is simply the Strong-Axiom over the space of binary problems, where a choice problem A is understood as the collection of binary problems that compose it.

Denote by P an acyclic binary relation on X, $P \subseteq X \times X$. That is, for any collection $x_1, \ldots, x_r \in X$, with $r > 1$, whenever $(x_i, x_{i+1}) \in P$ for all $i = 1, \ldots, r-1$, it is not true that $(x_r, x_1) \in P$. We will often refer to P as a *rationale*. For any $A \in \mathcal{P}(X)$, $M(A, P)$ refers to the set of maximal elements in A with respect to P, that is $M(A, P) = \{x \in A : (y, x) \in P \text{ for no } y \in A\}$. Let that $M(\emptyset, P) = \emptyset$.

In the classical notion of rationalizability, a choice function c is *rationalizable* if there is a rationale P such that, for any choice problem $A \in \mathcal{P}(X)$, $c(A) = M(A, P)$. A choice function c is *sequentially rationalizable* by the ordered collection of rationales $\{P_1, \ldots, P_K\}$ if, for every choice problem A, the sequential application of the rationales in that fixed order leads to the choice of c. Let $M_i^j(A)$ with $i \leq j$ denote

[1]In their original formulation all the structure that is imposed on the binary relations representing the criteria applied by the DM is asymmetry. We believe that the essence of sequential rationalizability lies in the incompleteness of the criteria, reflecting the crudeness of the binary relations sequentially applied by the DM in order to reach a choice. The use of cyclic criteria may be controversial from a behavioral perspective, since they may appear to be unrealistic. It seems then natural to study possibly cyclic choice behavior arising from the sequential application of acyclic binary relations. This is the position we adopt here.

[2]Salant and Rubinstein (2008) provide an alternative characterization of Rational Shortlist Methods within the framework of a 'limited attention' model.

the set $M_i^j(A) = M(M(\ldots M(M(A, P_i), P_{i+1}), \ldots, P_{j-1}), P_j)$.[3]

Sequential Rationalizability: A choice function c is sequentially rationalizable whenever there exists a non-empty ordered list $\{P_1, \ldots, P_K\}$ of rationales on X such that $c(A) = M_1^K(A)$ for all $A \subseteq X$.

3. Characterization

The classic notion of rationalizability of a choice function c deals with the issue of the existence of a rationale that explains choice behavior as the result of maximization. A well-known result establishes that a choice function c is rationalizable if and only if c satisfies the Independence of Irrelevant Alternatives (IIA) property:

Independence of Irrelevant Alternatives (IIA): For any $A \subseteq X$, $c(A) = c(A \setminus \{x\})$ for every $x \in A \setminus c(A)$.

IIA imposes that the choice from A should not vary when any alternative different to the chosen one in A is dropped. By this, IIA rules out menu effects. We offer a property, Independence of One Irrelevant Alternative (IOIA), that consists on a strict weakening of IIA.

Independence of One Irrelevant Alternative (IOIA): There is a rational binary selector f such that for any $A \subseteq X$, $c(A) = c(A \setminus \{x^*\})$, with $x^* = f(A) \setminus c(f(A))$.

According to IOIA, the DM simplifies the evaluation of a choice problem A by focusing on the pairs of alternatives that compose it. Then, when confronting A she selects a binary problem in A, and imposes that the choice from A should be independent of the dominated alternative in this binary problem. The DM selects the binary problem by following a rule f that is totally rational in the sense of satisfying the Strong-Axiom.

IOIA is particularly interesting from a behavioral perspective: it first selects a binary problem following a totally consistent rule across choice problems, and then imposes a mild congruence requirement on choice. The selection of the binary problem may represent, for example, the familiarity the DM has with the binary problems. That is, the DM may be more used to compare certain pairs of alternatives than others, and for every choice problem A identifies the most familiar pair in A. Relatedly, pairs of alternatives may differ in terms of how similar the alternatives are between them. Then, for any choice problem A, the DM may focus on that pair composed

[3]With a slight abuse of notation, we identify elements with sets containing only one element. Also, we will often suppress braces by writing $c(x, y)$ instead of $c(\{x, y\})$.

of the most similar alternatives. Also, the alternatives may be perceived by the DM in a particular physical order, and then the DM when evaluating a choice problem A she may select the first two alternatives in A according to the physical order.

