This paper studies contemporaneous relationship between S&P 500 index returns and log-increments of the market volatility index (VIX) via a nonparametric copula method. Specifically, we propose a conditional dependence index to investigate how the dependence between the two series varies across different segments of the market return distribution. We observe the following findings: (a) the two series exhibit strong, negative, extreme tail dependence; (b) the negative dependence is stronger in extreme bearish markets than in extreme bullish markets; (c) the dependence gradually weakens as the market return moves toward the center of its distribution, or in quiet markets. The unique dependence structure supports the VIX as a barometer of markets’ mood in general. Applying the proposed method to the S&P 500 returns and the implied variance (VIX²), we find that the nonparametric leverage effect is much stronger than the nonparametric volatility feedback effect, although, in general, both effects are weaker than the dependence relation between the market returns and the log-increments of the VIX.

**Keywords**: Conditional Dependence Index; Kendall’s Tau; Leverage Effect; Nonparametric Copula; Tail Dependence Index; Volatility Feedback Effect.

**JEL Classification**: C13; C22; G1

## 1 Introduction

Investors witnessed severe downturn in the U.S. stock market in the second half of year 2008 when the mood of the bearish market was often cited through an implied volatility index—the VIX, a trade mark held by the Chicago Board Options Exchange (CBOE), which is designed to retrieve market’s estimate of average S&P 500 index volatility over the subsequent 22 trading days. As the bearish market frequently observed counter-movements between the S&P 500 index prices and the VIX, the VIX earned itself a reputation of market barometer of investors’ fear (see Figure 1). Motivated by this observation, we join the traditional finance literature to study the leverage and volatility feedback effects via nonparametric method, where asymmetric GARCH-in-mean type of models are popularly used in such study (see Bekaert and Wu (2000) and references therein).

The leverage and volatility feedback effects are put forward to explain a stylized fact of stock markets, the asymmetric volatility: volatility responds more to a drop in the value of a stock...
(negative return) than an increase of equal amount in the value of the stock (positive return). Taking zero return as the reference point, this stylized fact documents how stock volatility behaves when the stock returns deviate from the reference value, zero. In this paper, we aim to document how the market implied volatility reacts to the &P 500 index returns as the market returns move across the return distribution. Evidently, our paper extends and compliments the stylized fact aforementioned.

The volatility asymmetry has been documented at the individual firm and the market level with the volatility measured by historical volatility (e.g. Black (1976) and Christie (1982)), conditional volatility (e.g. French, Schwert and Stambaugh (1987)), realized volatility, or implied volatility (e.g. Bollerslev and Zhou, 2006). Among historical volatility, conditional volatility, realized volatility and implied volatility, do they reveal the same amount of market information? This question has motivated noticeable research study of the information content of the four different volatility measures in the literature. Interested readers are referred to Anderson and Bollerslev (1998), Christensen and Prabhala (1998), Fleming (1998), Blair, et al. (2001), Poon and Granger (2003), Becker, Clements, and McClelland (2009), Jiang and Tian (2005), among many others. As the VIX is published by the CBOE almost continuously each trading day such that it is public information available to all investors, we believe that it will be a public interest to learn more about how the two publicly observable series interact with each other. Therefore, this paper aims to exam the relation between the S&P 500 index and its implied volatility index, VIX.

The current paper contributes to the existing literature in two folds—a new methodology and new empirical findings. In the aspect of a new methodology in studying the leverage and volatility feedback effects, we attach both effects to market specific conditions by proposing a nonparametric measure of the two effects, a conditional dependence index, given in Section 4. Take the leverage effect as an example. It is a common practice in the traditional finance literature that volatility asymmetry is linked to the sign of market returns (or the sign of market return innovations) in asymmetric GARCH-type models to specify the leverage effect, where a leverage parameter is used to capture the volatility asymmetry. Our proposed method, however, can be used to uncover the strength of the leverage effects across different market conditions, as we directly measure the dependence of the implied variances on the S&P 500 index returns given that the S&P 500 returns fall into different segments of the return distribution. As a result, readers are able to learn under what circumstances they should pay particular attention to the leverage effect of the market returns on the market expected future volatility. Here, the concept of the leverage effect is extended to the impact of the (contemporaneous and lagged) S&P 500 index returns on the implied variances.

One advantage of our research is that we attach the leverage effect with the performance of the S&P 500 index, while traditional research, using asymmetric GARCH-type models to study the leverage effect, tends to define the leverage effect with respect to a predetermined reference point, usually zero. Interestingly, we find that the leverage effects exhibit a W-shape curve across different segments of the S&P 500 index return distribution (see the red curve in Figure 5). This is an interesting new finding that has not been documented in the finance literature: when studying the leverage effects of market returns, one needs to looks beyond how market volatility reacts to positive or negative market returns.

The volatility feedback effect documented states that market returns are positively correlated with market volatility, and the returns are high (low) if the anticipated volatility increases (decreases). GARCH-in-mean type of models are usually used to test the volatility feedback effect (e.g. Portorba and Summers (1986), French, Schwert, and Stambaugh (1987), Campbell and Hentschel

\footnote{As an exception, Wu and Xiao (2002) studied the asymmetry of the volatility response curve via a generalized partially linear regression model of the VIX on S&P 100 index, which is a semiparametric approach.}
(1992), Glosten, Jagannathan and Runkle (1993)), where the coefficient in front of the volatility term measures the volatility effect, and this coefficient is assumed to be constant. Bekaert and Wu (2000) did allow the volatility to bear a varying risk premium when modeling the excess stock (index) returns from Japanese market by assuming a conditional version of the CAPM based on the riskless debt model; however, the volatility feedback effect is difficult to be estimated accurately as stated in their paper. In this paper, the conditional dependence index proposed in Section 4 is a model-free measure of the volatility feedback effect. We find that the volatility feedback effect (the green line in Figure 5) is a U-shaped curve as the squared VIX moves across different segment of its distribution. In contrast to Bekaert and Wu’s (2000) finding, but consistent with Engle and Ng (1993) and references in Bollerslev, Litvinova, and Tauchen (2006), we find that the volatility feedback effect is generally smaller than the leverage effect (see Figure 5).

