A Model of Technological Breakthrough in the Renewable Energy Sector

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Abstract:

Models of induced technological change often predict a gradual expansion of the renewable energy sector, and a substantial share of fossil fuels remaining in the energy mix through the end of our century. However, economic history provides examples where new products or technologies expanded rapidly and achieved a high output in just a few decades. This paper explores the possibility of a technological breakthrough in the renewable energy sector, using a partial equilibrium model of energy generation with endogenous R&D. Our results indicate, that due to increasing returns-to-scale, a multiplicity of stable states can arise. At the transition from the "lower" to the "upper" state, a regime change occurs, where output and R&D in the renewable energy sector rise discontinuously. A transition to the upper state can be triggered by a rise in world energy demand (e.g. due to economic growth), by a drop in the supply of fossil fuels, or by policy intervention. Under market conditions, the transition is likely to occur later than in the social optimum. Hence, we identify a market failure related to path-dependence, that can justify an active policy intervention. Paradoxically, well-intended energy-saving policies can lead to higher emissions, as they reduce the incentives to invest in renewable energies by having a cushioning effect on the energy price. Hence, they should be supplemented by policies that restore these incentives (e.g. a carbon tax).

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1. Introduction

Models of induced technological change often predict a gradual transition from a fossil energy based economy to a fossil and renewable based one.[1] Many authors conclude that even after several decades of capacity investments and R&D in renewable energies, a major share of world energy supply will still rely on fossil fuels.[2] However, looking back in history, there are various examples of rapid expansion of a new technology or a new type of product. E.g., who would have thought at the beginning of the 20'th century that cars might become the primary means of transportation just a few decades later? At the time, cars were slow, dangerous, and expensive. But due to ongoing innovations, they became cheaper and their quality improved, which paved the way for their massive proliferation. Or as another example, who would have predicted at the beginning of the 1980's that mobile phones would soon become a primary means of telecommunication in many countries of the world? The "walkie-talkies" that were available at the time were big, expensive, and of limited use for private communication.

Clearly, there are scale economies involved in the above examples of rapid expansion. But these are not merely economies of scale in production. It seems more likely that they are related to innovation. R&D is most valuable when it leads to an improvement of a technology that is applied to a large output quantity, which was originally not the case for cars or mobile phones. Only as output started to grow, major investments in R&D became profitable. Hence, production became cheaper and quality improved, which led to further output growth. It, thus, seems that self-enforcing processes in the interplay between production and innovation may explain the possibility of a technological breakthrough.

The present paper describes a microeconomic foundation of a technological breakthrough in the renewable energy sector. The details of the model are as follows. There are two sectors that supply energy: a fossil and a renewable energy sector. The renewable energy sector consists of two sub-sectors: an energy production sector, and an R&D sector for renewable energy technologies. Firms in the production sector invest in capacities for energy generation. The productivity of their investment depends on the technological standard of the firm, which is an increasing function of the amount of knowledge that the firm can apply. The input factor "knowledge" is supplied by the R&D sector (due to a lack of appropriability, some innovations leak to a pool of publicly available knowledge). The fossil energy sector is modeled by a simple linear supply curve. This captures the notion that the scope for cost-reducing innovations is presumably much smaller in this mature sector. World energy demand is also approximated by a linear curve. We apply a competitive equilibrium concept.

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[1] See e.g. Edenhofer, Bauer, and Kriegler (2005), Gerlagh and Lise (2005), Goulder and Schneider (1999).

[2] Gerlagh (2008), Goulder, Mathai (2000), Nordhaus, Boyer (2000), Nordhaus (2002), Popp (2004, 2006).

In the model, a multiplicity of equilibria arises. Intuitively, the incentives to innovate are increasing in the amount of capacity investments for which the innovations are finally used. However, due to perfect competition, the license fee for patented innovations *decreases* in the amount of capacity investments, which makes further investments profitable. Hence, there is *positive feedback*, giving rise to the possibility of multiple equilibria. If the investments in capacity are low, R&D firms have little incentive to produce innovations, and the license fee is high. The renewable energy sector can, thus, end up in a "low-investment trap" situation. Conversely, if the investments in capacity are high, the incentives to innovate are high, too, and the license fee is low. This makes the high capacity investments viable. Hence, two stable states can coexist: one with a higher share of renewables in the world energy mix, and one with a lower share. At the point of transition from the 'lower' to the 'upper' state, a *regime change* occurs. At this point, the share of renewables in the world energy mix *discontinuously* rises to a higher level, while the share of fossil fuels drops. This reflects our notion of a technological breakthrough. The regime change is related to increasing returns-to-scale in the renewable energy sector, based on efficiency gains through R&D.

The transition from the lower to the upper state can e.g. be triggered by an exogenous rise in world energy demand (due to economic growth)[3], or by a reduction in fossil energy supply (due to resource depletion). The exact location of the transition point depends on when private agents *coordinate* to switch to the other state. Hence, it can not be determined within our model. However, it seems plausible to assume that the transition occurs near the point where the lower state *ceases* to exist, because – once a stable state is reached, there is no reason why the energy sector would spontaneously switch to another state. The social optimum, however, requires an earlier transition. Hence, we identify a market failure related to *path-dependence*. It reflects an inefficient equilibrium selection when a multiplicity of equilibria occurs.

Apart from this, two other market failures are embedded in the model, that are related to inefficiencies in the generation and distribution of information. They prevent optimality even when the market failure related to path-dependence has already been addressed. One of these inefficiencies reflects imperfect appropriability of innovations. The other one is related to the non-rivalness property of knowledge. Whereas in the standard neoclassical world, each good can only be consumed once, innovations can be sold to *all* firms in the renewable energy sector. We, therefore, assume that licenses are issued that can be purchased by any firm. This deviates from the neoclassical approach, so the usual welfare theorems do not apply.

[3] Hoffert et al. (2002) point out that "Mid-century primary power requirements that are free of carbon dioxide emissions could be several times what we now derive from fossil fuels".

We discuss various policy instruments to eliminate the inefficiencies. We distinguish between policies to alter the equilibrium selection when multiple equilibria exist, and policies to eliminate the remaining inefficiencies in the generation and distribution of information. In some cases, the latter may actually be sufficient to overcome also the problem of equilibrium selection. Otherwise, a strong policy intervention may be required to push the energy sector into the "right" state. Once this state is reached, softer instruments can be used to eliminate the remaining sources of market failure.

Among the policy measures that can trigger a transition from the lower to the upper state, is a *subsidy* for the use of energy. Intuitively, a rise in energy demand induces upwards pressure on the energy price. This can trigger investments in the renewable energy sector and induce the transition. Conversely, well-intended energy-saving policies that are often used in practice (e.g. promotion of better heat insulation for houses or less fuel-demanding cars), have a cushioning effect on the energy price, and, thus, *reduce* the incentives to invest in renewable energies. Hence, energy-saving policies can actually be *harmful* to the climate, if they hinder the transition to the upper state. Therefore, they should be supplemented by other instruments that restore the incentives to invest in renewable energies.

In this paper, output and investment decisions over the next decades are condensed into a single period. Thus, we choose a *static* modelling framework. The static approach permits a more thorough understanding of the underlying effects that drive our results than a dynamic approach, where – due to the increased complexity – analytical results would be hard to obtain. The model is nevertheless rich enough to capture the notion of a technological breakthrough in the renewable energy sector.[4] Furthermore, we do *not* include an environmental damage function of carbon dioxide emissions. The inclusion of a damage function would not affect the market solution, as private agents generally neglect environmental externalities. However, the social optimum would then require an even earlier transition from the lower to the upper state – hence, strengthening our results further.

