Bank Networks: Contagion, Systemic Risk and Prudential Policy

Iñaki Aldasoro*  Domenico Delli Gatti†  Ester Faia‡

This draft: January 2014

[PRELIMINARY AND INCOMPLETE - PLEASE DO NOT QUOTE NOR DISTRIBUTE]

Abstract

We present a network model of the interbank market in which optimizing risk averse banks lend to each other and invest in liquid and non-liquid assets. Banks trade on both sides of the interbank market. Clearing in the interbank and in the non-liquid asset markets takes place through a price tâtonnement mechanism. In order to match traded quantities we consider three alternative algorithms: Maximum Entropy, Closest matching and Sequential matching with exposure limits. We analyze the resulting network configurations using various network centrality, input-output and systemic risk (Shapley value) metrics. The interbank network generated by the model replicates several features of empirical interbank networks. We exploit our model to assess the performance of two prudential policies (liquidity and capital requirements) on the stability and the efficiency of the system (the latter measured by overall investment). Overall we find that liquidity requirements unequivocally increase stability but reduce efficiency, while capital requirements tend to increase stability without reducing significantly overall investment.

Keywords: banking networks, centrality metrics, systemic risk

*Goethe University Frankfurt & SAFE. Email: JuanIgnacio.Aldasoro@hof.uni-frankfurt.de.
†Catholic University of Milan. Email: domenico.delligatti@unicatt.it.
‡Goethe University Frankfurt, CFS & SAFE. Email: faia@wiwi.uni-frankfurt.de
1 Introduction

The propagation of bank losses which turned a shock to a relatively small segment of the US financial system (the subprime mortgage market) into a large global banking crisis in 2007-8 is due to a large extent to an emerging property of the banking system worldwide, namely the increasing number and multifaceted nature of bank interlinkages. Losses were propagated through two main contagion channels: (i) fire sales ignited by the practice of mark-to-market accounting and (ii) direct cross-exposures. A bank hit by a negative shock to its assets would indeed propagate losses in two ways. First, by selling assets under distress with the purpose of satisfying capital requirements and/or VaR constraints individual banks affect the market value of those assets. As other banks have invested in similar assets, mark-to-market accounting leads to the emergence of losses in the balance sheets of these other banks. This is an indirect propagation channel that works primarily through market prices and pecuniary externalities (see ?). In recent decades, however banks, have also become highly interconnected, primarily through direct cross exposure on loans in interbank markets and through cross-investment in derivatives. A bank hit by a negative shock to its assets could for instance default on its interbank debt, thereby inflicting direct losses to lending banks. As a consequence of increasing connectivity, the criterion for identifying systemically important banks has shifted from size to connectivity: in the eyes of regulators and policy makers, systemically important institutions are not only too-big-to fail but (increasingly) too-interconnected-to fail. These developments pose new challenges to researchers: for instance, elaborating a new concept of systemic risk and of systemically important banks requires the design of new models that embed a role for interconnections. Supervisors and policy makers will also be challenged: prudential regulation shall now also aim at improving the efficiency and stability of the overall financial architecture.

Against this background we lay down and analyze a network model of the interbank market. The main purpose of our analysis is to derive various measures of systemic importance and systemic risk and to see how different prudential regulation measures can affect such indicators. Several novelties characterize our analysis. First, in this paper we do not adopt the convention - generally accepted in other disciplines such as physics, neurology and epidemiology - according to which (i) links among nodes are exogenous and probabilistic and (ii) nodes’ behaviour is best described by heuristic rules. On the contrary, we adopt the well established economic methodology according to which agents are optimizers, decisions are micro-founded and the price mechanism is endogenous (see also ? for a similar approach and ?). The modelling challenge in this context consists in merging the network externalities into the micro-economic decisions and to design the network evolution and the market mechanism by matching interacting decisional processes. The basic set up of our model is very simple. The model consists of $N$ risk averse banks which perform optimizing portfolio decisions constrained by VaR or regulatory constraints. Banks invest in non-liquid assets, which trade at common prices, and trade with each other in the interbank market, with banks being both borrowers
and lenders in equilibrium. Investment in a common non-liquid asset determines the emergence
of pecuniary externalities, while the optimal exposure in the interbank market contributes to the
direct propagation of losses via banks’ defaults on loans. The market equilibrium is reached through
a full-fledged tâtonnement process (see also ?, ?, ?) in which prices are endogenously determined
by sequential convergence of excess demand and supply. Once prices are determined, actual trading
among heterogenous banks takes place through a matching algorithm (see more generally matching
algorithms from ? and ?). In this context we analyze three alternative matching algorithms: the
maximum entropy, the closest matching and the sequential matching with exposure limits. The
comparison of the three algorithms is useful also since they deliver different network structures in
terms of topology, concentration and stability.