Most of researchers agree that the implied variance, $VIX^2$, has a long-memory of its past, while the S&P 500 market return has a very short memory of its past; see the sample autocorrelations of the implied variance and of the market return reported in Table 1 over the period of 01/01/1990 and 12/31/2008. We decompose the logarithm of the implied variance into two components: its previous day value and its daily increment (named $rVIX$ in this paper). The results in Table 1 show that the log-increment of the VIX has very short memory comparable with the market return. Since the relation between the market return and the implied variance is a balanced or net outcome of the relation of the market returns with each component of the implied variance, we then explore the instantaneous relation between the short-memory component of the implied variance and the market return. That is, we investigate not only the leverage and volatility feedback effects along the line of the traditional finance literature, but also study the relation between the log-increments of the VIX and the market returns. Our empirical findings are consistent with our intuition: we observe outstanding contemporaneous dependence between the S&P 500 index returns and the log-increment changes of the VIX (see the black curve in Figure 5), which is bigger than both the leverage and volatility feedback effects in terms of magnitude in general.

The strong daily, negative, asymmetric relation between the market returns and the increments of the market volatility is also found in Gibot (2005) and Hibbert, et al. (2009) in a simple linear regression model framework. Our analysis provides several additional noteworthy results: (a) the two series exhibit strong, negative, extreme tail dependency; (b) the negative dependency is stronger in extreme downturn markets than in extreme bullish markets; (c) the dependency gradually weakens as the market return moves toward the center of its distribution, or in quiet markets. These results imply that the simple linear regression model with a dummy to account for positive or negative market returns may not be sufficient to capture the extreme tail relation between the log-increments of the VIX and the S&P 500 index returns and that the average relation implied by the linear regression model may understate the relation of the two series in extreme market conditions.

The rest of the paper is organized as follows. Section 2 presents the data and summary statistics. Section 3 discusses the nonparametric estimation of copula joint densities and presents the tail dependence indexes of interest. In Section 4, we propose a conditional dependence index to study the leverage and volatility feedback effects and the relation between market returns and the log-increments of the VIX. To check on the robustness of the results, we conduct subsample analysis by splitting the data into four subsample periods. The last section concludes. All the tables and figures are delayed to the end of the paper.
2 Data and Descriptive Statistics

The Chicago Board Options Exchange (CBOE) started publishing the implied volatility index of the S&P 100 index since 1993, which was constructed from at-the-money S&P 100 index option prices using the Black-Scholes-Merton formula; see details in Whaley (1993, 2000). The VIX was introduced with two purposes in mind. First, it was intended to provide a benchmark of expected short-term market volatility. Second, it was intended to provide an index upon which futures and options contracts on volatility could be written. In September 2003, the CBOE replaced the old volatility index with the current volatility index, which is constructed via a model-free formula developed by Demeterfi, Derman, Kamal and Zou (1999) and originated from the seminal work of Breeden and Litzenberger (1978). The current volatility index is extracted from both at-the-money and out-of-the-money S&P 500 index option prices. Detailed information can be found at http://www.cboe.com.

In this study, we downloaded daily S&P 500 index prices from DataStream and the daily implied volatilities (VIX) from the CBOE. The data span from January 2, 1990 to December 31, 2008. The VIX is frequently cited as a barometer of investors’ fear and the market’s aggregate expectation of near-term market volatility. This view of the implied volatility has found strong popularity among investor community since its first debut at the CBOE in 1993. A high VIX beyond 40 is usually linked to a severe bear market and a low VIX value to a market with more confidence. The first time that the VIX passed the value of 40 was on August 31, 1998, a year marked with Russia’s currency devaluation and national debt moratorium and the collapse of the Long Term Capital Management in the U.S.A. For the sample period under consideration, the number of transaction days with the VIX value exceeding 40 is 15, 4, 10, and 64 in the year of 1998, 2001, 2002, and 2008, respectively. On November 20, 2008, the VIX reached its record high of 80.86, marking an unprecedented financial crisis faced by global financial markets. It is interesting to observe that the counter-movement between the VIX and the S&P 500 index did not become a dominant tone until August 1998 as shown in Figure 1. Of 76.76 percent of the total 2,259 transaction days that the S&P 500 index fell, the VIX gained; of 76.85 percent of the total 2,531 transaction days that the S&P 500 index gained, the VIX fell. In total, the two return series moved to opposite directions in 76.58 percent of transaction days for the data period examined in this paper, and this number increased to 84.92% and 88.93% in the year of 1998 and 2008, respectively. One evidently sees prominent increase of counter-movements between the market index and market volatility index in extremely bearish markets.

Let $P_t$ and $VIX_t$ be the S&P 500 index price and the implied variance at date $t$, respectively. We construct the daily S&P 500 index return and the log-increment of the VIX by

$$r_{sp_t} = 100 \times \ln \left( \frac{P_t}{P_{t-1}} \right) \quad \text{and} \quad r_{vix_t} = 100 \times \ln \left( \frac{VIX_t}{VIX_{t-1}} \right).$$

(1)

Table 1 reports the summary statistics of the implied variance, S&P 500 index returns, and log-changes of the VIX (both daily and month statistics for the latter two series). First, the log-increments of the VIX have a lower average but significantly higher variation than the S&P 500 index returns. We then split the data according to the sign of the S&P 500 returns and calculate the upside and downside average returns and sample standard deviations for both $r_{sp}$ and $r_{vix}$. Interestingly, we observe that both series exhibit stronger volatility in the downturn markets than in the upturn markets. In the downturn markets, the market index performed considerably worse than in the upturn markets, and the opposite holds true for the VIX index. Also, the implied variance ($VIX_t^2$) is on average lower and less volatile over days when the S&P 500 index prices

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3 The CBOE launched trading of VIX futures contracts in May 2004 and VIX option contracts in February 2006.
went up than when the S&P 500 index prices came down.\footnote{Some results are studied but not reported in the main text for brevity. We constructed two optimal portfolios based on minimum variance criterion and maximum Sharpe ratio criterion. The results show that the optimal portfolios enjoy much smaller volatility than the market index, but little improvement on average returns. In addition, the optimal portfolios allot higher percentage of investment to the VIX in bearish markets than in bullish markets.}

Next, to study the counter-movement between $rs$ and $VIX^2$ and between $rs$ and $rvix$, we calculate two statistics of dependence: Pearson’s correlation coefficient and Kendall’s tau. Kendall’s tau is given by

$$
\tau = \Pr[(X_1 - X_2)(Y_1 - Y_2) > 0] - \Pr[(X_1 - X_2)(Y_1 - Y_2) < 0] \\
= 2\Pr[(X_1 - X_2)(Y_1 - Y_2) > 0] - 1,
$$

where $(X_1, Y_1)$ and $(X_2, Y_2)$ are continuous random vectors drawn from the joint and marginal distribution $F(x, y)$, $F_X(x)$, and $F_Y(y)$, respectively (see Chapter 5 of Nelsen, 1999). Apparently, Kendall’s tau reveals a strong negative (or positive) association between the two series if it is close to negative (or positive) one, and a weak association if it is close to zero. Kendall’s tau equals zero, if the two variables are independent, but it may not hold true vice versa. In addition, we estimate the probability that the two indexes move to opposite directions, i.e. $\lambda = \Pr(rsp_t \times rvix_t < 0)$. The closer $\lambda$ is to one, the stronger is the negative association between the two indexes.