Related literature:

This work is related to Gerlagh and Lise (2005). Similar to these authors, we also assume that innovations are produced by an external R&D sector, and that knowledge accumulates in a linear way. One of the differences to our model is, that they treat energy from fossil and from renewable sources as *imperfect* substitutes. This assumption seems plausible e.g. in the transportation sector, or in the electricity sector when fossil energies are used to compensate

[4] In a dynamic model, the regime change we identify would translate into a discontinuous change in the *rate* of growth of the renewable energy sector.

fluctuations in the supply of renewable energy. However, it does not seem implausible to assume that some of the major technical issues concerning energy storage and conversion will be solved during the next decades. E.g., new types of batteries may stimulate market penetration of electric cars, and solar-thermal power plants may generate electricity even at night-time. Hence, fossil and renewable energy may become almost perfect substitutes in many sectors of the economy. Furthermore, Gerlagh and Lise (2005) assume that knowledge enters the production function for renewable energy in a multiplicative way, along with the current capital stock and labor. This implies that new knowledge makes old capacities (from previous periods) more productive, which is clearly not the case for wind or solar energy. We adopt an assumption made by Edenhofer, Bauer, and Kriegler (2005), namely that knowledge affects the productivity of *investments* in capacity.

Our discussion of an optimal policy mix (see Section 4) is in the spirit of Fisher and Newell (2008).[5] Among the differences between the two papers is, that we introduce a notion of technological breakthrough, whereas Fisher and Newell explicitly point out that their model is *not* suitable to analyze this type of technological change.[6] Furthermore, we offer a microeconomic foundation for this. Fisher and Newell use a technical simplification, namely the assumption of a 'representative firm'. This may be justified under certain conditions. However, in the presence of increasing returns-to-scale (due to R&D), we believe it is important to verify that the assumption of perfect competition is actually consistent with our model. If a natural monopoly situation arises, this assumption is no longer justified. Hence, for the purpose of our analysis, where increasing returns-to-scale play a crucial role, a representative firm approach seems inappropriate.

The remainder of this paper is organized as follows. Section 2 introduces a social planner version of the model, and characterizes the social optimum. The market version is introduced in Section 3. Section 4 discusses policies to implement the optimum. Section 5 concludes.

2. Social optimum

The capacity for energy generation in the renewable energy sector is denoted by K. To install a capacity of K, the planner uses two input factors: an investment good I, and knowledge a.

[5] Hart (2008) points out that if a carbon tax is the only instrument available, it should – under some conditions – be raised above the Pigouvian level to boost emissions-saving technology investments.

[6] The literature on this issue is surprisingly sparse. Unruh (2000) argues that: "industrial economies have been locked into fossil fuel-based energy systems through a process of technological and institutional co-evolution driven by path-dependent increasing returns to scale". Barett (2006) points out that: "R&D leading to breakthrough technologies exhibiting increasing returns can improve dramatically on the Kyoto approach, even when these technologies are otherwise inferior to the alternatives available".

A higher technological standard – formalized as a larger choice of a – implies that a *given* investment spending I yields *more* capacity. More specifically, we assume:

$$K = \kappa(a)I \equiv a^{\eta} I \qquad (1)$$

, where $\kappa(a)$ is an increasing function that reflects the productivity of capacity investments. Throughout the paper, we use the specification: $\kappa(a) = a^{\eta}$, where $\eta \in (0,1)$ is the elasticity of the productivity of knowledge (assumed constant).

By assumption, the installed capacity in the renewable energy sector is fully used for energy generation (there is no idle capacity). This reflects the idea that variable costs of energy generation are negligible, once the capacity is installed (think, e.g., of solar panels).[7]

Knowledge is generated via R&D investments, denoted by r. We do not assume any scale or saturation effects in the generation of knowledge.[8] Therefore, a linear relation obtains:

$$a = a_0 + \rho r \qquad (2)$$

ρ is the productivity of R&D investments, and a_0 is the initial amount of knowledge. Therefore, the amount of new knowledge generated via R&D is $a - a_0$. Note, that the planner never chooses $a < a_0$, since knowledge is productive and, thus, never wasted.[9]

The costs of renewable energy are the sum of the investment costs in capacity I, and R&D investments r. Using (1) and (2), they can be written as a function of K and a:

$$C^{ren}(K,a) = a^{-\eta} K + \frac{a - a_0}{\rho} \qquad (3)$$

The planner's problem can be divided into to tasks: 1. determine the optimal amount of capacity K in the renewable energy sector, and 2. determine the cost-minimizing combination of capacity investments I and knowledge a that yields this capacity. Let us proceed by backwards induction, and minimize the cost function $C^{ren}(K,a)$ over a first. Intuitively, (1) defines a set of isoquants (combinations of I and a that yield a fixed capacity K). The planner, thus, computes the cost-minimizing location on the isoquant that corresponds to the given target capacity K. The first-order condition (FOC) yields for the optimal amount of knowledge, given K (using (3)): $a = (\rho \eta K)^{\frac{1}{1+\eta}}$. This holds if K is sufficiently large so that $a \geq a_0$. Otherwise, the planner sets $a = a_0$. Hence:

[7] The capacity K can be interpreted as average capacity for electricity generation. Whether induced fluctuations of energy supply may be flattened by energy storage facilities.

[8] Saturation effects are already embedded in (1), since $\eta < 1$.

[9] Unless $K = 0$, but in this boundary case, we can safely assume that $a = a_0$.

$$a^*(K) = \begin{cases} (\rho\eta K)^{\frac{1}{1+\eta}} & \text{, if } K \geq a_0^{1+\eta}/\rho\eta \\ a_0 & \text{, otherwise} \end{cases} \tag{4}$$

Together with (1), (4) defines the optimal location on the isoquant in the I - a - space.[10] Substituting for a in (3) (using (4)), we obtain the following cost function: [11]

$$C^{ren}(K) = \begin{cases} (1+\eta)(\rho\eta)^{-\frac{\eta}{1+n}} K^{\frac{1}{1+\eta}} - \frac{a_0}{\rho} & \text{, if } K \geq a_0^{1+\eta}/\rho\eta \\ a_0^{-\eta}\mathrm{K} & \text{, otherwise} \end{cases} \tag{5}$$

The cost function of the renewable energy sector is, thus, *linear* in K if K is small. In this case, the planner uses only the existing stock of knowledge a_0, and does not invest in R&D. The cost function, then, reflects a constant returns-to-scale technology (K increases linearly in I – see (1)). However, when the target capacity K becomes sufficiently large, cost-reducing R&D becomes profitable, and the cost function becomes *concave* (as can easily be confirmed using (5)). This reflects *increasing* returns-to-scale, resulting from cost savings achieved via R&D activities.[12]

Let us now derive the optimal capacity K in the renewable energy sector. Let p be the world energy price. Optimality requires that K is chosen such that price equals marginal costs. Using (5), we obtain the following marginal cost function for the renewable energy sector:

$$MC^{ren}(K) = \begin{cases} (\rho\eta K)^{-\frac{\eta}{1+\eta}} & \text{, if } K \geq a_0^{1+\eta}/\rho\eta \\ a_0^{-\eta} & \text{, otherwise} \end{cases} \tag{6}$$

In a standard neoclassical model without R&D, the marginal costs are the additional costs of the next unit, while the production costs of the other units remain unchanged. Here, the optimal amount of knowledge a is embedded in the cost function. When K is marginally raised, a increases. Therefore, when an additional unit of capacity is produced, all other units become cheaper as well, due to the rise in a (unless $K < a_0^{1+\eta}/\rho\eta$: no R&D takes place).

To close the model, we need to define the supply of fossil energy and the world energy demand. Fossil energy supply is approximated by a linear supply curve:

$$S^{fos}(p) = A + \alpha p \tag{7}$$

[10] As K increases, a linear relation between a and I (on the isoquants) obtains: $a(I) = \rho\eta I$. Under market conditions (see Section 3), the location on the isoquants differs from this, as different types of market failure prevent optimality. Furthermore, the aggregate capacity K is suboptimal in the market case.

[11] It is straight-forward to verify that this cost function is continuous (there is no "jump" at $K = a_0^{1+\eta}/\rho\eta$).

[12] The linear technology in the production sector seems plausible as long as there is no scarcity of locations, e.g. for solar panels. This assumption may become problematic when the share of renewables in the world energy mix becomes large. For simplicity, we ignore this issue. However, the main effects predicted by the model would not be affected by including it, but the optimal share of renewables in the energy mix would be lower.