The contrast with IIA is immediate. IOIA requires the choice from A to be independent, not of every single alternative different from $c(A)$, like IIA requires, but only of a single such alternative. It is clear that IOIA represents a strict weakening of IIA, allowing for certain types of menu effects. We show next that this behavioral property completely characterizes sequential rationalizability.

Theorem 3.1. *c is sequentially rationalizable if and only if c satisfies IOIA.*

Proof of Theorem 3.1: We start by proving that IOIA is a sufficient condition for sequential rationalizability. IOIA implies that there is a rational binary selector f such that for every $A \subseteq X$, $c(A) = c(A \setminus \{x^*\})$, with $x^* = f(A) \setminus c(f(A))$. Since f satisfies the Strong-Axiom, it is immediate that there is an order $\lhd$ over the collection of binary problems $\mathcal{B}(X)$ such that for every $A \in \mathcal{P}_2(X)$, $f(A)$ is the first binary problem contained in A according to $\lhd$. Denote, then, the ordered collection of binary problems $\mathcal{B}(X)$ according to $\lhd$ by $\{a_i, b_i\}_{i=1}^{n(n-1)/2}$. Without loss of generality let $a_i = c(a_i, b_i)$, $i = 1, \ldots, n(n-1)/2$. Define $P_i = \{(a_i, b_i)\}$. Clearly, $\{P_i\}_{i=1}^{n(n-1)/2}$ is a collection of rationales.

We prove by induction over the cardinality of choice problems $A \subseteq X$ that $\{P_i\}_{i=1}^{n(n-1)/2}$ sequentially rationalizes c. It is obvious for the case of $|A| \leq 2$. Suppose the claim is true for $|A| = t$, we show it for $|A| = t+1$. By IOIA $c(A) = c(A \setminus \{x^*\})$, with $x^* = f(A) \setminus c(f(A))$. By the inductive hypothesis $c(A \setminus \{x^*\}) = M_1^{n(n-1)/2}(A \setminus \{x^*\})$, and by the definition of x^* and the construction of the rationales, $M_1^{n(n-1)/2}(A \setminus \{x^*\}) = M_1^{n(n-1)/2}(A)$. Therefore, $c(A) = M_1^{n(n-1)/2}(A)$, as desired.

In the other direction, we now show that if c is sequentially rationalizable, then IOIA holds. Let c be sequentially rationalizable by the ordered collection of rationales $\{P_1, \ldots, P_K\}$. First, construct the collection of rationales $\{P'_1, \ldots, P'_K\}$ from $\{P_1, \ldots, P_K\}$, as follows: for all $j = 1, \ldots, K$, $(x, y) \in P'_j$ if and only if $(x, y) \in P_j$ and there is no $i < j$ such that $(x, y) \in P_i$ or $(y, x) \in P_i$. Clearly, $\{P'_1, \ldots, P'_K\}$ is an ordered collection of rationales that sequentially rationalizes c. Assume, without loss of generality, that the constructed collection $\{P'_1, \ldots, P'_K\}$ is composed of non-empty rationales (otherwise, simply remove the empty rationales and re-number them).

Now, consider a rationale P'_j in the constructed collection of rationales $\{P'_1, \ldots, P'_K\}$, that contains more than one pair of alternatives. Since P'_j is acyclic, there is a pair of alternatives (a, b) in P'_j, such that $(b, c) \notin P'_j$ for every $c \in X$. We can split the rationale P'_j into two rationales, $\{(a, b)\}$ and $P'_j \setminus \{(a, b)\}$. We show that for every $A \subseteq X$, $M(A, P'_j) = M(M(A, \{(a, b)\}), P'_j \setminus \{(a, b)\})$. If a or b is not in A, then clearly $M(A, P'_j) = M(A, P'_j \setminus \{(a, b)\})$, and since $M(A, \{(a, b)\}) = A$, it follows that $M(A, P'_j \setminus \{(a, b)\}) = M(M(A, \{(a, b)\}), P'_j \setminus \{(a, b)\})$. Thus, $M(A, P'_j) =$

$M(M(A, \{(a, b)\}), P'_j \setminus \{(a, b)\})$. If, on the contrary, $a, b \in A$, then $M(A, \{(a, b)\}) = A \setminus \{b\}$. Given that (a, b) in P'_j and that $(b, c) \notin P'_j$ for every $c \in X$, $M(A, P'_j) = M(A \setminus \{b\}, P'_j)$. Hence, $M(A, P'_j) = M(M(A, \{(a, b)\}), P'_j \setminus \{(a, b)\})$, as desired. It then follows that the ordered collection of rationales $\{P'_1, \ldots, P'_{j-1}, \{(a, b)\}, P'_j \setminus \{(a, b)\}, P'_{j+1}, \ldots, P'_K\}$ also sequentially rationalizes c. By iterating this splitting process of rationales we end up in a collection of rationales $\{P^*_1, \ldots, P^*_K\}$, each one composed by one pair of alternatives, that sequentially rationales c.