The numerical results are reported in Table 2, where the sample correlation and Kendall’s tau of $(VIX^2, rsp)$ and of $(rvix, rsp)$, $\lambda$ between $rvix$ and $rsp$, and the average S&P 500 index returns are for the entire sample period and for each year from 1990 to 2008 as well. We find that the sample correlation and Kendall’s tau are both significantly negative for $(VIX^2, rsp)$ and for $(rvix, rsp)$ although the negative relation is more prominent between the S&P 500 index returns and the log-increments of the VIX than between the S&P 500 index returns and the implied variance ($VIX^2$) at any reasonable significance level. The overall lower negative relation between $rs$ and $VIX^2$ is not a surprise, given the fact that the $VIX^2$ is a fractionally integrated series while the $rs$ has very short serial correlation with itself.

As a highly persistent (usually characterized as a long-memory) process, the $VIX^2$ is expected to have a low correlation with a short-memory process $rs$; otherwise, the $rs$ would not be a short memory process. This is also the reason as to why our main interest in this paper is to stress the importance of the strong relation between the two short memory processes $rs$ and $rvix$, where $rvix$ equals a half of the log-increment of the implied variance.

We can see that the sample correlation between $rvix$ and $rs$ ranges from -0.450 in 1995 to -0.850 in 2007 and Kendall’s tau ranges from -0.295 in 1995 to -0.690 in 2008. The negative dependence was more prominent in the first decade of the 21st century than in the 1990s. During the sample period, there are 76.6 percent of chances that the S&P 500 index and the VIX moved to opposite directions, and this figure even reached 88.9 percent in 2008 and bottomed at 63.9 percent in 1995. Tables 1 and 2 indicate uneven negative relations between market returns and the log-increments of the VIX: the worse the market is, the stronger is the negative dependence, which implies that an overall negative association between the two series cannot tell the full story of how the two series relate. We therefore want to look at the joint distribution of the two series in the next section.

3 Copula Function and Tail-Dependence Index

To further our understanding of the dependence relationship between the S&P 500 returns and the log-increments of the VIX and between the S&P 500 returns and the $VIX^2$, we use the device of copula to decompose their joint PDFs. According to the Skalar’s theorem, the joint density of two
continuous random variables \( X \) and \( Y \) can be written as

\[
f(x, y) = f_X(x) f_Y(y) c(F_X(x), F_Y(y)),
\]

where \( f(x, y) \), \( f_X(x) \), and \( f_Y(y) \) are the joint and marginal probability density functions, respectively. The function \( c(F_X(x), F_Y(y)) \) is called the copula density function which completely summarizes the dependence structure between \( X \) and \( Y \). See Nelsen (1999) for a thorough treatment of the copula method and Cherubini, Luciano, and Vecchiato (2004) for applications in finance literature.

The commonly used parametric copulas are usually parameterized by one or two coefficients, and therefore might not be flexible enough for empirical studies. In this paper, we opt to estimate the copula density function of \( (\text{rspb}_t, VIX^2_t) \) and \( (\text{rspb}_t, \text{rvix}_t) \), using the Exponential Series Density Estimator (ESE) in Wu (2007). The estimator takes the form

\[
c(u, v) = \exp \left( \sum_{0<i+j<m} \theta_{ij} u^i v^j + \theta_0 \right),
\]

where \( m \) is a positive integer, and \( \theta_0 = -\ln \int_0^1 \int_0^1 \exp \left( \sum_{0<i+j<m} \theta_{ij} u^i v^j \right) dudv \) such that \( c(u, v) \) integrates to unity. This density estimator has an appealing information theoretic interpretation, being a maximum entropy density. Shannon’s information entropy is a central concept of information theory. For a continuous density \( p(x) \), it is defined as

\[
H(p) = -\int p(x) \log p(x) dx.
\]

Jaynes (1957) proposed the famous Maximum Entropy (ME) Principle. Suppose that only a number of moments are known about an unknown distribution. There may exist an infinite number of distributions satisfying those moments conditions. The ME principle states that one can construct a density estimator by maximizing Shannon’s entropy subject to known moments. In particular, the density (3) is constructed as

\[
\max_c -\int_0^1 \int_0^1 c(u, v) \log c(u, v) dudv
\]

subject to the following integration-to-unity condition and moment conditions

\[
\int_0^1 \int_0^1 c(u, v) dudv = 1
\]

\[
\int_0^1 \int_0^1 u^i v^j c(u, v) dudv = E(u^i v^j),
\]

where \( 0 < i + j \leq m \).

Note that when \( m = 2 \), the ESE copula estimator coincides with the popular Gaussian copula.

In practice, letting the number of moments increase with sample size at an appropriate rate and replacing the population moments in (6) with sample moments, we obtain a consistent nonparametric estimator of the underlying density function. The sample moments are sufficient statistics of the underlying distribution, and the MLE estimator of the ME density can be shown to be asymptotically efficient (Crain, 1974). The degree of the exponential polynomial \( m \) is selected according to the information criterion AIC. Unlike the kernel density estimator for \( f(x, y) \), the ESE is explicitly defined on the space of \([0, 1] \times [0, 1]\) and does not suffer from boundary biases.
Since we do not know the marginal cumulative distribution functions of \(rvix_t\), \(VIX_t^2\) and \(rsp_t\), denoted by \(u_{rvix,t} \equiv F_{rvix}(x)\), \(u_{VIX^2,t} \equiv F_{VIX^2}(x)\), and \(v_t \equiv F_{rsp}(x)\), respectively,\(^5\) we replace them by their empirical distribution functions, i.e. \(\hat{F}_{rvix}(x) = 1/T \sum_{t=1}^{T} I(rvix_t \leq x)\), \(\hat{F}_{VIX^2}(x) = 1/T \sum_{t=1}^{T} I(VIX_t^2 \leq x)\), and \(\hat{F}_{rsp} = 1/T \sum_{t=1}^{T} I(rsp_t \leq x)\), respectively. Several benefits could result from the one-to-one transformation of the variable of interest via its cumulative distribution function: a) it can effectively mitigate potential outlier problems in the nonparametric estimation; b) as a measure of the likelihood of the occurrence of an event, probability provides a direct way of capturing market relative status than the raw data value does across time, which is the upmost important in our study of the relationship between the two indexes in a quick-changing market. Furthermore, the study of the transformed data \((u,v)\), instead of the raw data, provides a key tool to consolidate historical study of similar issues so that we can discuss relation between two series according to event probabilities. This point will be illustrated in the next section in the discussion of full sample and subsample results.