, where A and α are parameters. Note, that a linear supply curve corresponds to *decreasing* returns-to-scale in the technology of fossil energy generation.[13] World energy demand is also approximated by a linear curve:

$$D(p) = B - \beta p \tag{8}$$

Market clearing on the world energy market requires that (using (7) and (8)):

$$B - \beta p = A + \alpha p + K \tag{9}$$

By appropriately rescaling the units of energy, we can normalize the sum of α and β to 1 ($\beta \equiv 1-\alpha$). Furthermore, let us define $Z \equiv B - A$. Z is, thus, the *excess* demand of energy when $K = 0$, at an energy price of zero.[14] Under these assumptions, (9) simplifies to:

$$p = Z - K \tag{10}$$

Note, that Z is also the equilibrium energy price when K is zero.

We are now ready to maximize total welfare. Using the optimality condition: "price = marginal cost", equations (6) and (10) yield for the case $K < a_0^{1+\eta} / \rho\eta$: [15]

$$K = Z - a_0^{-\eta} \tag{11}$$

This is the optimal capacity choice when no R&D investments are undertaken, given that the non-negativity constraint $K \geq 0$ is fulfilled. Otherwise, the optimal capacity is zero.

For the case $K \geq a_0^{1+\eta} / \rho\eta$, we obtain the following condition:

$$\left(\rho\eta K\right)^{-\frac{\eta}{1+\eta}} = Z - K \tag{12}$$

This is a non-linear equation in K. A closed-form solution can generally not be obtained[16], but the solutions can be derived numerically. (12) has at most *two* real-valued solutions.

Lemma 1: [17]

Whenever (12) has two real-valued solutions, the solution with the larger value of K is a local maximum of the welfare function. The other solution is a local minimum and, hence, never welfare maximizing.

To understand the intuition behind the above results, it is useful to visualize the marginal cost function of the renewable energy sector (6):

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[13] E.g. due to resource scarcity, increasing extraction costs, and capacity constraints in the fossil energy sector.

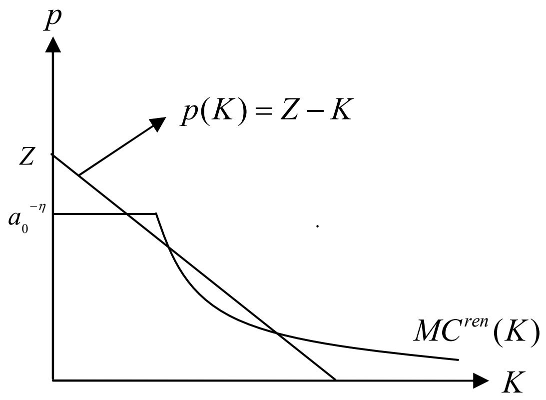
[14] Due to the linearization, fossil energy supply can be positive when the energy price is zero. This is an artifact of the model, but the main results do not depend on this. The linearization is for mathematical tractability.

[15] An alternative approach is to write down total welfare as a function of *K*, and to maximize it over *K*. This yields the same optimality conditions (11) and (12), but requires a few more steps in the derivation.

[16] Except for certain parameter values, e.g. $\eta = 0.5$.

[17] As the graphical intuition is obvious (see Figure 1, and the subsequent discussion), a formal proof is omitted.

Figure 1: Marginal cost function of renewable energy sector



The intersection points of the marginal cost curve $MC^{ren}(K)$ and the price schedule $p(K)$ are candidate solutions to the planner's maximization problem. Note, that $p(K)$ shifts upwards as Z increases. For low values of Z, no intersection point exists, and marginal costs in the renewable energy sector always exceed the price $p(K)$. Hence, in the optimum, the planner does not invest in the renewable energy sector. If Z increases, an intersection point of $p(K)$ and the horizontal segment of $MC^{ren}(K)$ at $a_0^{-\eta}$ emerges (see Figure 1). This corresponds to solution (11), with $r = 0$ (no R&D). In the following, we refer to this as the "lower solution". When Z rises further, two other intersection points emerge. These are the solutions to (12). The intersection point located in the middle is a *minimum*, because a rise in K leads to a situation where $p(K) > MC^{ren}(K)$, which implies that welfare increases in K (similarly for a reduction in K). The other intersection point is a maximum, and we refer to this as the "upper solution". It corresponds to an outcome where the planner invests both in capacities and in R&D for renewable energy generation. For some parameter values, the lower and the upper solution coexist. The following Proposition summarizes the planner's outcome for varying values of Z, that can be interpreted as changes in world energy demand (due to economic growth), or in the supply of fossil fuels (due to resource depletion).

Proposition 1:

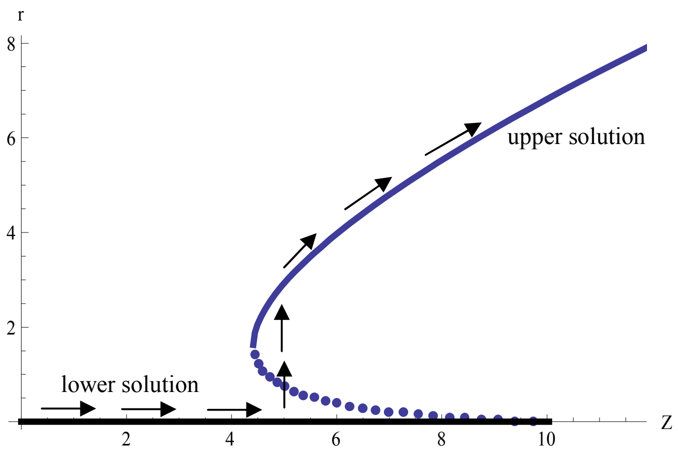
If Z is small, the planner satisfies the energy demand with fossil fuels only. If Z is in an intermediate range, the planner invests in capacities for renewable energy generation, but not in R&D.[18] If Z is sufficiently large, the planner invests in capacities and R&D for renewable

[18] For some parameter values, this intermediate range does not exist. The planner then switches directly from the lower solution with $K = r = 0$ to the upper solution with $K > 0$ and $r > 0$ (see the Appendix).

energies. Unless the initial stock of knowledge a_0 is sufficiently high, a discontinuous drop in fossil energy supply occurs, when R&D in the renewable energy sector sets in.

The technical details are in the Appendix. Here, we give a qualitative description of the planner's solution. Figure 2 shows the optimal amount of R&D investments r in the renewable energy sector, for different values of Z.

Figure 2: Optimal R&D effort r, plotted for $\rho = 0.05$, $\eta = 0.5$, $a_0 = 0.01$



The arrows in Figure 2 illustrate which type of solution the planner chooses for a given value of Z. The dotted curve shows the location of the local minimum. If Z is sufficiently small, so that the upper solution does not exist, the planner chooses the lower solution with $r = 0$, and $K \geq 0$ (depending on the parameter values). If Z is high, so that the lower solution does not exist, the planner chooses the upper solution, with capacity investments *and* R&D in the renewable energy sector. In the intermediate range where the lower and the upper solution coexist, the planner compares welfare in these two states. In the interior of this range, a critical value for Z exists, where welfare is *identical* in the two states (see the Appendix). At this point, the capacity and R&D investments in the renewable energy sector *discontinuously* rise to a higher level as Z increases. The vertical arrows in Figure 2 indicate the location of the discontinuity point.[19] At this point, the output of the fossil energy sector drops to a lower level. This is shown in Figure 3:

[19] The location of the discontinuity, as well as the solutions to (12) shown in Figure 2, have been computed numerically (using the software package Mathematica).

