Access regulation and investment in Next Generation Networks - a ranking of regulatory regimes

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February 2009

Abstract

This paper analyses how different types of access regulation to next generation networks affect investments and consumer welfare. The model consists of an investment stage with uncertain returns and subsequent quantity competition. The access price is a function of investment costs and the regulatory regime. A regime with fully distributed costs or regulatory holiday induces highest investments, followed by risk-sharing and long-run-incremental cost regulation. Risk-sharing creates most consumer welfare, followed by regimes with fully distributed costs, long-run-incremental costs and regulatory holiday, respectively. Risk-sharing benefits consumers as it combines relatively high ex-ante investment incentives with strong ex-post competitive intensity.

JEL Classification: L51, L96, L10, K23

Keywords: Regulation, competition, telecommunications, broadband, strategic investment.

*We would like to thank Michal Grajek, Lars-Hendrik Roeller and Tomasso Valletti for helpful comments. We are further grateful to audiences at the European Commission, Deutsche Telekom and the European School of Management and Technology for discussions and useful comments.

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1 Introduction

The telecommunication industry is currently at the midst of a disruptive technological development. Next generation networks (NGN) allow to increase data transmission speeds in the local fixed network from the current 16Mbit/s to - at least - 100MBit/s. This enables new applications such as IP based and high definition TV as well as interactive gaming and TV.

While the technology exists today, it is uncertain when and to what extent it will be deployed by operators. Indeed, with a number of XX NGN customers per 100,000 inhabitants, NGN rollout in Europe seems to lag behind the development in the US and Asia with YY and ZZ customers per 100,000 inhabitants, respectively (UPDATE OECD COMMUNICATION OUTLOOK 2009). European telecommunication incumbents quote a tight regulatory regime, in which the investor bears the risk alone but has to share potential benefits, as a major barrier to investment.

This paper analyses investment incentives and consumer surplus under long-run-incremental cost regulation (LRIC) and three regulatory alternatives. We suppose that under LRIC the incumbent may recoup investment costs through the access price; however, only to the extent that the investment reflects the most efficient technology in providing the service. NGN will be considered an efficient technology only if the consumers value NGN-based services as otherwise the copper network is (or would be) the cheapest way to provide old services.

We compare LRIC to three alternatives:

- Fully distributed costs regulation (FDC): Under this regime the in-

\[ \text{footnote}{1}{In 2008 LRIC or long-run-average-incremental cost regulation (LRAIC) was the approach most often applied to European markets for unbundled wholesale access (64%) and wholesale broadband access (54%). The second most important approach was FDC with a share of 32% and 46%, respectively. See European Regulatory Group (2008). With respect to investment in NGN, the Commission draft recommendation suggests a risk premium (see European Commission 2008). We discuss the risk premium approach in Section 3.}


\[ \text{footnote}{3}{An alternative interpretation is that, if there is no demand for NGN applications, the entrant continues to purchase cheaper access on the basis of the copper technology.} \]
cumbent may recoup NGN investment costs through the access price, regardless of the NGN’s market success. The entrant is forced to cover part of the investment costs, thereby reducing the potential downside for the incumbent.

- Risk-sharing: Telecom operators jointly deploy and share the costs of NGN. Each operator may use the NGN for a new NGN customer without any further access payment.4

- Regulatory holiday: The incumbent is not forced to give access to its NGN and it can set the access price without regulatory oversight (at least for a certain period). Consistent to Foros’ (2004) finding, we assume that the incumbent sets a prohibitive access price.5

Our results can be broken down into three layers: (i) competitive intensity for given investment levels, (ii) investment levels and (iii) consumer surplus (combining (i) and (ii)). First, for any given investment level, we show that risk-sharing is expected to induce highest competitive intensity in the product market, followed by LRIC, FDC and regulatory holiday, respectively. Second, under uncertainty, FDC or regulatory holiday induce highest investments, followed by risk-sharing and LRIC, respectively. Third, simulation results indicate that risk-sharing induces highest consumer surplus. This results occurs due to a combination of strong ex-post competitive intensity

4In Germany, for example, incumbent Deutsche Telekom jointly deploys fibre with partners. In particular, Deutsche Telekom deploys cities Bremerhaven, Wilhelmshaven, Emden and Stade whereas EWE Tel lays fibre in Leer, Vechta, Cloppenburg, Aurich and Delmenhorst. We understand that the first to acquire a fibre customer in one of these cities can utilise the corresponding NGN without further payments for access. See dsl-magazin, "Telekom baut VDSL zusammen mit Ewe Tel aus (Upd.)", 27.1.2009, www.dsl-magazin.de, available 2.2.2009. Under a similar arrangement, Deutsche Telekom and Vodafone jointly deploy NGA in Würzburg and Heilbronn, respectively. See Financial Times Deutschland, "Telekom und Vodafone bilden Allianz", 23.12.2008, www.ftd.de, available 2.2.2009.

5Brito et al. (2008) consider a model in which the incumbent decides on the access conditions for NGN whereas the regulator decides on access conditions to the existing (e.g. copper) network. The authors establish that the incumbent may grant access to the NGN, provided that the existing network is tightly regulated.
and yet reasonable investment incentives. FDC, LRIC and regulatory holiday, generate the second, third and least desirable outcome for consumers, respectively.

The contribution of this paper lies in the intersection of a new access price formulation, a comparative evaluation of different regulatory regimes and the introduction of uncertainty. Our basic set-up follows Foros (2004) in that we consider a two stage game where the incumbent first invests in infrastructure and then competes with an entrant à la Cournot. In contrast to our paper, Foros (2004) and many other papers\(^6\) do not capture the link between investments and the regulated access price. In reality, however, the incumbent is usually allowed to recoup investment costs through the access price and it will consider this link when determining investments. Next to our model Klumpp and Su (2008) appear to be the only (and first) who model the link between investment costs and access price. In particular, they suppose “revenue-neutral open access”, an access price that lets firms share the investment costs in proportion to their usage. In a risk-free environment, revenue-neutral open access induces more investment than regulatory holiday (e.g. monopoly). Under uncertainty, the result is reversed and the authors propose regulatory holiday as a remedy. However, this recommendation is based solely on the effect on investments; our results suggest that, even under uncertainty, regulatory holiday is the least preferable option for consumers. Further, Klumpp and Su (2008) do not analyse the cases of risk-sharing and FDC.