In Figure 2, we plot the estimated copula density functions for \((v_t,u_{rvix,t})\) with \(m = 6\) and for \((v_t,u_{VIX^2,t})\) with \(m = 7\), where \(m\)'s are selected to minimize the AIC. The preliminary results in Section 2 indicate strong negative association between \(rsp_t\) and \(rvix_t\) without identifying the sources of the observed relation. The left panel in Figure 2 suggests that the negative dependence between the S&P 500 index returns and the log-increments of the VIX is largely driven by the counter-movements of the two tails, since the bulk of the copula density is along the anti-diagonal line and spikes up at the two corners. In other words, the co-movements of the opposite tails of two marginal distributions contribute significantly to the negative dependence between the S&P 500 index returns and the log-increments of the VIX. In addition, the density at the upper left corner in this graph, corresponding to the case of low market index returns and high VIX changes, is larger than its counterpart associated with high market index returns and low VIX changes. Except for the two tails along the anti-diagonal, the copula density appears to be rather symmetric.

The right panel in Figure 2 plots the estimated copula density for \((rsp_t,VIX_t^2)\), where we observe that the S&P 500 index returns and the \(VIX^2\) are strongly dependent when the VIX level is extremely high or its cdf is close to one. The dependence is stronger when the market returns are extremely low and the VIX level is very high than when both the market returns and the VIX level are extremely high, but the dependence between the two variables flattens out when the VIX level locates between its 10th percentiles to its 80th percentiles from the graph. We do see the estimated copula density hump up a bit when the VIX level locates to its left tail.

The two joint copula densities consistently show that in a low volatility market environment, which usually accompanies with limited movement in the changes of the VIX level, the dependences between S&P 500 index returns and the implied variances and between the market returns and the log-increments of the VIX is less noticeable.

### 3.1 Tail Dependence Index Between \(rsp\) and \(rvix\)

The joint copula density of \((rsp,rvix)\) in Figure 2 clearly exhibits the left-right and right-left tail dependences between the S&P 500 index returns and the log-increments of the VIX. To quantify the prominent tail dependence between the two series, one naturally wants to the probability with which \(rsp\) (or \(rvix\)) lies to the lower or upper tail area when \(rvix\) (or \(rsp\)) resides in the opposite tail area. As the dependence occurs at the tails, such probability is usually called tail dependence index (TDI). This idea is not new and has been studied in different fields. For example, Poon, Rockinger, and Tawn (2004) studied one particular tail index using extreme value theory, although

\(^5\)We occasionally drop the subscript \(t\) from cdfs and variables when we see no confusion to simplify our notation.
they focus on the limit cases; that is, TDI$(\alpha) = \Pr \left( Y < F_Y^{-1}(\alpha) \mid X < F_X^{-1}(\alpha) \right)$ when $\alpha \to 1$ or $\alpha \to 0$, where $F_X^{-1}(\alpha)$ and $F_Y^{-1}(\alpha)$ are the $(100 \times \alpha)$th percentile of the random variables $X$ and $Y$, respectively. Taking clues from the estimated copula density seen in the left panel of Figure 2, we focus on the following two TDIs that capture the co-movements of opposite tails of the two series $(rsp, rvix)$:

$$
\text{TDI}_1(\alpha) = \Pr \left( rvix < rvix_\alpha \mid rsp > rsp_{1-\alpha} \right) = \Pr \left( u_{rvix} < \alpha \mid v > 1 - \alpha \right),
$$

$$
\text{TDI}_2(\alpha) = \Pr \left( rvix > rvix_{1-\alpha} \mid rsp < rsp_\alpha \right) = \Pr \left( u_{rvix} > 1 - \alpha \mid v < \alpha \right),
$$

where $rvix_\alpha$ and $rsp_\alpha$ are the $(100 \times \alpha)$th percentile of the return series $rvix$ and $rsp$, respectively.

Taking $\alpha = .01$ and $\alpha = .05$ respectively and basing on the estimated copula function, we obtain $\text{TDI}_1(.01) = .2292$, $\text{TDI}_2(.01) = .3542$, $\text{TDI}_1(.05) = .3687$, and $\text{TDI}_2(.05) = .4234$. If the two series were independent, we would have obtained $\text{TDI}_j(\alpha) = \alpha$ for $j = 1, 2$. Therefore, the fact that $\text{TDI}_j(\alpha)$ is substantially higher than $\alpha$ indicates strong negative tail dependence between $rsp$ and $rvix$ series. In particular, our results suggest that extreme movements in the S&P 500 index are associated with extreme movements of the VIX to the opposite direction with high probabilities.

In addition, the fact that $\text{TDI}_1(\alpha) < \text{TDI}_2(\alpha)$ for both $\alpha = .01$ and $\alpha = .05$ reveals that the VIX asymmetrically responds to the extreme movements in the S&P 500 index. The probability that the VIX jumps abruptly when the market index faces free-fall is much higher than the probability that the VIX falls back when the market index enjoys strong rebound. The asymmetry is consistent with the stylized fact that the market tends to respond more to bad news than to good news of equal magnitude. The fact that the tail dependence is more pronounced when the market is in turmoil explains why the VIX is dubbed the Investor Fear Gauge.