Construct the binary selector f as follows. For every $A \in \mathcal{P}_2(X)$, $f(A) = \{a, b\}$ if and only if $\{(a, b)\} = P^*_l$ and there is no $\{d, e\} \subseteq A$ with $\{(d, e)\} = P^*_m$ and $m < l$. It is immediate that f satisfies the Strong-Axiom. Given that c is sequentially rationalizable by the collection $\{P^*_1, \ldots, P^*_K\}$ and the construction of f it follows that $c(A) = c(A \setminus \{x^*\})$, with $x^* = f(A) \setminus c(f(A))$, and hence IOIA holds.□

Notice that a choice function c is rationalizable in the classical sense if for every choice problem A and every single binary problem $B \subseteq A$ the choice from A does not depend on the dominated alternative in B. On the other hand, the proof of Theorem 3.1 states that assessing whether a particular c is sequentially rationalizable limits to check whether there is a linear order over the binary sets such that for every choice problem A and for the first binary problem $B \subseteq A$ the choice from A does not depend on the dominated alternative in B. This makes the property IOIA particularly manageable for applications, and easy to use in practice.

4. Applications

4.1. **Rationalizability by Game Trees.** We begin this section by establishing the relation between sequential rationalizability and a rationalizability notion due to Xu and Zhou (2007): rationalizability by game trees. Xu and Zhou characterize those choice functions that can be rationalized by extensive games with perfect information. More specifically, we say that $(G; R)$ denotes a game tree whenever: (i) G is an extensive game with perfect information that has alternatives X as terminal nodes, such that each alternative in X appears once and only once as a terminal node of G (hence, X and G can be identified), and (ii) every node i in the tree G represents the possible choices of an agent i endowed with a linear order R_i over X. Thus, if there are K non-terminal nodes, denote $R = (R_1, \ldots, R_K)$.

Rationalizability by Game Trees: A choice function c is rationalizable by game trees whenever there exists a game tree $(G; R)$ such that

$$c(A) = SPNE(G|A; R) \text{ for all } A \subseteq X,$$

where $G|A$ is the reduced tree of G that retains all the branches of G leading to terminal nodes in A, and $SPNE(\Gamma)$ stands for the subgame perfect Nash equilibrium outcome of Γ.

The relation between rationalizability by game trees and sequential rationalizability is not clear a priori. On the one hand, the game tree structure of the preferences in the former is much richer than the linear structure of the rationales in the latter. On the other hand, the preferences of the players in rationalizability by game trees are imposed to be linear orders, whereas in sequential rationalizability the rationales are only imposed to be acyclic. The following result establishes a perhaps unexpected relation between sequential rationalizability and rationalizability by game trees: namely, that the latter is a strict refinement of the former.

Theorem 4.1. *If c is rationalizable by game trees, c is sequentially rationalizable. The converse is not necessarily true.*

Proof of Theorem 4.1: Let c be a choice function rationalizable by game trees. We will prove that c satisfies IOIA and is therefore sequentially rationalizable. Consider a game tree $(G; R = (R_1, \ldots, R_K))$ that rationalizes c. Suppose, without loss of generality, that players are indexed by a linear order $<$ such that $i < j$ if i is a successor of j in the game tree G. We say that i is resolute on $\{x, y\}$, if R_i determines the outcome over $\{x, y\}$. That is, whenever $SPNE(G|\{x, y\}; R) = SPNE(G|\{x, y\}; (R'_{-i}, R_i))$ for any vector of linear orders of players other than i, R'_{-i}. Given a set A in $\mathcal{P}(X)$, define

$$m(A) = \min\{i \in \{1, \ldots, K\} : i \text{ is resolute over a pair } \{x, y\} \text{ contained in } A\}.$$