Another strand of literature applies dynamic models to analyse the incentive to delay investment into (NGN) infrastructure. Hori and Mizuno (2007) employ a real options approach to compare investment incentives under service- and facility-based competition. Their results confirm the trade-off between the desire to induce early service based competition and stronger incentives to invest early under facility based competition.\(^7\) Our model, in


\(^7\)Similar results are obtained by Gavosto et al. (2007) who also provide an applied analysis of NGN investments under a real options approach.
contrast, supposes service based competition but accommodates different regu-
latory means to that end. Earlier dynamic models of regulated infrastruc-
ture investments include Gans (2001), Gans and King (2004), Bourreau and
Dogan (2005) and Hori and Mizuno (2006). These models, in contrast to
our, assume no link between investments and access prices. Further they
are more limited in terms of different regulatory regimes, most notably as
regards risk-sharing.

The remainder of this paper is organised as follows. Section 2 presents the
model. Section 3 discusses extensions and limitations. Section 4 concludes.

2 The Model

We first present the basic modelling framework. Then the different regula-
tory regimes, LRIC, FDC, risk-sharing and holiday, are explained in some
more detail. The third subsection analyses the product market (second-
stage) equilibrium, i.e. the competitive intensity for given investment levels
and depending on the regulatory scenario. The fourth subsection determines
optimal investments (first-stage equilibrium) and the fifth subsection com-
pares consumer welfare in different regulatory regimes.

2.1 The basic framework

Consider a two-stage setting. In the first stage, an incumbent, $I$, invests in
non-duplicable network infrastructure which we interpret, for concreteness,
as NGN infrastructure. In the second stage the incumbent and an entrant, $E$,
compete in the product market. The entrant’s access to the new infrastruc-
ture is regulated by a known regulatory regime. We describe each stage in
more detail below.

In the first stage, the incumbent determines the extent of NGN deploy-
ment, $x$. NGN deployment requires investments of the form $(\gamma/2)x^2$. The
convex form accounts for the fact that deploying a given number of fibre to

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8Guthrie (2006) summarises the earlier literature dealing with investments in regulated

9As NGN investments are only undertaken by the incumbent we avoid a subscript, $I$. 

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home connections becomes more expensive as NGN rollout is extended to rural, less densely populated areas. Market success of NGN is uncertain. While NGN enables higher quality and new services, it is only with probability $\beta$, $0 < \beta < 1$, that consumers’ willingness to pay increases by $x$. With probability $(1 - \beta)$ consumers’ willingness to pay does not increase, despite of higher quality and new services.\(^{10}\) With this formulation we specifically refer to regions in which there is uncertainty about NGN success. In other regions there may be certainty: in some urban regions NGN deployment might be relatively cheap and, hence, profitable due to operating cost savings alone while in some rural areas high NGN deployment costs may be prohibitive for all plausible expectations of demand. Our analysis is relevant for areas between these two extremes.

In the second stage the incumbent and the entrant learn about NGN success and compete à la Cournot. If NGN is successful, the incumbent faces (inverse) demand

\[
P^s_I = A + x - q_I - q_E,
\]

where superscript $s$ denotes the success case, $A$ is the reservation price, $x$ is the extent of NGN deployment and $q_I$ and $q_E$ denote the incumbent’s and the entrant’s output quantities, respectively. The entrant faces demand

\[
P^s_E = A + \phi x - q_I - q_E,
\]

where $\phi$ is a regulatory parameter that we specify below. If NGN is not successful, firms face demand

\[
P^f_i = A - q_i - q_j, \quad i = I, E, \quad i \neq j,
\]

where superscript $f$ denotes the failure case. In the failure case consumers’ willingness to pay does not increase beyond the pre-existing level. For example, consumers may not value the new services such as IPTV, HDTV or interactive gaming and TV.\(^{11}\)

\(^{10}\)Of course, in reality there would be a continuum of states rather than the two polar cases depicted by the model. However, the essential feature, i.e. that there is uncertainty about consumers’ willingness to pay, is captured by the model.

\(^{11}\)Note that demand equations (2) and (3) imply positive demand for services by the incumbent even if it does not get access to NGN ($\phi = 0$) or NGN fails ($x = 0$). In these cases the NGN entrant continues to provide copper based telecommunication services.
If the entrant gets access to the new technology ($\phi = 1$), it has to purchase the inputs from the incumbent.\footnote{The entrant may require more inputs from the incumbent. Our focus is, however, on the input associated to the incumbent’s new investments. We therefore assume that the entrant obtains all other inputs on a non-discriminatory basis, i.e. at the same marginal costs as the incumbent.} We propose an unit access price, $w$, that spreads investment costs over total second stage output quantities,

$$w^\ell = \frac{\alpha^\ell (\gamma/2) x^2}{q_I + q_E}, \quad \ell = s, f,$$

where $\alpha^\ell$ is a regulatory parameter depending on whether the new technology is successful, denoted by superscript $s$, or fails, denoted by superscript $f$. We further specify the different regulatory settings below. The access price formulation in (4) is relatively new to the literature. To the best of our knowledge, only Klumpp and Su (2008) have proposed a similar formulation. Previous models such as Foros (2004) and Kotakarpi (2006) typically assume that the access price can be freely set by the incumbent and then forecloses entrants, or that it is regulated down to marginal costs (or zero). However, these formulations seem to reflect rather extreme cases compared to actual regulatory practice. Indeed, regulated access prices are commonly a function of network costs, including depreciation, and hence investments. Both the incumbent and the entrant bear constant marginal costs of production and distribution, $c$.

We now have all elements at hand to specify the incumbent’s and the entrant’s maximisation problems in the second stage. If NGN turns out successful, the incumbent maximises

$$\pi^s_I = (P^s_I - c)q_I + \alpha^s (\gamma/2) x^2 \frac{q_I + q_E}{q_I + q_E}$$

with respect to $q_I$ and the entrant

$$\pi^s_E = (P^s_E - c)q_E - \alpha^s (\gamma/2) x^2 \frac{q_I + q_E}{q_I + q_E}$$

with respect to $q_E$. If NGN fails, the incumbent and the entrant maximise

$$\pi^f_I = (P^f_I - c)q_I + \alpha^f (\gamma/2) x^2 \frac{q_I + q_E}{q_I + q_E}$$

$$\pi^f_E = (P^f_E - c)q_E - \alpha^f (\gamma/2) x^2 \frac{q_I + q_E}{q_I + q_E}$$
and

$$\pi_I^f = (P_E^I - c)q_E - \alpha_f \frac{(\gamma/2)x^2}{q_I + q_E}$$  \hspace{1cm} (8)$$

with respect to $q_I$ and $q_E$, respectively.