Figure 3: Optimal output of fossil energy sector S^{fos}, plotted for $\rho = 0.01$, $\eta = 0.5$, $a_0 = 0.025$, $A = 0$, $\alpha = 1$

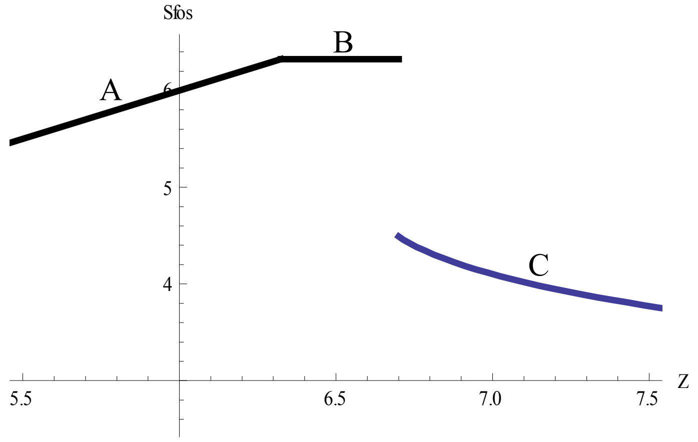


Figure 3 illustrates the transition of a fossil energy based economy to a fossil *and* renewable based one. If Z is sufficiently small (region A), the planner fulfills the entire energy demand with fossil fuels only, because at lower output levels, the marginal production costs of energy are low in this sector. However, they increase with the output level. In region B, they become as high as in the renewable sector, that operates with a constant returns-to-scale technology when no R&D investments are undertaken. R&D in the renewable energy sector becomes profitable when the capacity investments in this sector are sufficiently large. When R&D sets in, the optimal capacity level rises *discontinuously*. This leads to a drop of fossil energy supply, visible at the transition from region B to C in Figure 3. If Z rises further, the output of the fossil energy sector declines, as renewable energies become increasingly cheap.[20]

3. Market solution

In the previous section we have seen that the welfare function can have several local maxima. The social planner compares them and chooses the *global* maximum. In a market economy, the existence of more than one stable state can lead to an inefficiency. Agents will not generally be able to coordinate on the welfare maximizing state. Once a *stable* state is reached, there is no reason why the energy sector would switch to another state. Hence, there is a coordination problem related to path dependence. Furthermore, even if the "correct" state is chosen, there are other sources of market failure that prevent optimality.

[20] The share of fossil fuels in the world energy mix approaches zero if Z goes to infinity.

Let us introduce a market version of the model. Energy demand and fossil energy supply are modeled as in Section 2. However, we add a fossil energy tax τ_f, and a tax on energy consumption τ_e to the model. Therefore, (7) and (8) become:

$$S^{fos}(p) = A + \alpha(p - \tau_f) \tag{13}$$

$$D(p) = B - \beta(p + \tau_e) \tag{14}$$

Using $\beta \equiv 1-\alpha$ and $Z \equiv B - A$ as in Section 2, and $\tau \equiv \alpha\tau_f - \beta\tau_e$, (10) becomes:

$$p = Z - K + \tau \tag{15}$$

In the following, we discuss the two subsectors of the renewable energy sector.

3.1 Renewable energy production sector

Firms in this subsector are indexed by j. I_j is firm j's investment in capacity for energy generation, and K_j the resulting capacity.[21] The productivity of firm j's investment, $\kappa(a_j)$, depends on the amount of knowledge a_j applied by firm j:

$$K_j = \kappa(a_j)I_j = a_j^{\ \eta} I_j \tag{16}$$

Let s_r be an output subsidy for renewable energies. Firm j's revenues are, thus, $(p+s_r)K_j$, and the costs are I_j, plus the license fees for patented innovations. Let a_j^{priv} be the number of licenses for patented innovations purchased by firm j. The license fees are, by assumption, *linear* in the investment sum.[22] Firm j's profit, thus, equals: $\pi_j = (p+s_r)K_j - I_j - \theta a_j^{priv} I_j$, where θ denotes the price per license. Using (16), we obtain:

$$\pi_j = \left(p + s_r - a_j^{\ -\eta}(1+\theta a_j^{priv})\right)K_j \tag{17}$$

Let a be the total stock of knowledge. It is the sum of public knowledge a^{pub} and private knowledge a^{priv}. Public knowledge reflects expired patents, publicly funded R&D, and spillovers. Since it is free, all firms use the entire stock ($a_j^{pub} = a^{pub} \ \forall j$). When determining the demand for private knowledge a_j^{priv}, firms take the license fee θ as given. Maximizing π_j over a_j^{priv}, we obtain the following first-order condition (FOC):

$$(1+\theta a_j^{priv})\eta \le \theta a_j, \text{ with equality if } a_j > a^{pub} \tag{18}$$

Using (18), we can derive firm j's total demand for knowledge:

[21] Existing capacities from previous periods are zero. This is a useful approximation, given the currently small share of renewable energies in the world energy mix.
[22] Think e.g. of software where a license must be purchased for each computer.

$$a_j(\theta \mid a^{pub}) = \begin{cases} \dfrac{\eta}{1-\eta}(\theta^{-1} - a^{pub}) & \text{, if } \theta \le \eta / a^{pub} \\ a^{pub} & \text{, otherwise} \end{cases} \tag{19}$$

By (19), all firms use the *same* amount of knowledge, as the right-hand-side is independent of the index j. Furthermore, the optimal amount of knowledge is independent of the capacity K_j that the firm builds.[23] (19) reveals yet another interesting effect, namely that firm j's *total* demand for knowledge a_j is *decreasing* in the amount of public knowledge a^{pub}, unless θ is so large that the firm does not use any private knowledge ($\theta > \eta / a^{pub}$).[24]

Knowledge that is obsolete is not produced by the R&D sector. Therefore, in equilibrium, the total amount of private information matches its demand:

$$a_j^{priv} = a^{priv} \text{ , and } a_j = a \quad \forall j \tag{20}$$

Firms, thus, purchase licenses for *all* existing private innovations.[25]

In the maximization of π_j over K_j (see (17)), we obtain the following equilibrium condition:

$$p + s_r \le a^{-\eta}(1 + \theta a^{priv}) \text{ , with equality if } K > 0 \tag{21}$$

Finally note, that the assumption of *perfect competition* in the renewable energy sector is consistent with the increasing returns-to-scale technology (see (16)), because the price of the factor "knowledge" is non-linear (the fees for private innovations a_j^{priv} are linear in θ *and* in the investment I_j). The model, thus, diverts from the standard neoclassical approach, which reflects the *non-rivalness* property of knowledge (the usage of an innovation by one firm does not exclude the usage by another firm). The sector is, thus, *not* a natural monopoly.[26]

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[23] Intuitively, by choosing a_j, the firm decides over the capacity *per* investment spending. Think e.g. of wind turbines with a lower or a higher capacity: the firm first chooses the type of turbine, and then decides over I_j.

[24] The intuition is as follows. Using $a_j = a_j^{priv} + a^{pub}$, firm j's costs are: $C_j(I_j, a_j) = (1 - \theta a^{pub})I_j + \theta a_j I_j$. Hence, for a given choice of I_j and a_j, a^{pub} has similar effects as a *subsidy* on the investment I_j. Therefore, given a target capacity K_j, the firm has an incentive to use *less* knowledge, and invest more in I_j.

[25] Each private innovation is rented to *all* firms in the renewable energy production sector, as this yields the highest revenue to the innovator.

[26] To see this, note that firm j's marginal cost is: $a_j^{-\eta}(1 + \theta a_j^{priv})$ (see (17)), and (by (19)) independent of K_j. Therefore, firm j's profit is linear in K_j, so a horizontal supply curve $K_j(p \mid \theta, a^{pub})$ is obtained, as in a standard neoclassical model with a *constant* returns-to-scale technology.

3.2 R&D-sector for renewable energies

Firms in this subsector (indexed by i) produce innovations that improve the productivity of investments in capacity for energy generation. Let r_i be firm i's R&D expenditure, and let a_i^{priv} be the number of patents held by firm i. As in Section 2, we assume that knowledge accumulates in a linear way. Therefore, (2) remains valid.[27] However, we assume that private innovators can not fully appropriate the rents generated by their innovations. Let σ be the rate of appropriability.[28] We, thus, obtain for firm i: $a_i^{priv} = \sigma \rho r_i$. Aggregation over i yields:

$$a^{priv} = \sigma \rho r \qquad (22)$$

Let us introduce an R&D subsidy s_k (k for "knowledge"), that covers the share s_k of an innovator's R&D expenditure. Given that all firms in the renewable energy production sector purchase firm i's licenses (see Section 3.1), firm i's profits are: $\pi_i = \theta I a_i^{priv} - (1-s_k) r_i$, where $I = \sum_j I_j$ is the aggregate investment in capacity. Using $a_i^{priv} = \sigma \rho r_i$, this becomes:

$$\pi_i = \left(\sigma \rho \theta I - (1-s_k) \right) r_i \qquad (23)$$

The maximization over r_i (given θ and I) yields the following equilibrium condition:

$$\sigma \rho \theta I \leq 1 - s_k \text{ , with equality if } r_i > 0 \qquad (24)$$

$\sigma \rho \theta I$ is the marginal revenue of R&D, and the marginal cost is $1 - s_k$.[29] When $r_i > 0$, there is an *inverse* relationship between the license fee θ and the aggregate capacity investment I. If I is large, each innovation is used to build a large amount of new capacity. Since the marginal innovation costs are constant, and perfect competition drives θ down to a level where firms earn zero profits, the license fee *falls* when I increases. However, a reduction in θ triggers additional capacity investments, as the investment costs are lowered. Therefore, there is *positive feedback*, giving rise to the possibility of multiple equilibria.