### 3.2 Tail Dependence Index Between $rsp$ and $VIX^2$

As the right panel of Figure 2 exhibits prominent dependence between the S&P 500 index returns and the implied variances when the latter reside at the right tail of its distribution, we introduce the following four TDIs:

$$
\tilde{\text{TDI}}_1(\alpha) = \Pr \left( rsp > rsp_{1-\alpha} \mid VIX^2 > VIX_{1-\alpha}^2 \right) = \Pr \left( v > 1 - \alpha \mid u_{VIX^2} > 1 - \alpha \right)
$$

$$
\tilde{\text{TDI}}_2(\alpha) = \Pr \left( rsp < rsp_{\alpha} \mid VIX^2 > VIX_{1-\alpha}^2 \right) = \Pr \left( v < \alpha \mid u_{VIX^2} > 1 - \alpha \right)
$$

$$
\tilde{\text{TDI}}_3(\alpha) = \Pr \left( VIX^2 > VIX_{1-\alpha}^2 \mid rsp > rsp_{1-\alpha} \right) = \Pr \left( u_{VIX^2} > 1 - \alpha \mid v > 1 - \alpha \right)
$$

$$
\tilde{\text{TDI}}_4(\alpha) = \Pr \left( VIX^2 > VIX_{1-\alpha}^2 \mid rsp < rsp_\alpha \right) = \Pr \left( u_{VIX^2} > 1 - \alpha \mid v < \alpha \right)
$$

where $VIX_\alpha^2$ is the $(100 \times \alpha)$th percentile of the $VIX^2$ series. $\tilde{\text{TDI}}_1(\alpha)$ and $\tilde{\text{TDI}}_2(\alpha)$ measure the probability that the S&P 500 index returns fall to the respective left and right tails of the return distribution while the implied variances take values from the right tail of the volatility distribution. $\tilde{\text{TDI}}_3(\alpha)$ and $\tilde{\text{TDI}}_4(\alpha)$ measure the probability that the implied variance lies to its upper tail part of its distribution while the S&P 500 index returns fall to the left and right tails of the return distribution. By formula, $\tilde{\text{TDI}}_1(\alpha)$ and $\tilde{\text{TDI}}_3(\alpha)$ are two conditional probabilities with switched interest, so are $\tilde{\text{TDI}}_2(\alpha)$ and $\tilde{\text{TDI}}_4(\alpha)$. This is necessary as the direction of the dependence matters.

Again, taking $\alpha = .01$ and $\alpha = .05$ respectively and basing on the estimated copula function, we obtain $\tilde{\text{TDI}}_1(.01) = .0838$, $\tilde{\text{TDI}}_2(.01) = .1268$, $\tilde{\text{TDI}}_1(.05) = .1933$, $\tilde{\text{TDI}}_2(.05) = .3063$, $\tilde{\text{TDI}}_3(.01) = .0869$, $\tilde{\text{TDI}}_4(.01) = .1343$, $\tilde{\text{TDI}}_3(.05) = .2124$, and $\tilde{\text{TDI}}_4(.05) = .3292$. These numbers show that $\tilde{\text{TDI}}_j(\alpha) > \alpha$ for all $j$ and both $\alpha = .01$ and $\alpha = .05$, which reveals the existence of tail dependence between the market returns and the market implied variance, although
the TDIs are weaker then the TDIs between the market returns and the log-increments of the VIX. In addition, $TDI_1(\alpha) < TDI_2(\alpha)$ and $TDI_3(\alpha) < TDI_4(\alpha)$ for both $\alpha = .01$ and $\alpha = .05$ indicates asymmetric tail dependences between the market returns and the implied variance—the TDIs are stronger when the market returns lie to the left tail of than to the right tail of the return distribution. Moreover, $TDI_1(\alpha) < TDI_3(\alpha)$ and $TDI_2(\alpha) < TDI_4(\alpha)$ for both $\alpha = .01$ and $\alpha = .05$ document that the implied variance reacts more to the market return than vice versa.

3.3 Contemporaneous and Lagged Conditional Distribution

The tail dependence index only describes a probability number of the occurrence of one rare event depending on the occurrence of another rare event. In this section, we want to extract more information from the data by estimating the conditional cumulative distribution (cdf) functions via the nonparametric copula method. Specifically let $A$ be a subset of $[0,1]$. We are interested in estimating the conditional cdfs of $u_{vix,t}$ given $v_{t-h} \in A$, $v_t$ given $u_{vix,t-h} \in A$, $u_{VIX_t}$ given $v_{t-h} \in A$, and $v_t$ given $u_{VIX_t}$, and $u_{VIX_t}$ are defined above. When $h = 0$, we have contemporaneous conditional cdfs; when $h = 1$, we have lag-one conditional cdfs. In addition, we set $A = [0,.05], [.45, .55], [01]$. Therefore, the variable conditioned on lies to the lower 5% tail, around its median, and the upper 5% tail of its distribution. If the two variables of interest are independent, we expect the resulted conditional cdf to coincide with the 45-degree line in a $[0,1] \times [0,1]$ space.

Figure 3 contains six graphs. The first row plots the conditional cdfs of $u_{vix,t}$ given $v_{t-h} \in A$, and the second row plots the conditional cdfs of $v_t$ given $u_{vix,t-h} \in A$, and the solid lines are for $h = 0$, and the dashed lines are for $h = 1$. For all the six graphs in Figure 3, the deviations between lagged conditional cdfs and the 45-degree line are very small. Therefore, one-day lagged information in the log-increments of the VIX (or the market returns) is not of much help in learning future market returns (or future changes in implied volatility). Also, the deviations between the conditional cdfs and the 45-degree line when $A = [.45, .55]$ are much smaller than when $A = [0, .05]$ and [.95, 1]. It means that we have more predictive power over the movement of the VIX when the market price makes extreme movement than when the market return is around its median level, and vice versa.

Reading the first row of Figure 3 in detail, we observe the following findings as well: (i) When the market return lies to its lower 5% tail, the contemporaneous conditional cdf of the log-change of the VIX lies completely below the 45-degree line. The conditional cdf flattens out first before the 80-percentile of the log-change of the VIX, then quickly grows to one. With $TDI_2(.05) = .4234$, the market expects more than 40% chance that the market will revise upward its expectation of average near-future market volatility when the market price sinks to its bottom. (ii) When the market return lies to its upper 5% tail, the contemporaneous conditional cdf of the log-increments of the VIX lies above the 45-degree line. A strong market performance is associated with a bet on very high chance that the market expected average near-future market volatility will be reduced significantly. The second row of Figure 3 also evidently shows the counter-movements of the market returns and the log-increments of the VIX in the presence of volatile markets.