### 2.2 Regulatory settings

The main purpose of this paper is to analyse how different regulatory policies affect incumbents’ investment incentives and consumer surplus. The framework introduced above allows us to capture essential elements of the following regulatory policies: i) long-run-incremental cost regulation (LRIC), ii) fully distributed costs regulation (FDC), iii) risk-sharing and iv) regulatory holiday. In the first three regimes the entrant gets access to the infrastructure, $\phi = 1$, whilst under regulatory holiday, it does not, $\phi = 0$. Our framework allows for additional regulatory settings that will be discussed in Section 3.

We consider LRIC as the current regulatory counterfactual.\(^{13}\) One essential element of LRIC is that the incumbent may recoup investment costs through its access price but only if the investment reflects the most efficient means of providing certain services. Within our framework this means, if NGN is successful, the incumbent may pass-on investment costs to the entrant via the access price, $\alpha^s = 1$. On the contrary, if NGN is not successful, we model the regulatory outcome via $\alpha^f = 0$.

As a second regulatory option we consider FDC that allows the incumbent to recoup NGN investment costs in both the success case, $\alpha^s = 1$, and the failure case, $\alpha^f = 1$. The latter case could be implemented, for example, if the incumbent solely wholesales fibre based access (where it is available), so as to recoup its costs. Alternatively, the incumbent might be given some discretion in its access prices as long as a range of certain products does not exceed an average price cap (price cap baskets). In a basket containing copper and fibre based access, the incumbent could price these equally and thereby fully distribute its costs of NGN deployment.

Risk sharing is an option that is currently discussed by operators and regulators alike. Under this mode the incumbent and the entrant decide

\(^{13}\)See footnote 1.
jointly to deploy a certain region and to share the costs and risks of this investment. Practically, a joint-venture might deploy a region and whoever wins a customer has the right to utilize NGN. Alternatively one could require the entrant to commit to a certain number of NGNs ex ante. Under this option it makes sense to assume that the incumbent (or whoever carries out the actual investment) invests, so as to maximise firms’ joint profits, i.e. $E(\Pi_I + \Pi_E)$, where $E(\Pi_i), i = I, E$, are the incumbent’s and the entrant’s expected profits in the investment stage. We assume here that the incumbent and the entrant agree ex-ante how they share the costs and benefits of the investment; there are no ex-post access arrangements, $\alpha^s = 0$ and $\alpha^f = 0$. We discuss alternative scenarios in Section 3.

Under regulatory holiday the incumbent obtains the right to exploit its investments exclusively.\textsuperscript{14} We can capture this option in setting $\phi = 0$ in (2), so that the entrant cannot offer the high quality services. Of course, the entrant had not to cover any investment costs, $\alpha^s = 0$ and $\alpha^f = 0$. Table 1 summarises the regulatory settings considered alongside with the relevant model parameters.

<table>
<thead>
<tr>
<th>Setting, $\phi$</th>
<th>Model parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRIC, $L$</td>
<td>$\phi = 1$, $\alpha^s = 1$, $\alpha^f = 0$, $\max E(\Pi_I)$</td>
</tr>
<tr>
<td>FDC, $FD$</td>
<td>$\phi = 1$, $\alpha^s = 1$, $\alpha^f = 1$, $\max E(\Pi_I)$</td>
</tr>
<tr>
<td>Risk-sharing, $RS$</td>
<td>$\phi = 1$, $\alpha^s = 0$, $\alpha^f = 0$, $\max E(\Pi_I + \Pi_E)$</td>
</tr>
<tr>
<td>Regulatory holiday, $H$</td>
<td>$\phi = 0$, $\alpha^s = 0$, $\alpha^f = 0$, $\max E(\Pi_I)$</td>
</tr>
</tbody>
</table>

Table 1: Regulatory regimes and model parameters

2.3 Second stage equilibrium

In the second stage firms maximise profits with respect to output quantities, given the extent of NGN deployment, the regulatory regime and NGN success.

Consider first the cases in which NGN turns out successful, $\beta = 1$, and the entrant gets access to the new infrastructure, $\phi = 1$ (i.e. cases $L, FD, RS$).

\textsuperscript{14}In practice exclusivity might be limited to a certain period of time.
By (5) and (6) the first-order conditions are

\[
\frac{\partial \pi^s_i}{\partial q_i} = A - c - 2q_i - q_j + x - \frac{\alpha^s \gamma x^2}{2(q_i + q_j)^2} = 0, \quad i = I, E, i \neq j.
\] (9)

In order to make the setup relevant, we assume parameters so that NGN investments increase equilibrium output quantities in the success case

\[
\frac{\partial^2 \pi^s_i}{\partial q_i \partial x} = 1 - \frac{\alpha^s \gamma x^2}{2(q_i + q_j)^2} > 0 \iff \gamma < \frac{(q_i + q_j)^2}{\alpha^s q_j x}, \quad i = I, E, i \neq j.
\] (10)

According to (10) we require the investment cost parameter \(\gamma\) to be sufficiently small so that the demand enhancing effect of NGN dominates the marginal cost increasing effect. This assumption is probably never binding, because a larger \(\gamma\) will decrease the equilibrium \(x\) in the denominator of (10). However, its notion is helpful to show that second-order conditions are satisfied (see appendix for details). The first-order conditions (9) can be solved for

\[
q^s_{i\varphi} = \frac{8(A - c + x) + \sqrt{64(A - c + x)^2 - 96\alpha^s \gamma x^2}}{48}, \quad i = I, E, \varphi = L, FD, RS.
\] (11)

Next, if NGN is successful, \(\beta = 1\), and the entrant does not get access, \(\phi = 0\) (i.e. case \(H\)), maximisation of (5) and (6) imply equilibrium quantities of the form

\[
q^s_{I\varphi}^H = \frac{A - c + 2x}{3},
\] (12)

\[
q^s_{E\varphi}^H = \frac{A - c - x}{3}.
\] (13)

Equation (11) reveals two interesting insights. First, NGN deployment, \(x\), increases output quantities, as expected, because the demand curve shifts outwards. However, for \(\alpha^s > 0\), there is also a countervailing effect of \(x\) because investment costs are recouped through the access price and thereby increase the entrant’s marginal costs. Second, whenever the entrant has access to the new technology, \(\phi = 1\), by (11) output quantities are symmetric. This might surprise at first glance because by (5) and (6) investment costs, which are spread over the access price \(w\), enter the incumbent’s profits as a marginal revenue and the entrant’s as a marginal cost. However, an access
to NGN not sold to the entrant bears an opportunity cost in form of an additional investment fraction not born by the entrant. Therefore, using a NGN bears the same economic costs for the incumbent and the entrant alike; the equilibrium is symmetric. This result is consistent to Klumpp and Su’s (2008) access price formulation that supposes revenue neutrality. By (12) and (13) NGN investment causes an asymmetric market structure in the case of regulatory holiday.