Consider the following linear order $\lhd$ on $\mathcal{B}(X)$. Let $\{a, b\}, \{d, e\} \in \mathcal{B}(X)$, with $a = c(a, b)$ and $d = c(d, e)$,

$$\{a, b\} \lhd \{d, e\} \Leftrightarrow \begin{cases} m(\{a, b\}) < m(\{d, e\}) \text{ or }, \\ m(\{a, b\}) = m(\{d, e\}), (b, e) \in R_{m(\{a,b\})}. \end{cases}$$

We now show that IOIA holds with respect to the constructed linear order $\lhd$. By rationalizability of game trees, $c(A) = SPNE(G|A; R)$, and also $c(A \setminus \{x^*\}) = SPNE(G|(A \setminus \{x^*\}); R)$, where x^* is the dominated alternative in the first binary problem in A according to $\lhd$. We show $SPNE(G|A; R) = SPNE(G|(A \setminus \{x^*\}); R)$. By definition of subgame perfect Nash equilibrium, $SPNE(G|A; R) = SPNE(G|(A \setminus \{z\}); R)$, where z is every dominated alternative in A by the minimal resolute player in A. Clearly, by construction x^* satisfies these conditions, and hence the claim follows.

We now show by way of an example that the inclusion is strict. Let $X = \{1, \ldots, 4\}$ and c be such that: $\mathcal{A}_c[1] = \{(1, 3, 4), (1, 3)\}$, $\mathcal{A}_c[2] = \{(1, 2, 4), (1, 2), (2, 4)\}$, $\mathcal{A}_c[3] = \{(1, 2, 3, 4), (1, 2, 3), (2, 3, 4), (2, 3), (3, 4)\}$ and $\mathcal{A}_c[4] = \{(1, 4)\}$. It can be easily checked that the following two rationales sequentially rationalize c: $P_1 = \{(3, 4), (2, 4), (2, 1), (3, 2)\}$ and $P_2 = \{(1, 3), (4, 1)\}$. To see that c is not rationalizable by game trees, we show that c does not satisfy the *divergence consistency* property, which Xu and Zhou prove to be a necessary condition for rationalizability by game trees.[4] Note that in c, 1 diverges before 3 and 4, and 3 diverges before 1 and 2. At the same time, we have that $c(1,3) = 1$ but $c(2,4) = 2$, which contradicts divergence consistency. Therefore, c is not rationalizable by game trees and the theorem follows.$\square$

4.2. **Agenda Rationalizability.** Let us assume that the n elements in X are linearly ordered by $<$. This order may be interpreted as, say, a particular physical presentation of the objects. For any choice problem A in X, write the l elements in A ordered by $<$ as $a(1) < a(2) < \cdots < a(l)$. Consider a tournament T.[5] The DM makes a selection from A according to the following elimination process. First she makes a selection between $a(1)$ and $a(2)$ using T, then compares the selected element from $a(1), a(2)$ with $a(3)$ and makes a new selection according to T. The DM continues in this ordered manner until the surviving element is compared with the last element $a(l)$; this last choice determines the choice in A. Denote by $e(<, T, A)$ the alternative chosen from A by this process, given the agenda $<$, and the tournament T.

Similar choice by ordered elimination procedures are studied in the choice-theoretic literature. The models studied in Rubinstein and Salant (2006) and Salant and Rubinstein (2008), for example, include this one as a special case. See also Masatlioglu and Nakajima (2008). But the binary choices between alternatives may also be the result of majority voting, for example. Therefore voting mechanisms such as those based on successive elimination are also connected to the above (see Dutta, Jackson, and Le Breton, 2002).

Consider now the following notion of rationalizability.

Agenda Rationalizability: A choice function c is agenda rationalizable whenever there exists a linear order $<$ over the set of alternatives (an agenda) and a tournament T such that, for every $A \subseteq X$, $c(A) = e(<, T, A)$.

Theorem 4.2 establishes the relation between agenda rationalizability and sequential rationalizability.

Theorem 4.2. *If c is agenda rationalizable, c is sequentially rationalizable. The converse is not necessarily true.*

Proof of Theorem 4.2: Suppose that c is agenda rationalizable by by the agenda $<$ and the tournament T. We now define a linear order $\lhd$ on $\mathcal{B}(X)$. Let $\{a, b\}, \{d, e\} \in$

[4]For any triple x, y, z, x *diverges* before y and z, if $c(x, y) = x, c(y, z) = y$, and $c(z, x) = z$, and $c(x, y, z) = x$. *Divergence consistency*: for any four alternatives x_1, x_2, y_1, y_2, if x_1 diverges before y_1 and y_2, and y_1 diverges before x_1 and x_2, then $c(x_1, y_1) = x_1$ if and only if $c(x_2, y_2) = y_2$.