In Figure 4, the first row plots the estimated conditional cdfs of $u_{VIX_t}$ given $v_{t-h} \in A$, and the second row plots the estimated conditional cdfs of $v_t$ given $u_{VIX_t}$, where $h = 0$ and 1. We observe the following features: (i) There is a very small difference among the contemporaneous conditional cdfs and the lagged ones, implying that the conditional relation between the implied variances and the S&P 500 index returns are rather persistent. (ii) The conditional cdf of $u_{VIX_t}$ given $v_{t-h} \in A$ lies well below the 45-degree line when the market return lies to its lower and upper 5% tails, although the conditional cdf curve is closer to the 45-degree line when the market...
return lies to its upper 5% tail than the lower 5% tail. This finding suggests that extreme market price movements, up and down, increase the chance of pushing up the implied market variance to a very high level, although we are more certain on a very high VIX level for an extremely bearish market than an extremely bullish market. (iii) When the market return is around its median, the conditional cdf of the market variance lies above the 45-degree line, but its deviation from the 45-degree line is much smaller than the other two cases explained above.

If we extend the leverage effect to the nonlinear dependence of $VIX^2$ on past $rsp$, we find that leverage effect is more distinctive at the both tails, or when market prices experienced unusual movements than at the middle of the return distribution. Therefore, the leverage effect will be understated if one estimates such effect from a linear mean regression model as the mean regression model aims to explain average responses.

The second row of Figure 4 shows that the estimated conditional cdfs do not deviate much from the 45-degree line, especially when $u_{VIX^2,t-h}$ falls to the interval $[.45, .55]$, and the estimated conditional cdfs cross the 45-degree line when the market variance takes unusually low and high value. When the market variance falls to its lower 5% tail, this reduces the chance that the market return goes to either its tail, and when the market variance grows to its upper 5% tail, this enhances the chance that the market goes to extremely bearish and bullish cases. Evidently, the volatility feedback effect is much weaker than the leverage effect mentioned above.

If we extend the volatility feedback effect to the nonlinear dependence of $rsp$ on $VIX^2$, we find that the volatility feedback effect is more distinctive when the implied variance is very high or very low than around its median. Therefore, the volatility feedback effect also will be understated if one estimates such effect from a simple GARCH types models. Figure 4 seems to indicate a stronger leverage effect than the volatility feedback effect.

To summarize, we find strong left-right and right-left tail dependence between $rsp$ and $rvix$, significant leverage effect of market returns on the $VIX^2$, and volatility feedback effect of market variance on future returns. Among the three dependence relations, which one is the strongest? To answer this question, we propose a conditional dependence index to formally quantify these dependence relations.

### 4 Conditional Dependence Index

Figures 3 and 4 plot several estimated conditional distribution functions of $u$ given $v \in A$, where $A$ is a nonempty subinterval of the interval $[0, 1]$. If $u$ and $v$ are independent of each other when $v \in A$, we have $F(u|v \in A) = F(u)$ for all $u \in [0, 1]$ so that knowing the information set $\{v \in A\}$ does not help us make better predictions about $u$. On the other hand, the further is the conditional cdf away from the 45-degree line, the higher is the dependency between $u$ and $v \in A$. Therefore, it is natural to use the area between the conditional cdf and the 45-degree line as a proxy of the predictive power of $v \in A$ on $u$. In doing so, we are able to learn where $u$ and $v$ are most dependent as $v$ moves across its distribution function. Consequently, we can make inference on the relation between the pair of variables of interest conditional across different market status.

Specifically, we propose a conditional dependence index which equals twice of the area between the conditional CDF and the 45-degree line, conditional on the information set $\{v \in A\}$. Thus, the index is defined as a functional of $A$:

$$G(A) = 2 \int_0^1 |F(u|v \in A) - u| \, du = 2E \left[ |F(u|v \in A) - u| \right]. \tag{13}$$

Apparently, for any given subset $A \subset [0, 1]$, $0 \leq G(A) \leq 1$. This can be considered as a conditional
version of the Gini index.

Partitioning the [0,1] interval into twenty equal-width intervals, we calculate \( G(A) \) for each interval and report the results in Table 3 for both \( h = 0 \) and \( h = 1 \).

We shall illustrate the estimation method and the test for zero \( G(A) \) using the leverage effect as an example, and the method also applied to other cases. The null hypothesis of \( G(A) = 0 \) means that the movements of the implied variance do not response to the fact that the market return lies in interval \( A \), and the alternative hypothesis is \( G(A) > 0 \). To simplify the test statistic, we replace the unknown conditional cdf \( F(u|v_{t-h} \in A) = \Pr \left( u_{VIX_t^2} \leq u, v_{t-h} \in A \right) / \Pr (v_{t-h} \in A) \) by its empirical conditional distribution, i.e.

\[
\hat{F}(u|v_{t-h} \in A) = \frac{n^{-1} \sum_{t=1}^{n} I \left( u_{VIX_t^2} \leq u, v_{t-h} \in A \right)}{n^{-1} \sum_{t=1}^{n} I (v_{t-h} \in A)}
= \frac{n^{-1} \sum_{t=1}^{n} I \left( VIX_t^2 \leq F^{-1}_{VIX_t^2} (u), rsp_{t-h} \in F^{-1}_{rspt_{-h}} (A) \right)}{n^{-1} \sum_{t=1}^{n} I (rsp_{t-h} \in F^{-1}_{rspt_{-h}} (A))},
\]

(14)

where \( n=4790 \) is the total sample size, \( I (\cdot) \) is the indicator function, and \( F_{VIX_t^2} (\cdot) \) and \( F_{rspt_{-h}} (\cdot) \) are the unconditional cdfs of \( VIX_t^2 \) and \( v_{t-h} \), respectively. We denote by \( \hat{G}(A) \) the estimator of \( G(A) \). Under the null hypothesis, we have \( \hat{F}(u|v_{t-h} \in A) = n^{-1} \sum_{t=1}^{n} I \left( VIX_t^2 \leq F^{-1}_{VIX_t^2} (u) \right) \), the empirical cdf of \( VIX_t^2 \).

Under the null hypothesis, we can show that \( \sqrt{n} \left( \hat{F}(u|v_{t-h} \in A) - u \right) \) converges to a normal random variable with zero mean and finite variance. Under the alternative hypothesis, we expect \( \sqrt{n} \left( \hat{F}(u|v_{t-h} \in A) - u \right) = O_p (\sqrt{n}) \). Therefore, we expect \( \sqrt{n} \hat{G}(A) = O_p (1) \) under the null hypothesis and \( \sqrt{n} \hat{G}(A) = O_p (\sqrt{n}) \) under the alternative hypothesis. However, to conduct the test, we need to obtain proper critical values. As the distribution of \( \hat{G}(A) \) under the null hypothesis does not have a simple formula, we propose to use bootstrap critical values.