If NGN turns out non-successful, \( \beta = 0 \), the incumbent and the entrant maximise (7) and (8) with respect to \( q_I \) and \( q_E \) respectively. We obtain

\[
q_i^* = \frac{8(A - c) + \sqrt{64(A - c)^2 - 96\alpha^2\gamma x^2}}{48}, \quad i = I, E, \quad \varrho = L, FD, RS, H.
\]  

(14)

Again, the equilibrium is symmetric for reasons pointed out above.\(^{15}\)

By (11), (12), (13) and (14) the various regulatory settings imply different degrees of competitive intensity for any given level of NGN deployment. Specifically, let \( E(Q^\varrho(x)) \) denote expected industry output given investment \( x \) and regulation \( \varrho = L, FD, RS, H; \)

\[
E(Q^\varrho(x)) = \beta(q_i^\varrho + q\varrho_E) + (1 - \beta)((q_i^I + q_E^I), \quad \varrho = L, FD, RS, H. \]  

(15)

We can state:

**Proposition 1**  
Expected industry output \( E(Q^\varrho(x)) \) for a given level of NGN deployment, \( x \), and, \( 0 < \beta < 1 \), satisfies

\[
E(Q^{RS}(x)) > E(Q^L(x)) > E(Q^{FD}(x)),
\]

and

\[
E(Q^L(x)) > E(Q^H(x)) \iff x < \frac{2(A - c)}{3}\gamma - 2.
\]

Proof. See appendix.

The first inequality, \( E(Q^{RS}(x)) > E(Q^L(x)) \), is driven by the fact that a risk-sharing regime induces more output than LRIC, provided that NGN is successful. This results because the LRIC regime allocates investment costs to firms’ second stage marginal costs in the success case and thereby reduces

\(^{15}\)The second order conditions are discussed in the appendix.
competitive intensity relative to risk-sharing. Yet, risk-sharing and LRIC induce the same industry output if NGN is not successful because neither regime allocates any investment costs to second stage marginal costs.

The second inequality, \( E(Q^L(x)) > E(Q^{FD}(x)) \), arises because LRIC induces more output than FDC, provided that NGN is not successful. Recall that the FDC regime allocates investment costs to firms’ second stage marginal costs even if the investment is not successful. Therefore FDC reduces competitive intensity relative to LRIC in the failure case. In contrast, the regimes are identical if NGN is successful, either regime allocating investment costs to second stage marginal costs.

Third, \( E(Q^L(x)) > E(Q^H(x)) \), follows because, in the success case, the symmetric market structure implied by LRIC induces more output than the asymmetric (concentrated) market structure induced by regulatory holiday. For large given investment levels, however, this result would be reversed. In such cases, if they would arise, output reduction through firms’ large marginal costs as implied by LRIC would be stronger than through the asymmetric market structure caused by regulatory holiday (with no marginal costs of NGN deployment). As we will see later equilibrium NGN investment will not exceed the critical threshold given by Proposition 1. As regards the non-success case, notice that the LRIC and the holiday regime induce the same output.

2.4 First stage equilibrium

Let \( P_i^e \) denote \( P_i^s \) where equilibrium quantities, \( q_i^e \), are substituted for \( q_i \) and, respectively, \( w_i^e \) denote \( w_i \) given equilibrium quantities. The incumbent’s and the entrant’s first stage expected profits are

\[
E(\Pi^I_E) = \beta \left[ (P^e_I - c)q_i^e + \alpha^e w_i^e q_i^e - \frac{\gamma}{2} x^2 \right] + (1 - \beta) \left[ (P^e_I - c)q_i^e + \alpha^e w_i^e q_i^e \right], \quad \varrho = L, FD, RS, H. \tag{16}
\]

and

\[
E(\Pi^E_E) = \beta \left[ (P^e_E - c)q_E^e - \alpha^e w_i^e q_i^e \right] + (1 - \beta) \left[ (P^e_E - c)q_E^e - \alpha^e w_i^e q_i^e \right], \quad \varrho = L, FD, RS, H. \tag{17}
\]
Notice that the incumbent’s and entrant’s expected profits differ in that i) the incumbent earns access revenues whilst the entrant incurs access costs and ii) only the incumbent incurs NGN investment costs.

The incumbent’s maximisation problem depends on the regulatory regime. Consider first the cases of LRIC and FDC. By (16) and keeping in mind that \( q_c^l = q_c^e \), \( l = s, f \), therefore \( q_c^l / (q_c^l + q_c^e) = 1/2 \) and \( \partial(q_c^l / (q_c^l + q_c^e)) / \partial x = 0 \), we can write the first-order condition as

\[
\frac{\partial E(\Pi^0)}{\partial x} = \beta \left[ (A - c + x - 4q^s) \frac{\partial q^s}{\partial x} + q^s \right] \\
+ (1 - \beta) \left[ (A - c - 4q^f) \frac{\partial q^f}{\partial x} \right] \\
- \left[ 1 - \frac{1}{2} (\beta \alpha^s + (1 - \beta) \alpha^f) \right] \gamma x = 0, \quad \varrho = L, F.D. \tag{18}
\]

The first two terms of (18) refer to the incumbent’s marginal gain from NGN investment. The third term reflects marginal costs of NGN deployment. As can be seen, under LRIC regulation (\( \alpha^sL = 1, \alpha^fL = 0 \)) marginal investment costs are effectively reduced by 1/2, but only in the success case, with probability \( \beta \). Under FDC (\( \alpha^sFD = 1, \alpha^fFD = 1 \)) the incumbent is ensured that also in the failure case, with probability \( (1 - \beta) \), the entrant bears its share of investment costs.