[5]A tournament T is a binary relation that is asymmetric and connected.

$\mathcal{B}(X)$, with $a = c(a, b)$ and $d = c(d, e)$,

$$\{a, b\} \lhd \{d, e\} \Leftrightarrow \begin{cases} a < d \text{ or }, \\ a = d, b < e. \end{cases}$$

We prove that for every $A \subseteq X$, $c(A) = c(A \setminus \{x^*\})$ where x^* is the dominated alternative in the first binary problem in A according to $\lhd$. By construction, and by agenda rationalizability $x^* = a(1)$ if $(a(2), a(1)) \in T$ and $x^* = a(2)$ if $(a(1), a(2)) \in T$. Hence, it follows that $c(A) = e(<, T, A) = e(<, T, A \setminus \{x^*\}) = c(A \setminus \{x^*\})$, as desired.

Consider now the choice function c defined in the proof of Theorem 4.1. We showed that c is sequentially rationalizable. To show that c is not agenda rationalizable note that since 1 diverges before 3 and 4, any agenda $<$ that rationalizes c should be of the form $3 < 1$ and $4 < 1$. Furthermore, since 3 diverges before 1 and 2, the agenda should also be of the form $1 < 3$ and $2 < 3$. But this is obviously absurd, and therefore, there is no agenda rationalizing c.$\square$

4.3. **Status-Quo Bias.** There is a large literature supporting the view that DMs typically value an alternative more highly when it is regarded as the status quo, than they would otherwise. This is the so-called status-quo bias. Masatlioglu and Ok (2005) provide a characterization of choice behavior that allows for the presence of a status-quo bias.[6]

In Masatlioglu and Ok's setting, a choice problem is a pair (A, x) where A is the set of alternatives, and $x \in A$ or $x = \diamond$. When $x \in A$, the pair (A, x) represents a choice problem with a status quo, while if $x = \diamond$, the choice problem is standard in the sense that it is without a status quo. Then, a choice correspondence is defined over the collection of all choice problems (A, x). Masatlioglu and Ok introduce a set of properties on choice behavior that characterize the following choice model of the status-quo bias. The DM evaluates the alternatives by means of a vector-valued utility function u, in a multi-criteria style. If the DM confronts a choice problem without a status quo, then she simply maximizes an aggregation h of these criteria. If there is a status quo, then the DM compares the status quo with all the alternatives in the set using all the criteria. She will stay with the status quo unless there is an alternative that dominates the status quo in terms of *all* decision criteria. This represents a marked status-quo bias. If there are alternatives that dominate the status quo in all the criteria, then the DM chooses among them using the same aggregator h as above.

We can adapt Masatlioglu and Ok's representation to our setting with single-valued choice functions defined on the collection of problem sets $\mathcal{P}(X)$. Consider the following definition, which is a reformulation of the characterization in Theorem 1 of

[6] See Apesteguia and Ballester (2008) for a characterization of choice behavior dependent on reference points in general, and for references to the empirical and theoretical literature on the status-quo bias in particular.

Masatlioglu and Ok.

Status-Quo Biased Choice Function: A choice function c is status-quo biased if there exists an element $\overline{x} \in X$, a positive integer q, an injective function $u : X \to \mathbb{R}^q$ and a strictly increasing map $h : u(X) \to \mathbb{R}$ such that:

(1) For all $A \subseteq X$ with $\overline{x} \notin A$, $c(A) = \arg\max_{x \in A} h(u(x))$.

(2) For all $A \subseteq X$ with $\overline{x} \in A$:

$$c(A) = \begin{cases} \overline{x} & \text{if } A \cap \{x \in X : u(x) > u(\overline{x})\} = \emptyset, \\ \arg\max\limits_{y \in A \cap \{x \in X : u(x) > u(\overline{x})\}} h(u(y)) & \text{if } A \cap \{x \in X : u(x) > u(\overline{x})\} \neq \emptyset. \end{cases}$$

Theorem 4.3 shows that status-quo biased choice functions are sequentially rationalizable.[7]

Theorem 4.3. *If c is a status-quo biased choice function, c is sequentially rationalizable. The converse is not necessarily true.*

Proof of Theorem 4.3: Take a status-quo biased choice function c. Define the following two classes of binary problems: $\mathcal{B}_1(X) = \{\{\overline{x}, y\} \in \mathcal{B}(X) :$ it is not true that $u(y) > u(\overline{x})\}$ and $\mathcal{B}_2(X) = \mathcal{B}(X) \setminus \mathcal{B}_1(X)$. Take any linear order $\lhd$ on $\mathcal{B}(X)$ such that for every $\{a, b\} \in \mathcal{B}_1(X)$ and $\{c, d\} \in \mathcal{B}_2(X)$, $\{a, b\} \lhd \{c, d\}$.