3.3 Characterization of the market solution

As in the social planner's case, we distinguish between two types of solutions: an "upper solution" with positive capacity and R&D investments in the renewable energy sector, and a "lower solution" with $r = 0$ and $K \geq 0$ (depending on the parameter values). As we show in the Appendix, the market solution has qualitatively the same properties as the solution to the

[27] For simplicity, we assume that all existing knowledge a_0 is public.

[28] Unappropriated innovations leak to the pool of public knowledge.

[29] Due to constant-returns-to-scale (see (23)), firm i's supply function is fully price-elastic when $r_i > 0$.

planner's problem.[30] The main difference is, that – whereas the planner compares welfare in the upper and in the lower solution when both solutions coexist – under market conditions, the equilibrium selection is a matter of *coordination*. Hence, the exact location of the point of transition from the lower to the upper solution can *not* be determined within our model. However, once a stable state is reached, there is no reason why the energy sector would switch to another state. Reflecting the idea of *path-dependence*, it, therefore, seems plausible to assume that a transition to the upper state (see Figure 2) occurs near the point where the lower solution *ceases* to exist.

For the characterization of the market solution, it is useful to define: $X \equiv Z + \tau + s_r$. X combines Z (energy demand at a price of zero, net of fossil energy supply) with the policy parameters τ and s_r. Using this definition, Proposition 1 extends readily to the market case:

Proposition 2:

Under market conditions, the energy demand is satisfied exclusively by fossil fuels if X is sufficiently small. If X is in an intermediate range, there are investments in capacities for renewable energy generation, but not in R&D.[31] If X is sufficiently large, there are capacity investments for renewable energies and R&D. Unless the initial stock of knowledge a_0 is sufficiently high, a discontinuous drop in fossil energy supply occurs, when R&D in the renewable energy sector sets in.

The fossil energy sector is a mature sector of the economy. In contrast, the renewable energy sector is in its infancy. It, thus, seems plausible to assume that a large potential for cost reductions remains. Hence, speaking in the language of our model, it is possible that the renewable energy sector may be in a "low-investment trap" situation, while another stable state with high output may coexist.[32] Without active policy intervention, the low-investment trap may persist until world energy demand becomes so high (due to economic growth), or fossil energy supply so low (due to resource depletion), that a transition to the upper state will automatically be triggered. Due to path-dependence, this may happen close to the point where the lower state ceases to exist, while the *social optimum* requires an earlier transition.

[30] We also show in the Appendix that the model of Section 3 can be rewritten in a reduced form that corresponds to a standard learning-by-doing problem. This is a useful insight, because stable relations between cumulated capacity and costs have been identified empirically (IEA 2000). These "learning curves" are often associated with learning-by-doing, while technological progress in our model is based on explicit R&D activities.

[31] As in the planner's case, this intermediate range can be the empty set (depending on the parameters).

[32] Liquidity constraints may explain why an individual firm can not bridge the unstable region on its own and transfer the energy sector to the upper state with high output in the renewable energy sector.

Corollary 2:

Well-intended energy-saving policies can lead to higher emissions, if they hinder or postpone a transition from the lower to the upper state.[33]

Corollary 2 points out that – when governments are not willing or unable to introduce effective policy measures (e.g. a carbon tax) that can trigger a transition to the upper state – well-intended energy-saving policies (such as heat insulation for houses or less fuel-demanding cars) may achieve quite the opposite: by exerting downward pressure on the energy price, they can hinder the expansion of the renewable energy sector. Paradoxically, energy-saving policies can, thus, lead to *higher emissions*. Hence, they should be supplemented by other policies that restore the incentives to invest in renewable energies.

4. Optimal policy mix

There are three types of market failure that can be identified in the above model. The first, and perhaps most crucial one is the one related to path-dependence. This market failure reflects an inefficient equilibrium choice when multiple equilibria coexist. The other two market failures are related to inefficiencies in the generation and distribution of information. One is the well-known problem of imperfect appropriability of innovations, captured by the parameter σ. Furthermore, there is a market failure related to non-rivalness in the use of knowledge. R&D firms are able to sell (appropriated) innovations to *all* firms in the renewable energy production sector. Hence, licenses are issued, which can be bought by any firm. This deviates from the standard neoclassical framework, where each good is sold only once, and prices are linear in quantity. Therefore, the standard welfare theorems do not apply.

To implement the social optimum, we distinguish between policy measures to alter the equilibrium selection when multiple equilibria exist, and policies to eliminate the remaining inefficiencies in the generation and distribution of information.

Policies to alter the equilibrium selection:

As pointed out in Section 3, the transition point to the upper solution can not be determined within this model, as it depends on when private agents coordinate to switch from one stable state to another. However, reflecting the idea of path-dependence, it is plausible to assume that the transition occurs near the point where the lower state ceases to exist. Hence, any policy that pushes the energy sector beyond this point is sufficient to implement the upper

[33] A formal proof is omitted, but the graphical intuition is obvious (see Figure 2).

solution. A sufficient condition for the existence of the lower solution is derived in the Appendix (see (31)). By inspection of (31), we immediately obtain the following result:

Proposition 3:

The following policy instruments (or some combination) can be used to trigger a transition to the upper state: an output subsidy s_r or an R&D subsidy s_k for renewable energies, a fossil fuel tax τ_f, or a *negative* energy tax τ_e (hence, a subsidy for energy consumption).

The instruments s_r, τ_f, and τ_e are all embedded in the parameter X, that combines Z (energy demand at a price of zero, net of fossil energy supply) with the policy parameters τ and s_r. A (policy-induced) rise in X beyond the point where the lower state ceases to exist pushes the energy sector into the upper state.[34] The instrument s_k does not affect X, but it lowers the critical value of X where the lower solution ceases to exist. Hence, it is also an effective policy measure to alter the equilibrium selection.

Policies to eliminate inefficiencies in the generation and distribution of information:

When the inefficiency related to path-dependence has been eliminated, two sources of inefficiency remain (outlined above). Hence, (at least) two *independent* policy tools are required to reach optimality.[35] If estimates for the parameters of the model are available, one can compute the social optimum, and implement it by appropriately adjusting the instruments. We introduce an alternative approach, that does *not* require parameter estimates. The idea is to bring an equation that characterizes the market solution mathematically into the same form as the one that determines the social optimum ((12), when the *upper* state is optimal)[36].

We show in the Appendix that the market version of the model can be rewritten in a reduced form that corresponds to a standard formulation of a learning-by-doing problem. To this end, we expressed firm j's profit in the renewable energy production sector (given an optimal usage of the factor knowledge) as a function of the aggregate capacity K. A closed-form

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[34] Remember, that $\tau = \alpha\tau_f - \beta\tau_e$. A subsidy for energy consumption has similar effects as a rise in world energy demand or a drop in the supply of fossil fuels. They lead to upwards pressure on the energy price, which can trigger investments in the renewable energy sector. When environmental damages are taken into consideration, a fossil fuel / a carbon tax may be superior to the other instruments, as it has stronger effects upon emissions.

[35] E.g., τ and s_r are both embedded in the parameter X. Hence, they are *not* independent policy instruments, as both affect the energy price p and the capacity K in the same way.