Consider next the case of risk-sharing. As the incumbent and the entrant negotiate the terms and conditions of risk-sharing ex ante, they can deploy NGN so as to maximise their joint expected benefits. Keeping in mind the assumption that investment costs are not allocated to second stage marginal costs, \( \alpha^sRS = 0 \) and \( \alpha^fRS = 0 \), we have by (16) and (17)

\[
\frac{\partial (E(\Pi^RS) + E(\Pi^RS_E))}{\partial x} = 1/9(4\beta(A - c + x) - 9\gamma x) = 0, \tag{19}
\]

which can be solved for

\[
x^{RS} = \frac{4\beta(A - c)}{9\gamma - 4\beta}. \tag{20}
\]

In the case of regulatory holiday, the incumbent maximises (16) with respect to \( x \), where \( \alpha^sH = 0 \) and \( \alpha^fH = 0 \),

\[
\frac{\partial E(\Pi^H)}{\partial x} = 1/9(4\beta(A - c + 2x) - 9\gamma x) = 0, \tag{21}
\]
which can be solved for
\[ x^H = \frac{4\beta(A - c)}{9\gamma - 8\beta}. \] (22)

Now we are interested in the comparative degree of NGN deployment under the various regulatory regimes. In particular we have:

**Proposition 2** Suppose \( \gamma > 2 \), equilibrium NGN deployment satisfies
\[
\begin{align*}
&x^{FD}, x^H > x^{RS}, \\
&x^{RS} > x^L \quad \text{for all } 0 < \beta \lesssim 0.85.
\end{align*}
\]

Proof. See appendix.

Proposition 2 states that FDC and regulatory holiday induce more investment than risk-sharing (provided \( 0 < \beta < 1 \)). Furthermore, when risk matters, \( 0 < \beta \lesssim 0.85 \), FDC, regulatory holiday and risk-sharing all induce more investment than LRIC regulation. The assumption of a sufficiently high cost parameter \( \gamma \) ensures concavity of the incumbent’s first stage profit-function and is standard in the literature. However, \( \gamma > 2 \) is slightly more than is actually needed for second-order conditions. We make this assumption as it simplifies the formal proofs substantially.

The intuition behind Proposition 2 is as follows. First, FDC induces more investments than risk-sharing because the latter does not allow for the firms to allocate investment costs to their second stage marginal costs. Indeed, under risk-sharing, NGN deployment costs are entirely sunk in the second stage, leading to intensive product market competition and, consequently, somewhat modest first stage investment incentives. This result, however, is likely to rely on our assumption that using a NGN involves no money transfers among firms, once the investment is made. While this is a possible implementation of risk-sharing, there are alternatives. We discuss possible implications of these alternatives in Section 3.

Second, regulatory holiday dominates risk-sharing because, if NGN is successful, investments under holiday create a competitive advantage to the incumbent. Driven by this possible advantage, the incumbent invests intensively.

Third, all modes lead to more investments than LRIC regulation, provided that risk matters, \( 0 < \beta \lesssim 0.85 \). In particular, LRIC induces lower
investments than risk-sharing, as under LRIC the incumbent has to share the benefits of success but bears the costs alone in the failure case. Risk-sharing, in contrast, allows firms to share the benefits and costs, thereby stimulating investments. The intuition with respect to FDC and regulatory holiday, as explained above, holds respectively.

2.5 Consumer welfare

The previous two subsections revealed that risk-sharing induced both stronger competition for any given level of investment and more investment than LRIC regulation. Thereby consumer surplus is unambiguously higher under risk-sharing than under LRIC. However, with respect to other modes of regulation, competitive intensity and investment incentives go in opposite directions. This section consolidates the afore separate measures of competitive intensity and investment incentives into a consumer surplus analysis.

Unfortunately, as we cannot solve for investment levels under LRIC and FDC, we cannot substitute these for $x$ in (11) - (14). By the same token we cannot provide an analytical consumer surplus comparison. However, we offer a numerical assessment. The qualitative results as stated below do not depend on the remaining parameters $(A - c)$ and $\gamma$. For the sake of brevity we do not include the sensitivity check here. It is available upon request.

Expected consumer surplus is

$$E(CS^\varphi) = \frac{\beta}{2} (q_i^{s_i}(x^\varphi) + q_E^{s_E}(x^\varphi))^2$$

$$+ \frac{1 - \beta}{2} (q_i^{f_i}(x^\varphi) + q_E^{f_E}(x^\varphi))^2, \quad \varphi = L, F, RS, H, \tag{23}$$

where we substitute the numerical solutions to (18), (19) and (21), $x^\varphi$, into the respective output quantities, $q_i^\varphi, i = I, E$, as given by (11), (12), (13) and (14). Below we provide a comparison of expected consumer surplus in the LRIC, FDC, risk-sharing and holiday mode, for different degrees of success probability, $\beta$. The example assumes the other parameters at $A = 100$, $c = 20$ and $\gamma = 5$.

Figure 1 below displays expected consumer surplus under the regulatory alternatives, FDC, risk-sharing and holiday relative to the LRIC counterfac-
Figure 1: Additional consumer surplus from regulatory alternatives risk-sharing (RS), FDC (FD) and holiday (H) relative to LRIC (L). 

Based on Figure 1 and sensitivity checks across the relevant parameter range we state

**Remark 1** Expected consumer surplus satisfies

\[ E(CS^{RS}) > E(CS^{FD}) > E(CS^{L}) > E(CS^{H}). \]

The intuition behind Remark 1 is as follows. Risk-sharing yields highest expected consumer surplus due to a combination of strong competitive intensity (see Proposition 1) and yet reasonable investment incentives (Proposition 2). Strong competitive intensity stems from investment costs not increasing firms’ second stage marginal costs while risk-sharing allows firms to jointly internalise all costs and benefits associated with the risky investment. Noteworthy, risk-sharing remains superior even in a certain environment, \( \beta = 0 \), due to the strong competitive intensity implied by it.

FDC yields higher expected consumer surplus than LRIC (and regulatory holiday). The incumbent is ensured to share investment costs not only in
the success but also in the failure case. Here the positive effects from higher investments dominate the fact that FDC results in lower competitive intensity than LRIC in case the investment fails.