**Bootstrap critical values.** Under the null hypothesis, the distribution for \( \hat{G}(A) \) essentially does not depend on the market return series. Therefore, we randomly shuffle the market returns and the bootstrap samples become the raw data on \( VIX_t^2 \) and the shuffled market return data. From the bootstrap samples, we calculate \( \hat{G}(A) \) using (14). We do 1,000 bootstrap replications and find the p-values of the proposed test.

In Table 3, we report \( \hat{G}(A) \) for six cases: the CDIs of \( rvix_t \) given \( rspt_{-h} \), \( VIX_t^2 \) given \( rspt_{-h} \), \( rspt \) given \( VIX_t^2_{t-h} \) for \( h = 0 \) (capturing contemporaneous dependence) and for \( h = 1 \) (capturing one day lagged dependence).\(^6\) We divide the interval \([0,1]\) into twenty equal increment of .05, and the average bootstrap critical values across all the twenty intervals is .074 at the 5% significance level and .0964 at the 1% significance level. In Table 3, we marked the insignificant CDI estimates at the 5% level with an asterisk. The fifth column of Table 3 indicates little dependence of the log-increments of the VIX on previous day’s market index performance. Combining the second and fifth columns, we see close contemporaneous but less noticeable lagged relation between the changes of the VIX and of the market returns. In contrast, the relation between implied variances and market returns are rather persistent but become weaker in general over time, where the persistent relation may result from the long-memory properties of the implied variance.

\(^6\) We calculated but decided not to report the CDIs of \( rspt \) given \( rvix_{t-h} \) as the extra results do not add more information to the relation of the market returns and the log-increments of the VIX, and the intuition of which can be seen from Figure 3. However, these results can be obtained from the authors upon request.
To enhance the readability of the results given in Table 3, we plot the contemporaneous dependence in Figure 5. The solid line shows how the distribution of the log-increments of the VIX depends on the market return as the market return moves from its lower 5% tail, (.05,.10], (.10,.15],..., to its upper 5% tail, where each probability interval contains equally 5% data. It shows a general U-shape curve and bottoms at the interval of (.45,.50]—around the median of the market returns. The dependence of the distribution of the log-increments of the VIX on the market returns grows as market returns go farther away from its median, although the dependences grow faster with steeper slope when the market return falls below its median value than when the market return grows above its median value (for the full sample, this number is 0.04584%). At the extreme market cases, the CDI of the log-increments of the VIX takes the highest value .790 when the market return falls below its lower 5% tail, which is higher than .670 when the market return grows beyond its upper 5% tail. The asymmetry findings reflects market’s asymmetric attitude toward an extreme downmarket and an extreme upper market: investors in general feel more nervous in the former than in the latter situation.

**The leverage effect.** The red line in Figure 4 shows how the distribution of $VIX_t^2$ depends on the market return as it moves from the lower 5% tail to the upper 5% tail. The CDIs exhibit a W shape. The leverage effect is strongest when the market return falls below its 5% lower tail. If we call the volatility resulting from market’s expectation of dismal and bright future as good volatility and bad volatility from market’s nervous selloff, the dependence of the good volatility on market return is evidently lower than that of the bad volatility. The noteworthy leverage effect when the market return falls into its [.5,.55] probability interval dominates the relation of the log-increments of the VIX on the market return in the same probability interval.

**The volatility feedback effect.** The green line in Figure 5 shows how the distribution of $rsp_t$ depends on $VIX_t^2$, where we see a much flatter and lower concave curve than the solid curve. The fourth column in Table 3 shows that the volatility feedback effects are insignificant when the market return falls into the probability interval of [.35,.4] to [.7,.75) except for [.5,.6). It means that we would not find noticeable volatility feedback effect if we fit the data with a mean regression model. This result may be used to explain why empirical works cannot find volatility feedback effects with GARCH-in-mean model.

Comparing the three curves, we find noticeably higher dependence between market returns and changes of the VIX than the leverage and volatility feedback effects, except for a higher leverage effect when the market return is moving around its medium value. This result encourages the econometric modelling of the market index return and the change of the VIX as well as the leverage and volatility feedback effect. In addition, the volatility feedback effect is weaker than the leverage effects with some excepts. This result may support the finding in Christie (1982) and Bekaert and Wu (2000) that neither the leverage effect nor the volatility feedback effect can be used to solely explain the volatility asymmetry observed from stock markets.

To sum up, we find strong dependence between $rvix$ and $rsp$ and the dependence is stronger in volatile market periods than in relatively quiet market periods. As the VIX reveals market’s expectation on the future 30-day volatility, our results indicate that investors make sharp revision on their belief of market risks during extreme volatile market periods, and that the revision is less noticeable during tranquil market periods. It again confirms that the negative association between the S&P 500 index and the VIX mainly come from tail events.

### 4.1 Subsample Analysis

To check on how robust our findings are, we also conduct subsample analysis, where we split the sample period into four subperiods: January 2, 1990 to December 31, 1994; January 1, 1995 to...
December 31, 1999; January 1, 2000 to December 31, 2004; January 1, 2005 to December 31, 2008. Look at the rightmost two columns in Table 2. The first subperiod (01/02/1990-12/31/1994) has lowest frequency of counter movements between the S&P index and VIX among the four subsample periods. The second subperiod (01/01/1995-12/31/1999) enjoyed the highest annual market returns among the four subsample periods, although the frequency of counter movements between the market index and VIX are well above the average of .766 in 1998 and 1999. The third subperiod (01/01/2000-12/31/2004) exhibited both higher average VIX level and higher frequency of counter movements between the S&P 500 index and the VIX. The last subperiod had higher than average frequency of counter movements between the two indexes except year 2006, but lower average VIX levels except for 2008.\footnote{Of course, readers can split the whole data period differently from ours.}

Figure 6 plots the estimated CDIs for the four subsample periods. In general, the subsample results are similar to the full sample results shown in Figure 5, although the asymmetric of the CDI of $rvix_t$ conditional on $rsp_t$ almost vanishes in the subsample periods January 1, 2000 to December 31, 2004 and January 1, 2005 to December 31, 2008, where the market experienced high volatilities.