[36] We focus on this case. When the lower state is optimal, no policy intervention is required, as in this case, the market outcome *coincides* with the planner's solution.

solution for $a(K)$ could only be derived for the special case $a_0 = 0$. In this case, firm j's marginal cost can be written as: $c(K) = (1-\sigma\eta)^{-1}\left(\frac{\sigma\eta}{1-\sigma\eta}\frac{\rho K}{1-s_k}\right)^{-\frac{\eta}{1+\eta}}$ (see (36)). Since perfect competition drives the energy price down to a level where firms do not earn profits, we have in equilibrium: $p + s_r = c(K)$. Use (15) and $X = Z + \tau + s_r$ to find:

$$c(K) = X - K \tag{25}$$

This is a non-linear equation in a single variable (K), which characterizes the market solution. Comparing (25) with (12), we see that they are mathematically of the same form. To reach optimality, set $\tau = s_r = 0$ (so $X = Z$). Furthermore, we must identify and adjust two other policy instruments, such that the following conditions are fulfilled: 1. $c(K) = (\rho\eta K)^{-\frac{\eta}{1+n}}$, and 2. $a(K) = (\rho\eta K)^{\frac{1}{1+\eta}}$ (see (4) and (35)). The first condition assures that, in a market equilibrium, the socially *optimal* capacity K^* is implemented. In addition, it must hold that this capacity is built with the optimal combination of capacity investments I and knowledge a.[37] This is assured by the second condition.

Unfortunately, the situation is more complicated when $a_0 > 0$. In this case, a closed-form solution for the optimal amount of knowledge $a(K)$ can *not* be derived. However, by eliminating variables, it is still possible to derive a non-linear equation in a single variable that characterizes the market solution. This equation is mathematically *not* of the same form as (12), because the existence of public knowledge a^{pub} distorts the *total* use of the factor knowledge (see (19)). To eliminate this distortion, suppose, the regulator can charge a *fee* for the use of public knowledge, equal to the fee for private knowledge. Hence, in (17), a_j^{priv} is replaced by a_j (total use of knowledge). Following the same steps as before, we find that the marginal cost in the renewable energy production sector is now: $c(K) = (1-\eta)^{-1}\left(\frac{\eta}{1-\eta}\frac{\sigma\rho K}{1-s_k}\right)^{-\frac{\eta}{1+\eta}}$. Using $p + s_r = c(K)$, we obtain once more the same mathematical form as in (12). This allows us to derive an optimal policy mix.

[37] Hence, the optimal position on the isoquant in the I - a - space is reached.

In the optimal policy mix, the R&D subsidy s_k is used to correct for the lack of appropriability. Hence: $s_k = 1-\sigma$. The other policy tool that is used to reach optimality is an *investment subsidy* s_I for capacity in the renewable energy sector: [38]

Proposition 4:

When the upper solution is socially optimal and the problem of equilibrium selection has already been addressed, an optimal policy mix is given by: $s_k = 1-\sigma$, $s_I = \eta$, and a fee for public knowledge equal to the one for private knowledge.[39]

Intuitively, η is the elasticity of the productivity of knowledge. The non-rivalness problem leads to a distortion in the usage of capacity investments and information, as firms neglect the positive externality of an increase in I_j, namely a reduction in the patent fee θ.[40] For higher values of η, the distortion becomes more severe, hence, the subsidy on capacity investments s_I must be larger in order to correct for the inefficiency.

Note, that an optimal policy mix that eliminates inefficiencies in the generation and distribution of information, may sometimes be sufficient to overcome also the problem of equilibrium selection. However, this is not generally true. Otherwise, a strong policy intervention may be required to push the energy sector into the "right" state. Once this state is reached, softer instruments can be used to eliminate the remaining sources of market failure.

5. Conclusion

We explored the possibility of a technological breakthrough in the renewable energy sector. It has been shown that – due to increasing returns-to-scale – there can be a multiplicity of stable states. The "lower state" is characterized by lower investments in the renewable energy sector, and by a higher share of fossil fuels in the world energy mix. The "upper state" is characterized by a higher share of renewables. At the transition from the lower to the upper state, a discontinuous drop in the supply of fossil fuels occurs. The transition can be triggered

[38] Firm j's costs of capacity investments are, thus, $(1-s_I)I_j$. Note, that s_I is *not* equivalent to an output subsidy s_r, as the investment I_j affects the patent fees that firm j has to pay. To derive the optimality condition $s_I = \eta$, follow the steps outlined above. For the sake of brevity, the details are not shown.

[39] The fee for public knowledge is needed to assure that the equilibrium condition in the market case is mathematically of the same form as (12).

[40] This is similar to the market failure in a learning-by-doing problem, where a larger output of a firm reduces the costs of other firms. η is similar to a 'learning rate', hence, the subsidy should increase in η.

by increasing world energy demand, a reduction in the supply of fossil fuels, or by various policy measures. Under market conditions, it is plausible to assume that the transition will take place near the point where the lower state ceases to exist. This reflects the notion of path-dependence. However, the social optimum requires an earlier transition. Hence, we identified a market failure that reflects an inefficient equilibrium selection when multiple stable equilibria coexist. Paradoxically, well-intended energy-saving policies can be harmful to the climate, as they have the potential to postpone the transition to the upper state by having a cushioning effect on energy prices. Hence, energy-saving policies should be supplemented by other policy instruments that restore the incentives to invest in renewable energies.

A positive relationship between energy prices and innovative activity in the energy sector has been identified empirically by Popp (2002). However, the identified relationship is surprisingly weak: "Even during the peak of the energy crisis, energy prices result in just a 3.14-percent increase in patents." This finding is, however, *not* a contradiction to the results of this paper. Our model predicts that – as long as the energy sector is in the "lower state" – a rise in energy prices should have *no* effect upon innovative activities.

Appendix:

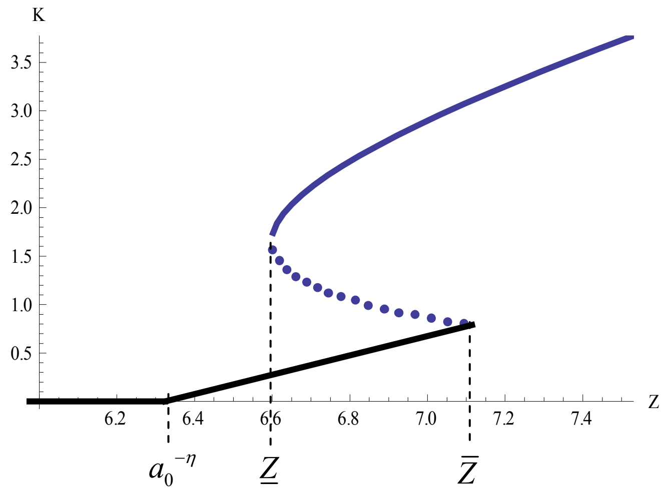
Characterization of the social optimum (Proposition 1):

We define three characteristic points along the Z - axis that are useful in the description of the planner's solution. Let $\underline{Z}$ be the critical value of Z, above which (12) has two solutions. To compute $\underline{Z}$, note that the tangency point of $MC^{ren}(K)$ and $p(K)$ (see Figure 1) is defined by $\frac{dp(K)}{dK} = \frac{dMC^{ren}(K)}{dK}$, which yields (using (6)): $K = \frac{1}{\rho\eta}\left(\frac{\rho\eta^2}{1+\eta}\right)^{\frac{1+\eta}{1+2\eta}}$. Insert this into (12) to get: $\underline{Z} = \frac{1+2\eta}{1+\eta}\left(\frac{1+\eta}{\rho\eta^2}\right)^{\frac{\eta}{1+2\eta}}$. Let $\overline{Z}$ be the critical value of Z, above which the lower solution and the local minimum cease to exist. (11) is valid if $K < a_0^{1+\eta}/\rho\eta$ holds (when $Z > \overline{Z}$, the condition is violated). Therefore, to compute $\overline{Z}$, equalize $K = Z - a_0^{-\eta}$ with $a_0^{1+\eta}/\rho\eta$, to obtain: $\overline{Z} = a_0^{-\eta} + \frac{a_0^{1+\eta}}{\rho\eta}$. The third critical is $a_0^{-\eta}$.[41] When $Z < a_0^{-\eta}$, solution (11) is negative

[41] This is the inverse of the productivity $\kappa(a)$, hence, the marginal cost of capacity investments for $a = a_0$.

and, thus, violates the non-negativity constraint $K \geq 0$. In this case, a corner solution with $K = 0$ obtains. Let us visualize (11) and (12), with the help of these critical points:

Figure 4: Optimal capacity in the renewable energy sector, for $\rho = 0.01$, $\eta = 0.5$, $a_0 = 0.025$



The dotted curve in Figure 4 is the local minimum. The Figure illustrates that when $Z < a_0^{-\eta}$, the lower solution yields $K = 0$. When $Z \in (\underline{Z}, \overline{Z})$, there are two candidate solutions: the lower solution with no R&D and $K \geq 0$ (depending on $a_0^{-\eta}$ relative to $\underline{Z}$), and the upper solution with $r > 0$ and $K > 0$. In the interval $(\underline{Z}, \overline{Z})$, there is a critical value for Z, denoted by Z^{disc} (for "discontinuity"), where welfare in the upper solution is as high as in the lower solution. If $Z > Z^{disc}$ ($Z < Z^{disc}$), the planner chooses the upper (lower) solution. When Z reaches Z^{disc}, K and r rise *discontinuously*, and the fossil output S^{fos} falls. An analytical expression for Z^{disc} can not be derived unless a closed-form solution to (12) exists. However, we can show that Z^{disc} lies in the *interior* of the interval $(\underline{Z}, \overline{Z})$. To see this, remember that the solution to (12) with the lower value of K is a minimum. However, at $Z = \underline{Z}$, the local minimum *coincides* with the upper solution. Hence, welfare *decreases* in K and is, thus, unambiguously higher in the lower solution. Similarly, at $Z = \overline{Z}$, the local minimum coincides with the lower solution. Hence, welfare is unambiguously higher in the upper solution. Note, that the interval $(\underline{Z}, \overline{Z})$ *can* be the empty set. This occurs if a_0 is sufficiently large. In that case, the technological standard of the renewable energy sector is fairly high to begin with. The discontinuity, then, disappears, and K is continuous in Z.

Characterization of the market solution (Proposition 2):

To reduce the number of variables, solve (2) for r and insert into (22) to obtain: $a^{priv} = \sigma(a - a_0)$. Use (1), that is valid also in the market case (by aggregation of (16)) in (15) to get: $I = (Z - p + \tau)a^{-\eta}$. For notational convenience, let $\varphi \equiv \sigma\eta$. φ, thus, replaces σ, that (below) will often appear as $\sigma\eta$. Finally, use (18), (20), (21), and (24) to obtain a reduced set of conditions that determines the variables of the model:

$$(p + s_r)a^{\eta} \leq 1 + \theta\sigma(a - a_0) \text{ , with equality if } K > 0 \tag{26}$$

$$\theta \geq \frac{\eta}{(1-\varphi)a + \varphi a_0} \text{ , with equality if } a > a_0 \tag{27}$$

$$\sigma\rho\theta(Z - p + \tau) \leq (1 - s_k)a^{\eta} \text{ , with equality if } r > 0 \tag{28}$$

Technically, the upper solution is an interior solution. It requires that (26) - (28) hold with equality. The lower solution is a corner solution with $r = 0$, so (27) and (28) are not binding.

Let us characterize the lower solution. Since $K > 0$, (26) holds with equality. Using $a = a_0$, we obtain: $p = a_0^{-\eta} - s_r$. The equilibrium capacity K is obtained using $I = (Z - p + \tau)a^{-\eta}$ and (1): $K = Z + \tau + s_r - a_0^{-\eta}$. Using $X = Z + \tau + s_r$ as in the main text, this simplifies to: $K = X - a_0^{-\eta}$. The patent fee θ is not fully determined, but it must be sufficiently large so that (27) is fulfilled. Using $a = a_0$, we find that: $\theta > \eta / a_0$. However, θ must not be too large so that (28) remains fulfilled (otherwise, R&D investments are triggered). Using $p = a_0^{-\eta} - s_r$, we obtain: $\theta < \frac{(1-s_k)a^{\eta}}{\sigma\rho(X - a_0^{-\eta})}$. Combining these two inequalities, we obtain a *sufficient* condition for the existence of a lower solution with $r = 0$ (after rearranging):

$$X < a_0^{-\eta} + \frac{1 - s_k}{\varphi\rho} a_0^{1+\eta} \equiv \overline{X} \tag{29}$$

By $K = X - a_0^{-\eta}$, the non-negativity constraint $K \geq 0$ is only fulfilled if: $X \geq a_0^{-\eta}$. Hence, if $X < a_0^{-\eta}$, a corner solution with $r = 0$ and $K = 0$ is obtained.

Let us turn to the characterization of the upper solution ((26) - (28), thus, hold with equality). It is convenient to eliminate θ, and to discuss the resulting equations in a and p: [42]

$$\hat{p} = \frac{a^{1-\eta}}{(1-\varphi)a + \varphi a_0} \equiv \hat{p}(a) \quad , \quad a = \frac{\varphi\rho}{1 - s_k}\hat{p}(X - \hat{p}) \equiv a(\hat{p}) \tag{30}$$

[42] Mathematically, this appears to be the simplest way of expressing the equilibrium conditions for an interior solution. Furthermore, this approach is convenient to discuss the *stability* of the solutions (see below).

, where $\hat{p} \equiv p + s_r$ is defined for notational convenience ($p = \hat{p}$ when $s_r = 0$). (30) defines two curves in the a - p - space ("$a(p)$" and "$p(a)$"). The intersection points are (candidate) interior solutions. $a(p)$ is a quadratic function (plotted against the vertical p - axis). $p(a)$ is quasi-concave and hump-shaped (see Figure 5, below). There are at most three intersection points with positive values of a and p.[43] However, the intersection point with the lowest value of a always violates the non-negativity constraint $r \geq 0$. It is, thus, *not* an equilibrium. To see this, compute the first derivative of $p(a)$ using (30) to find that the unique maximum is located at: $a = \frac{(1-\eta)\varphi a_0}{(1-\varphi)\eta} \equiv a_{\max}$. To show that $r < 0$ holds at the left intersection point, it suffices to show that $r(a_{\max}) < 0$. Insert $a_{\max}$ into (2) to obtain: $r(a_{\max}) = -\frac{(1-\sigma)a_0}{(1-\varphi)\rho} < 0$.

Hence, two candidates for an interior solution remain. However, as we show below, the intersection point of $a(p)$ and $p(a)$ located in the middle (see Figure 5) is *unstable*.[44]

To characterize the market solution, one can define characteristic points along the X - axis as in the planner's case. $\overline{X}$ is given by (29). Due to the increased complexity, a closed-form solution for $\underline{X}$ (critical value of X above which the upper solution exists) can not be obtained. Nevertheless, we can derive a condition for the coexistence of two stable solutions. To this end, consider the boundary case where $\underline{X} = \overline{X}$ holds exactly. Hence, X must be so large that the intermediate intersection point of the curves $a(p)$ and $p(a)$ yields $r = 0$ (hence $a = a_0$: this is how $\overline{X}$ is defined), *and* the two curves must be tangent (this defines $\underline{X}$). Use (30) to compute the slope of $a(p)$. Use $X = \overline{X}$ (from (29)), and $p = a_0^{-\eta} - s_r$ to obtain: $\frac{da(p)}{dp} = a_0^{1+\eta} - \frac{\varphi\rho}{1-s_k} a_0^{-\eta}$. Use (30) to compute the slope of $p(a)$. Using $a = a_0$, it becomes: $\frac{dp(a)}{da} = \frac{\varphi - \eta}{a_0^{1+\eta}}$. Now equalize the inverse of $\frac{da(p)}{dp}$ (since $a(p)$ is defined relative to the p - axis) with $\frac{dp(a)}{da}$ to obtain the following condition for the coexistence of two stable states:[45]

$$a_0 < \left(\frac{(\varphi - \eta - 1)(1 - s_k)}{\varphi\rho(\varphi - \eta)} \right)^{-\frac{1}{1+2\eta}} \tag{31}$$

[43] The intersection point at $a = p = 0$ is an artifact of a step in the derivation of $a(p)$ which requires that $p > 0$. It is, thus, not an equilibrium.

[44] We show that our stability criterion is fulfilled for the lower solution with $r = 0$, and for the intersection point of $a(p)$ and $p(a)$ with the larger value of a ("upper solution"), but not for the other intersection point.

[45] This is a necessary condition. For sufficiency, X must lie in the interval from $\underline{X}$ to $\overline{X}$.