Yet, LRIC leads to a better outcome for consumers than regulatory holiday. Regulatory holiday provides strong investment incentives but driven by the prospect of higher market power ex-post. From the consumers’ perspective the positive effects of high investment do not make up for the negative effects caused by the incumbent dominating the new technology.

3 Discussion and extensions

The main ingredients of our model are i) the regulated access price being linked to investment costs and ii) uncertainty. These features allow us to compare the effects of different regulatory approaches within one consistent framework. Yet the analyses carried out in this paper are by no means complete. Below we discuss extensions in terms of additional policy approaches as well as towards less restrictive assumptions.

3.1 Additional regulatory policies

Next to the policies analysed in this paper, recently, the risk-premium concept has attained much interest by regulators.\textsuperscript{16} To compensate the investor for bearing the risk of failure alone, it may be allowed to charge a higher access price in the success case. This policy could be captured by $\alpha_s = (1 + \rho)$ and $\alpha_f = 0$, where $\rho$ refers to the risk-premium as a percentage of the risk-free access price (where $\phi = 1$). A risk-premium may remove the structural disadvantage of investing. However, we suspect it is less likely to serve regulators’ objectives. If the success probability is rather low, a risk-premium has a low investment leverage because it is unlikely to become effective. If the success probability is relatively high, a risk-premium has a strong leverage; however, it then facilitates an asymmetric ex-post market structure exactly when investment incentives were rather high in the first place. That is, by

its very concept of rewarding risk-taking if and only if the investment is successful, a risk-premium may either turn out non-effective or competitively disruptive.

Further we focus on ex-ante regulation while, in practice, the incumbent’s access prices are also prone to ex-post regulation. Suppose a FDC regime, subject to a non margin squeeze obligation. In the failure case the incumbent is allowed to recoup its investment costs through a higher access price for NGN. As demand expectations have not materialised, however, the incumbent may have limited incentives to set a high retail price for NGN. In such situations the incumbent might have an incentive to margin squeeze without an anti-competitive intention. If competition law prevents the incumbent from doing so, it will revise its investment plans in the first place. This raises the question of whether a non margin squeeze obligation is beneficial for consumers in the context of investments under uncertainty.

Finally, we propose a risk-sharing arrangement in which the incumbent and the entrant do not compensate each other for NGN usage, once the investment is undertaken. While this formulation seems justified in light of the anecdotal evidence,\textsuperscript{17} it is by no means the only possibility to implement risk-sharing. First, firms may undertake a formal joint venture to deploy NGN. The joint venture in turn might be required to be profitable (e.g. for tax reasons). Second, the incumbent and the entrant might also determine the joint venture’s access prices so as to maximise their joint expected profits. A joint venture that sets high access prices for NGN might relax retail competition to the detriment of consumers. Hence, our positive risk-sharing result is likely sensitive as to how it is actually implemented.

\section{Model assumptions}

Our framework is based on some assumptions that might be critical. First, the assumption of Cournot competition restricts the possible intensity of product market competition. Consumer surplus is determined by the interplay of competitive intensity and investment incentives (as a function of, among other, competitive intensity). We cannot exclude, therefore, that al-

\textsuperscript{17}See footnote 4.
ternative assumptions on competitive intensity might affect the comparative performance of the regulatory regimes. The sensitivity of our results could be checked within a Hotelling framework or by introducing a substitution parameter and \( n \) firms within the given Cournot framework.

Second, indeed not only for the purpose of reflecting different degrees of competition, an extension to \( n \) firms would be desirable. Intuitively, there might be an optimal number of entrants that trades off ex-post competitive intensity against sufficient investment incentives. We suspect, however, that within the given Cournot framework, for any given regulatory regime, an increase in the number of entrants monotonically increases consumer surplus (i.e. the effect from additional ex-post competition is always stronger than the effect of additional investment).

Third, other than the exogenous investor / non-investor role, we impose no asymmetry on the incumbent and the entrant. However, when endogenising the investor / non-investor role, one may find that non-investors (or entrants) may suffer from smaller customer bases, higher risks and weaker financial positions.\(^{18}\) As a matter of fact, many entrants continue to have lower market shares than incumbents on the legacy network. We should, thus, like to explore whether asymmetries, e.g. in terms of market shares, affect our qualitative results.

While all the above extensions are interesting and desirable they will further complicate analytical solutions. We believe that, at least for the purpose of regulatory practice, numerical solutions might shed further light on the likely performance of different regulatory regimes.

4 Conclusions

This paper analysed network investment incentives and consumer surplus under various regulatory regimes. We show that a regime of fully distributed costs or regulatory holiday induces most investments. However, in combining strong competitive intensity with reasonable investment incentives, a risk-

sharing approach induces highest consumer surplus. Our analysis departs from previous assessments\textsuperscript{19} as it models the regulated access price as a function of the incumbent’s investments. Further we suppose that returns on investment are uncertain.

Our results appear relevant for the currently heated debate on how to regulate wholesale access to next generations networks. European regulators, by and large, seem to adhere to the existing regulatory framework that, with respect to NGN deployment, often implies a higher risk for the investor than for the non-investor. Investors, in contrast, proposed alternatives such as regulatory holiday and risk-sharing. Our results suggest that regulators may dismiss regulatory holiday for good reason whilst they might consider risk-sharing arrangements a priori positively or even encourage them. One critical question open for future research is how to set access conditions (if any) for (late) entrants that do not participate in a risk-sharing agreement. In this context it seems pivotal that, first, a risk-sharing consortium allows all interested parties to get on board ex ante and, second, a too favourable ex post access obligation does not jeopardise the very idea of risk-sharing.

\textsuperscript{19}As noted earlier with the exception of Klumpp and Su (2008).
Appendix

Second stage second-order conditions.

For cases $L$, $FD$ and $RS$, by (1), (2), (5), (6) and (3), (7), (8), respectively, we need to have

$$\frac{\partial^2 \pi_i'^\ell}{\partial q_i^2} \bigg|_{q_i=q_j=q} = -2 + \frac{\alpha^\ell \gamma x^2}{8q^2} < 0 \iff \gamma < \frac{4q}{\alpha^\ell x}, \quad \ell = s, f, \quad q = L, FD, RS,$$

which is satisfied by (10). For case $H$ the second-order condition is straightforward, satisfied and hence omitted.