5 Conclusions

Both leverage and volatility feedback hypotheses are developed to explain the stylized fact that volatility reacts asymmetrically to positive and negative stock returns. We re-examine the relationship between daily S&P 500 index returns and implied variance via nonparametric method. By proposing a nonparametric conditional dependence index, we document three findings. First, nonparametric leverage effect exhibits a W-shape curve as the implied variance moves from the left to the right tail of its distribution. Second, nonparametric volatility feedback exhibits a U-shape curve as the S&P 500 index returns moves across the return distribution. Third, the nonparametric leverage effect in general is higher than the nonparametric volatility feedback effect, except in relatively quite market conditions.

The VIX index squared, as a risk-neutral measure of market volatility, is market’s best estimate of average future realized volatility over ensuing 22 trading days plus a volatility risk premium, as documented by Todorov (2009), Bollerslev and Zhou (2005), among many others for other implied volatility indexes than S&P 500 implied volatility. Bakshi and Madan (2006) provide a theoretical model to explain that the VIX squared (or the implied variance) depends on historical skewness and kurtosis of return distributions and market risk aversions. Therefore, the log-increments of the VIX may reflect market’s revision on risk aversion and average future realized volatility as the S&P 500 index price changes. The empirical results in this paper indicate that the contemporaneous dependence between market’s revision on risk aversion and average future realized volatility and the market returns are stronger than the leverage and the volatility feedback effects when the market’s movement deviates from its medium range.

References


Appendix: Tables and Figures

Table 1: Summary Statistics (01/02/1990-12/31/2008)

<table>
<thead>
<tr>
<th>Data Frequency</th>
<th>Variable</th>
<th>$\bar{x}$</th>
<th>$\bar{x}_-$</th>
<th>$\bar{x}_+$</th>
<th>$\bar{\sigma}$</th>
<th>$\bar{\sigma}_-$</th>
<th>$\bar{\sigma}_+$</th>
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<td>Daily</td>
<td>$VIX^2$</td>
<td>450.178</td>
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a. $\bar{x}$=average return, $\bar{x}_-$=downside average return over times when $rsp < 0$,
$\bar{x}_+$=upside average return over times when $rsp \geq 0$;
b. $\bar{\sigma}$=sample standard deviation, $\bar{\sigma}_-$=downside sample standard deviation over times when $rsp < 0$, $\bar{\sigma}_+$=upside sample standard deviation over times when $rsp \geq 0$.
c. $\rho(h)$ is the sample autocorrelation of lag $h$. 
Table 2: Sample Correlation, Kendall’s τ, λ, Average Compound Return of the S&P 500 Index, and Average VIX

<table>
<thead>
<tr>
<th>Year</th>
<th>Sample Correlation (VIX², rsp)</th>
<th>Kendall’s τ</th>
<th>Sample Correlation (rvix, rsp)</th>
<th>Kendall’s τ</th>
<th>λ</th>
<th>S&amp;P 500 Return</th>
<th>VIX</th>
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<td>-.067</td>
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The compound return of the S&P500 index is the log-difference of market indexes observed at the ending and starting date of the period under consideration multiplied by 100; λ gives the relative frequency that the market index and market volatility index moved to opposite directions for the period of time under consideration.
Table 3: Conditional Dependence Indexes

| $A$    | $rvix_t|r_{sp_t}$ | $VIX^2_t|r_{sp_t}$ | $r_{sp_t}|VIX^2_t$ | $rvix_t|r_{sp_{t-1}}$ | $VIX^2_t|r_{sp_{t-1}}$ | $r_{sp_t}|VIX^2_{t-1}$ |
|--------|------------------|-------------------|-------------------|----------------------|----------------------|----------------------|
| (0,.05) | .790             | .685              | .227              | .127                 | .669                 | .149                 |
| (.05,1) | .670             | .380              | .187              | .079                 | .364                 | .194                 |
| (.1,.15)| .536             | .181              | .146              | .106                 | .176                 | .122                 |
| (.15,.2)| .420             | .082              | .161              | .060*                | .084                 | .138                 |
| (.2,.25)| .343             | .068*             | .108              | .028*                | .059*                | .114                 |
| (.25,.3)| .251             | .108              | .090              | .037*                | .116                 | .077                 |
| (.3,.35)| .219             | .123              | .096              | .065*                | .130                 | .073*                |
| (.35,.4)| .221             | .190              | .071*             | .057*                | .186                 | .083                 |
| (.4,.45)| .138             | .235              | .061*             | .032*                | .235                 | .062*                |
| (.45,.5)| .132             | .241              | .039*             | .023*                | .244                 | .027*                |
| (.5,.55)| .170             | .294              | .043*             | .100                 | .275                 | .032*                |
| (.55,.6)| .194             | .265              | .085              | .031*                | .276                 | .043*                |
| (.6,.65)| .222             | .240              | .048*             | .075                 | .224                 | .070*                |
| (.65,.7)| .277             | .149              | .066*             | .090                 | .137                 | .093                 |
| (.7,.75)| .317             | .157              | .075*             | .030*                | .151                 | .076                 |
| (.75,.8)| .338             | .052*             | .119              | .043*                | .054*                | .077                 |
| (.8,.85)| .393             | .060*             | .148              | .031*                | .059*                | .121                 |
| (.85,.9)| .444             | .048*             | .155              | .041*                | .049*                | .166                 |
| (.9,.95)| .594             | .184              | .215              | .029*                | .187                 | .223                 |
| (.95,1)| .670             | .526              | .277              | .046*                | .534                 | .277                 |

*" marks the CDIs that are insignificant at the 5% level.
Figure 1. Raw Data Plot (01/02/1990-12/31/2008)

(Black line: S&P 500 Index; Red line: the VIX)

Figure 2. Estimated Joint Copula Density Function
Figure 3. Conditional CDFs: (rvix,rsp)

The first row plots the estimated conditional cdf of $u_{rvix,t}$ given $v_{t-h}$ lying in interval A, and the second row plots the estimated conditional cdf of $v_{t}$ given $u_{rvix,t-h}$ lying in interval A. In each graph, the solid line (not the 45-degree line) results from $h=0$, and the dashed lines is for $h=1$.

Figure 4. Conditional CDFs: (VIX^2,rsp)

The first row plots the estimated conditional cdf of $u_{VIX^2,t}$ given $v_{t-h}$ lying in interval A, and the second row plots the estimated conditional cdf of $v_{t}$ given $u_{VIX^2,t-h}$ lying in interval A. In each graph, the solid line (not the 45-degree line) results from $h=0$, and the dashed lines is for $h=1$. 
Figure 5. The Conditional Dependence Indexes (CDIs): Full Sample Results

Figure 6. The Conditional Dependence Indexes (CDIs): Subsample Results