Stability analysis of market solutions:

Suppose, a firm in the renewable energy sector unilaterally deviates from its equilibrium capacity investment, while the other firms maintain their choices of I_j. Hence, the aggregate investment I differs from its equilibrium value. To obtain tractable results, we assume that all other variables (in particular the demand and supply of innovations) adjust optimally to the new value of I.[46] *Stability* requires that an increase (a reduction) in I_j implies that a reduction (an increase) in I becomes *profitable* – hence, a deviation in the *reverse* direction. The same must hold for the R&D efforts r_i. Otherwise, the equilibrium is unstable.[47]

Stability of the solutions to (30):

Write firm j's profit in the renewable energy production sector as a function of I_j and a_j^{priv}, using (16): $\pi_j = \left((p+s_r)a_j^{\eta} - 1 - \theta a_j^{priv}\right)I_j$. Maximizing over a_j^{priv}, we obtain (using (20)): $\theta = (p+s_r)\eta a^{\eta-1}$. If I_j equals its equilibrium value, then $\pi_j = 0$. Hence, by (20):

$$(p+s_r)a^{\eta} - 1 - \theta\sigma(a-a_0) = 0 \tag{32}$$

Inserting $\theta = (p+s_r)\eta a^{\eta-1}$, and using $\hat{p} \equiv p+s_r$, we obtain once more the definition of $p(a)$ as in (30). In the R&D sector, we maximize π_i over r_i as in the main text. Hence, (24) remains valid (with equality, since $r>0$). Using $I = (Z-p+\tau)a^{-\eta}$, $\theta = (p+s_r)\eta a^{\eta-1}$, $\hat{p} \equiv p+s_r$, and $X \equiv Z+\tau+s_r$, we obtain once more the definition of $a(p)$. The difference to the earlier derivation is, that we maximized π_j over I_j (instead of K_j). This allowed us to derive the function $a(p)$ *without* using the equilibrium condition for I, namely (32). Therefore, when I differs from its equilibrium value, we move *along* the $a(p)$- curve (this is not true when we consider a change in K)[48]. The advantage of this approach is, that we can analyze deviations in I and r, using the curves $a(p)$ and $p(a)$.

Consider an out-of-equilibrium choice of I. As I increases, we move *down* the $a(p)$- curve (towards lower prices p). (32) is, thus, violated (remember, that only along the $p(a)$- curve,

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[46] The main difficulty in the stability analysis for the market case is that analytical expressions for the solutions with $r>0$ can not be obtained. We circumvent this problem, using the functions $a(p)$ and $p(a)$ (see below).

[47] Deviations in the demand for private knowledge are neglected. Note, however, that a unilateral deviation to a higher a_j^{priv} by a firm in the renewable energy production sector is not feasible at the given license fee θ, because this requires that the additional knowledge is supplied by an R&D firm. But the R&D firm can not recover its innovation costs, unless it sells its innovations to *all* firms in the production sector.

[48] To see this, note that in the earlier derivation of the curve $a(p)$, using conditions (18) and (21) (both with equality), one needs to use the function $p(a)$ to simplify $a(p)$ and bring it into the form in (30).

(32) holds, as it *defines* this curve). If the equilibrium under consideration is the one with the lower value of a (the intersection point of $a(p)$ and $p(a)$ in the middle – see Figure 5, below), an increase in I implies that the price p is becomes *greater* than the one that leads to a profit of zero in the renewable energy production sector, as the $a(p)$- curve lies *above* the $p(a)$- curve in this range. Therefore, the expression in (32) is greater than zero, which implies that a *further increase* in I is profitable. The equilibrium is, thus, *unstable*.[49]

If the equilibrium under consideration is the one with the larger value of a (see Figure 5), an increase in I implies that p is *below* the price that leads to $\pi_j = 0$, as the $a(p)$- curve lies *below* the $p(a)$- curve in this range. A deviation in the reverse direction becomes profitable. To assure stability, we must also check that a deviation in r is not profitable. Starting from the equilibrium, an increase in r implies that we move along the $p(a)$- curve to the right. Hence, the value of a is now *greater* than the one that fulfills the zero-profit condition in the R&D sector, which defines the curve $a(p)$. This leads to a patent price θ below its equilibrium value, and firms in the R&D sector incur losses. Hence, a reduction in r becomes profitable, so the equilibrium is *stable*. Figure 5 illustrates these findings.

Figure 5: Deviations in I and r from the solutions to (30)

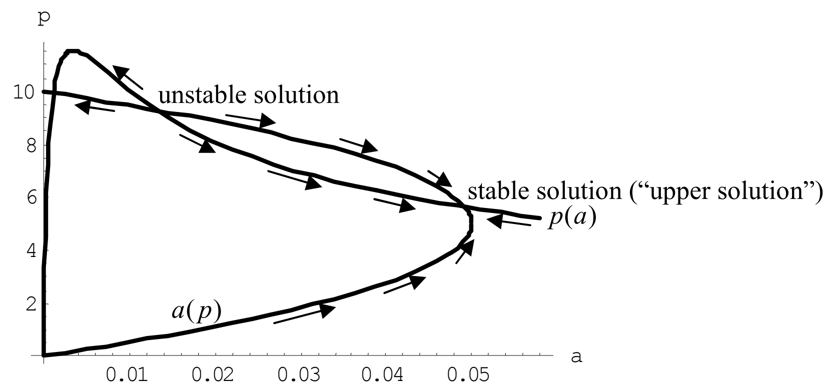

Figure 5 shows that, in the upper solution, changes in I or r make deviations in the *reverse* direction profitable. The equilibrium is stable. The other solution is unstable.

Stability of the lower solution (with $r = 0$*)*:

For $r = 0$, condition (28) is fulfilled with *inequality*. Hence, by (23), π_i becomes *negative* when r is raised above zero. Deviations in r are, thus, not profitable. Now consider a deviation in I, given the case $I > 0$ in equilibrium (so $X > a_0^{-\eta}$). Since $I > 0$, condition (32)

[49] We focus on upwards deviations in I and r. Analog results are obtained for downwards deviations.

holds. Using $a = a_0$ (since $r = 0$), it yields $p = a_0^{-\eta} - s_r$. However, an increase in I leads to a *reduction* in p (by $I = (Z - p + \tau)a^{-\eta}$). Therefore, we have: $p < a_0^{-\eta} - s_r$, which implies that $\pi_j < 0$. Hence, the deviation is not profitable.[50] Finally, consider a deviation in I, given the case $I = 0$ in equilibrium ($X < a_0^{-\eta}$). In this case, $p < a_0^{-\eta} - s_r$ holds already in equilibrium, so it holds *a fortiori* that a deviation in I is not profitable.

Relation between our model and learning-by-doing approach:

The market version of the model introduced in this paper (Section 3) can be rewritten in a reduced form that corresponds to a standard learning-by-doing problem (this holds for the upper solution with $r > 0$). To this end, use (1) in (24) (with equality), to obtain: $\theta = \frac{1 - s_k}{\sigma \rho K} a^{\eta}$. Eliminate θ using (27) (with equality), to obtain a non-linear equation in a and K. The goal is to derive an expression for a as a function of the aggregate capacity K. To obtain a *closed-form* solution for a, we must assume $a_0 = 0$.[51] After rearranging, we obtain:

$$a(K) = \left(\frac{\varphi}{1-\varphi} \frac{\rho}{1-s_k} K \right)^{\frac{1}{1+\eta}} \tag{33}$$

Use (33) and $a^{priv} = \sigma a$ (since $a_0 = 0$) in (17) to obtain after rearranging:

$$\pi_j = (p + s_r) K_j - c(K) K_j \tag{34}$$

, where $c(K) \equiv (1-\varphi)^{-1} \left(\frac{\varphi}{1-\varphi} \frac{\rho}{1-s_k} K \right)^{-\frac{\eta}{1+\eta}}$ is a decreasing function of the aggregate capacity K. This corresponds to the standard formulation of a learning-by-doing problem.

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[50] The same holds true for a reduction in I_j.

[51] When $a_0 > 0$, the correspondence to the learning curve formulation is still valid, as the equation obtained after eliminating θ *implicitly* defines a function $a(K)$. This is all that is required.

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