Proof of Proposition 1.

$$E(Q^{RS}(x)) - E(Q^{L}(x)) = \frac{1}{24}\beta \left[ \frac{8(A - c + x) + \sqrt{64(A - c + x)^2}}{-8(A - c + x) \sqrt{64(A - c + x)^2} - 96\gamma x^2} \right]$$

> 0, for all $\beta > 0$

$$E(Q^{L}(x)) - E(Q^{FD}(x)) = \frac{1}{24}(1 - \beta) \left[ \frac{8(A - c) + \sqrt{(8A - 8c)^2}}{-8(A - c) \sqrt{(8A - 8c)^2} - 96\gamma x^2} \right]$$

> 0, for all $\beta > 0$

$$E(Q^{L}(x)) - E(Q^{H}(x)) = \frac{1}{3} \beta \left[ \frac{A - c + x + \frac{1}{8} \sqrt{64(A - c + x)^2} - 96\gamma x^2}{-(2(A - c) + x)} \right]$$

= 0, for $\beta = 0$

and

$$\text{sign} \frac{\partial(E(Q^{L}(x)) - E(Q^{H}(x)))}{\partial\beta} = \text{sign}(2(A - c + x) - 3\gamma x)$$

> 0 $\iff x < \frac{2(A - c)}{3\gamma - 2}$

First stage second-order conditions.

Cases $RS$ and $H$ are by (20) and (22) first stage profit functions are strictly concave if $\gamma > 4/9$ and $\gamma > 8/9$, respectively. For case $L$ the fact that $x^L < x^{RS}$ for all $0 < \beta \leq 0.85$ and that $\partial E(\Pi^L_f)/\partial x \big|_{x=0} > 0$ implies that the
incumbent’s maximisation problem in case $L$ leads to at least one (interior) profit maximum in $x < x^{RS}$ for all $0 < \beta \leq 0.85$. The remaining concern hence regards case $FD$. In what follows we restrict ourselves to show that case $FD$ has an interior profit-maximum as well (the proof of strict concavity would be somewhat tedious and is not necessary for our claims).

Differentiating (16) with respect to $x$ gives

$$\frac{\partial^2 E(\Pi_{FD}^i)}{\partial x^2} = \beta \left[ (A - c + x - 4q^{sFD}) \frac{\partial^2 q^{sFD}}{\partial x^2} + 2 \frac{\partial q^{sFD}}{\partial x} - 4 \left( \frac{\partial q^{sFD}}{\partial x} \right)^2 \right] + (1 - \beta) \left[ (A - c - 4q^{fFD}) \frac{\partial^2 q^{fFD}}{\partial x^2} - 4 \left( \frac{\partial q^{fFD}}{\partial x} \right)^2 \right] - \frac{1}{2} \gamma.$$ 

Consider the success case in the first bracketed term. If $(A - c + x - 4q^{sFD}) \geq 0$, then by

$$\frac{\partial^2 q^{sFD}}{\partial x^2} = -\frac{6\sqrt{2}(A - c)^2 \gamma}{(2A^2 + 2c^2 - 4A(c - x) - 4cx + x^2(2 - 3\gamma))^{3/2}} \leq 0,$$

we have that

$$\frac{1}{2} \gamma > 2 \frac{\partial q^{sFD}}{\partial x}$$

suffices that the second-order condition is negative. Observe (by the fact that (11) decreases in $\alpha^{*e}$) that

$$q^{*M} = \frac{A - c + x}{3} \geq q^{sFD},$$

and

$$\frac{\partial q^{*M}}{\partial x} = \frac{1}{3} \geq \frac{\partial q^{sFD}}{\partial x}.$$ 

The latter inequality means that $\gamma > 4/3$ ensures that the second order-condition is satisfied. Next, if $(A - c + x - 4q^{sFD}) < 0$, then by (18) and (10), $(A - c + x - 4q^{sFD})\partial q^{sFD}/\partial x < 0$ and

$$\frac{1}{2} \gamma > \frac{\partial q^{sFD}}{\partial x} = \frac{1}{3} \iff \gamma > \frac{2}{3}$$

ensures an interior solution in $x$.

Consider next the failure case in the second bracketed term of the above second-order condition. First, if $(A - c - 4q^{sFD}) \geq 0$, then

$$\frac{\partial^2 q^{fFD}}{\partial x^2} = -\frac{(A - c)^2 \gamma}{\sqrt{2}(2(A - c)^2 - 3\gamma x^2)^{3/2}} \leq 0$$.  

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ensures that the second-order condition is satisfied. Next if 
\((A - c - 4q^{FD}) < 0\), the sign of second-order condition above is not
obvious. Notice that in the failure case the only effect that \(x\) has on \(E(\Pi^I)\)
is to reduce output quantities
\[
\frac{\partial q^{FD}}{\partial x} = -\frac{2x\gamma}{\sqrt{64(A-c)^2 - 96\gamma x^2}} \leq 0
\]
and thereby the only potentially profitable effect of \(\partial E(\Pi^I)/\partial x\) is due to
a price increase caused by the output contraction. This potentially positive
effect of an output contraction, however, will turn negative at some point.
In particular define
\[
q^{FM} \equiv \frac{A - c}{6} \leq q^{FD}
\]
as the smallest possible output quantity by (14) and note that
\((A - c - 4q^{FM}) > 0\) which contradicts \((A - c - 4q^{FD}) < 0\) for \(q^{FD} = q^{FM}\). Hence there must be an interior solution of the first-order condition in \(x\).

**Proof of Proposition 2.**

**First claim:** \(x^{FD} > x^{RS}\), for \(0 < \beta < 1\), \(\gamma > 2\). We evaluate the first-order condition for the FDC case (16), substituting optimal NGN deployment in the risk-sharing case, \(x^{RS}\), for \(x\), i.e.