Take any $A \in \mathcal{P}_2(X)$ and let x^* denote the dominated alternative in the first binary problem in A according to $\lhd$. In showing that c satisfies IOIA we distinguish between four cases.

Case 1: $\overline{x} \in A$ and $\{\overline{x}, y\} \in \mathcal{B}_1(X)$ for every $y \in A$. Given that $A \cap \{x \in X : u(x) > u(\overline{x})\} = (A \setminus \{x^*\}) \cap \{x \in X : u(x) > u(\overline{x})\} = \emptyset$, then $c(A) = c(A \setminus \{x^*\}) = \overline{x}$.

Case 2: $\overline{x} \in A$ and $\{\overline{x}, y\} \in \mathcal{B}_2(X)$ for every $y \in A$. Since $A \cap \{x \in X : u(x) > u(\overline{x})\} = A \setminus \{\overline{x}\} \neq \emptyset$, then $c(A) = \arg\max_{y \in A \setminus \{\overline{x}\}} h(u(y))$. The latter being obviously $c(A \setminus \{\overline{x}\})$.

Case 3: $\overline{x} \in A$ and there are two alternatives $y, z \in A$ such that $\{\overline{x}, y\} \in \mathcal{B}_1(X)$ and $\{\overline{x}, z\} \in \mathcal{B}_2(X)$. By construction $x^* \neq \overline{x}$, and $f(A) = \{\overline{x}, x^*\} \in \mathcal{B}_1(X)$. Then $A \cap \{x \in X : u(x) > u(\overline{x})\} = (A \setminus \{x^*\}) \cap \{x \in X : u(x) > u(\overline{x})\} \neq \emptyset$, then $c(A) = \arg\max\limits_{y \in A \cap \{x \in X : u(x) > u(\overline{x})\}} h(u(y)) = \arg\max\limits_{y \in (A \setminus \{x^*\}) \cap \{x \in X : u(x) > u(\overline{x})\}} h(u(y)) = c(A \setminus \{x^*\})$.

[7] It can be proved that the set of status quo biased choice functions is contained in the set of agenda rationalizable choice function, that at the same time the latter is contained in the set of choice functions that are rationalizable by game trees. In order to illustrate the applicability of our property, IOIA, we have chosen to offer direct links between sequential rationalizability and these notions of choice.

Case 4: $\overline{x} \notin A$ then, obviously $c(A) = c(A \setminus \{y\})$ for every $y \neq c(A)$, and hence the claim follows.

To show that the inclusion is strict, consider the example in Theorem 4.1. If $\overline{x} \in \{2, 4\}$, there is a pairwise cycle in $X \setminus \{\overline{x}\}$ contradicting (1) in the definition of status quo biased choice functions. If $\overline{x} \in \{1, 3\}$, then $3 = c(1, 2, 3) \notin \{1, 2, 3\} \cap \{x \in X : u(x) > u(\overline{x})\} \neq \emptyset$, contradicting (2) in the mentioned definition.□

5. Final Remarks

In this paper we have studied the behavioral structure of sequential rationalizability, and its relation to other choice models. We have proposed a simple relaxation of the classical IIA property, IOIA, and shown that it completely characterizes the set of choice functions that are sequentially rationalizable. Further, we have shown that our property is useful in the sense of facilitating investigation of the relationships between different notions of rationalizability. We have shown that the notions of rationalizability by game trees and agenda rationalizability are both strict refinements of sequential rationalizability. Furthermore, we have established that choice functions exhibiting a status-quo bias, are also sequentially rationalizable.

Our property IOIA suggests new promising lines for future research. Concretely, it is natural to investigate the nature of the process by which binary problems are selected in certain environments. That is, to ascertain what is the essence of the rule f that makes a binary problem salient over the others. Relatedly, it looks particularly interesting to study the role of marketing strategies or of a principal in shaping the f of a consumer in a market interaction environment.

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