\[
\left. \frac{\partial E(\Pi^I)}{\partial x} \right|_{x^{RS}} = -\beta \gamma (A - c)^2 \begin{bmatrix}
-12\beta X + 12\beta^2 X + 27\gamma X - 27\beta \gamma X \\
-12\beta XY + 27\gamma XY \\
+4\sqrt{3}\beta^2 Y - 27\sqrt{3}\gamma Y + 9\sqrt{3}\beta \gamma Y
\end{bmatrix}
\]

\[
= -\beta \gamma (A - c)^2 \begin{bmatrix}
-12\beta X(1 - \beta) + 27\gamma X(1 - \beta) \\
XY(-12\beta + 2\gamma) \\
+Y(4\sqrt{3}\beta^2 - 27\sqrt{3}\gamma + 9\sqrt{3}\beta \gamma)
\end{bmatrix}
\]

\[
= -\beta \gamma (A - c)^2 \begin{bmatrix}
(-12\beta + 27\gamma)((1 - \beta)X + XY) \\
+\sqrt{3}Y(4\beta^2 - 27\gamma + 9\beta \gamma)
\end{bmatrix}
\]

\[
= -\beta \gamma (A - c)^2 \begin{bmatrix}
3\sqrt{(27\gamma - 8\beta^2)\gamma((1 - \beta) + Y)} \\
+\sqrt{3}Y(4\beta^2 - 27\gamma + 9\beta \gamma)
\end{bmatrix},
\]

where

\[
X = \sqrt{\frac{(27\gamma - 8\beta^2)\gamma}{(4\beta - 9\gamma)^2}}.
\]
\[ Y = \sqrt{1 - \frac{24\beta^2\gamma}{(4\beta - 9\gamma)^2}}. \]

For \( x^{FD} > x^{RS} \), we need to show that the bracketed expression is negative, where the first term of the bracketed term is positive and the second one is negative. It is hence sufficient to show the bracketed expression is negative if, for the first term, we replace \( Y \) by the upper bound value of \( Y \), i.e. \( Y = 1 \), and, for the second term, we replace \( Y \) by the lower bound value of \( Y \), i.e. \( \sqrt{Y} \bigg|_{\beta=1, \gamma=2} = \sqrt{37}/7 \). It is further sufficient for our claim if we replace \((27\gamma - 8\beta^2)\gamma\) in the first term by \(27\gamma^2\). With these manipulations the bracketed expression becomes

\[
9\sqrt{3}(2 - \beta)\gamma + \sqrt{111}/7(4\beta^2 - 27\gamma + 9\beta\gamma) \\
\approx 31.18\gamma - 15.58\beta\gamma + 6.02\beta^2 - 40.63\gamma + 13.55\beta\gamma < 0, \text{ for } \gamma > 2.
\]

This proofs the first claim.

**Second claim:** \( x^H > x^{RS} \). Straightforward by (20) and (22).

**Third claim:** \( x^{RS} > x^L \) for \( 0 < \beta \leq 0.85 \). We evaluate the first-order condition for the LRIC case (16), substituting optimal NGN deployment in the risk-sharing case, \( x^{RS} \), for \( x \), i.e.

\[
\frac{\partial E(\Pi^L)}{\partial x} \bigg|_{x=x^{RS}} = \frac{-\beta(A-c)\gamma}{(3\sqrt{3}(4\beta - 9\gamma)^2\Phi)} \\
\times \left[ (A(4\beta^2 - 27\gamma + 9\beta\gamma) + 27\gamma(c + 3\sqrt{3}\Phi)) \\
+ 4\beta^2(-c + 8\sqrt{3}\Phi) - 9\beta(c\gamma + 4\sqrt{3}\Phi(1 + 2\gamma)) \right]
\]

where

\[
\Phi = \sqrt{(A-c)^2(27\gamma - 8\beta^2)\gamma/(4\beta - 9\gamma)^2}.
\]

The first factor is negative and hence the derivative is negative as long as the bracketed expression is positive. Re-arranging terms, the bracketed expression can be simplified to

\[
(A - c)(4\beta^2 - 27\gamma + 9\beta\gamma) + \sqrt{3}(-9 + 8\beta)(4\beta - 9\gamma)\Phi \\
= (A - c)\left[ 4\beta^2 - 27\gamma + 9\beta\gamma - \sqrt{3}(-9 + 8\beta)\sqrt{(27\gamma - 8\beta^2)\gamma} \right].
\]
Now suppose $\beta = 0.85$, and note that the bracketed expression is positive,

\[ [\ast]_{\beta=0.85} \approx 2.89 - 19.35\gamma + 3.81\sqrt{(27\gamma - 5.78)}\gamma > 0, \quad \text{for all } \gamma > 1, \]

as the second derivative of $[\ast]_{\beta=0.85}$ with respect to $\gamma$ is negative

\[ \frac{\partial^2 [\ast]_{\beta=0.85}}{\partial \gamma^2} \approx -31.83 \frac{((27\gamma - 5.78)\gamma)^{3/2}}{((27\gamma - 5.78)\gamma)^{3/2}} < 0, \]

while the first derivative of $[\ast]_{\beta=0.85}$ with respect to $\gamma$, for $\lim \gamma \rightarrow \infty$, is still positive

\[ \lim_{\gamma \rightarrow \infty} \frac{\partial [\ast]_{\beta=0.85}}{\partial \gamma} \approx 0.45 > 0, \]

and therefore the derivative of $[\ast]$ with respect to $\gamma$ is positive for all $\gamma > 1$. Now we evaluate $[\ast]$ at its lower bound $\gamma = 1$,

\[ [\ast]_{\beta=0.85, \gamma=1} = 1.09 > 0, \]

and are ensured that the bracketed expression is strictly positive for all $\gamma > 1$; hence $\partial E(\Pi^B) / \partial x|_{x=x^RS, \beta=0.85} > 0$ for all $\gamma > 1$. Next the fact that the bracketed expression $[\ast]$ decreases in $\beta$, completes this part of the proof. We have

\[ \frac{\partial [\ast]}{\partial \beta} = 8\beta + 9\gamma - 8\sqrt{3}\sqrt{(27\gamma - 8\beta^2)}\gamma + \frac{8\beta(-9 + 8\beta)\gamma}{\sqrt{-8\beta^2/3 + 9\gamma^2}} < 0. \]

The inequality follows because the first negative term, $-8\sqrt{3}\sqrt{(27\gamma - 8\beta^2)}\gamma$, increases in $\beta$ and is hence maximised for $\beta = 1$, i.e. $-8\sqrt{3}\sqrt{(27\gamma - 8)\gamma}$, where $-8\sqrt{3}\sqrt{(27\gamma - 8)\gamma} < -8\sqrt{3}\sqrt{19\gamma^2} = -8\sqrt{3}\sqrt{19\gamma}$, for all $\gamma > 1$, and still, $8\beta + 9\gamma - 8\sqrt{3}\sqrt{19\gamma} < 0$. As the last term of the above inequality is always negative this completes the proof of the claim, $x^{RS} > x^L$ for $0 < \beta \lesssim 0.85$. ■
References