Within this model we analyze various measures of banks’ systemic importance and of systemic
risk. In this respect we borrow from various branches of the literature analyzing measures of
systemic risk. Our measures, which will be described in greater detail later on, can be classified in
three main categories: centrality measures, input-output, Shapley values. The centrality measures
are more traditionally linked to the too-big-to-fail concept, as they identify banks to which most
banks in the system are connected. The input-output measures, which follow the tradition of
the Leontief methodology, describe the degree of sparseness versus concentration of the network
topology (see also ? and ? for related measures). The first two measures can be used as metrics of
systemic importance. The last, the Shapley value, has been borrowed from the literature on both
cooperative and non-cooperative games (see ? and also ?) and is used to measure the contribution
of each bank to the overall default probability of the financial system. Overall a comparison of all
our synthetic measures with their empirical counterpart suggest that our model can re-produce a
realistic banking network.

All those measures can also be used for normative purposes. They can indeed be confronted to
study the stability (lower systemic risk and lower contribution of each bank to it) and efficiency (the
latter measured by overall investment in the system) of the banking network. With those efficiency
and stability criteria at hand we can therefore confront the performance of various prudential
regulatory policies. The constraints characterizing the banks’ optimization process are indeed
affected by various regulatory parameters, such as capital and liquidity requirements as well as risk
charges. We can vary those policy parameters and evaluate the impact that they have on the various
stability and efficiency criteria. We find that increasing the liquidity requirement unequivocally
reduces systemic risk and the contribution of each bank to it. As banks are forced to hoard liquidity
they engage less in interbank lending. This reduces all network measures (density, closeness, average
degree, etc.), thereby reducing the scope for network externalities in the transmission of risk. As
banks are required to hold more liquidity they also reduce their share of investment in non-liquid
assets. This reduces also the scope for fire sale externalities, but also reduces the efficiency of
the system. An increase in the capital requirements instead has mixed effects. Overall interbank
liquidity falls and this reduces the extent of network links, but overall investment in non-liquid assets increases thereby increasing the scope for fire sale externalities.

2 Related literature

Interbank borrowing/lending relationships establish a network of connections among banks on the interbank market. The most influential model of the spreading of "financial contagion" in the interbank network is the seminal paper by ?

Increasing the number of credit relationships in an interbank network allows a bank to diversify the risk of a negative shock - e.g. a self-fulfilling panic (bank run) - but it also entails the propagation of financial distress to connected banks, i.e. financial contagion in the wording of Allen and Gale. In this context, systemic risk consists not only of the diffusion but also of the amplification of financial distress until the collapse of the financial system. Each one of the interdependent banks, in fact, is susceptible to go bankrupt so that the network as a whole is potentially exposed to the risk of a systemic failure. The literature has focused on two types of propagation of financial distress or channels of contagion: (1) the loss of market value of a common asset (fire sale of asset or asset price contagion), see ?, ?; (2) interlocking credit exposures, see ?, ?. These contagion channels may interact during the emergence of a financial crisis. For instance, a run on a bank is a negative shock which may trigger a fire sale of non-liquid assets (type (1) contagion channel) in an attempt to raise liquidity and satisfy the withdrawal of households’ deposits and an avalanche of interbank deposit withdrawals at other banks and liquidity evaporation on the interbank market (type (2) contagion channel). Focusing on the last channel, in their pioneering contribution ? reach the conclusion that if the interbank network is a credit chain – in which each agent is linked only to one neighbor along a ring – the probability of a collapse of each and every agent (a bankruptcy avalanche) in case a node is hit by a shock is equal to one. As the number of partners of each bank increases, the risk of a collapse of the bank hit by the shock goes asymptotically to zero, thanks to risk sharing. Systemic risk is at a minimum when the credit network is complete, i.e. when agents fully diversify individual risks. In other words, there is a monotonically decreasing relationship between systemic risk and the degree of connectivity of the credit network. In the wake of the global financial crisis, the awareness has gained ground that the benefits of distress propagation - i.e. risk sharing - are not without limits. An alternative view has emerged in the literature according to which as connectivity increases a trade off emerges between decreasing individual risk - due to risk sharing - and increasing systemic risk - due to the amplification of financial distress. The larger the number of connected neighbors, the smaller the risk of an individual collapse but the higher systemic risk may be and therefore the lower network resilience. As a consequence, the relationship between connectivity and systemic risk is not monotonically decreasing as in ?, but hump shaped, i.e.

\[^{1}\text{See also ?}, ?, ?, ?, ?, ?\]
decreasing for relatively low degree of connectivity and increasing afterwards, see ?. The positive correlation between connectivity and systemic risk is due to the fact that the evolution over time of financial fragility is intrinsically subject to positive feedbacks. In other words, the financial distress of a bank hit by a shock today is likely to lead to additional financial distress for the same bank in the future through interaction with other banks. The larger the number of connected banks, the more sizable the positive feedback on individual (and therefore on systemic) financial fragility. Suppose for instance (see e.g. in ?), that bank A is hit by a shock and is forced to fire-sell some of the securities in order to pay its debt. If the securities are sold below the market price, the asset side of the balance sheet is decreasing more than the liability side and the leverage of the bank is unintentionally increased. This situation can lead to a spiral of losses and increasing fragility. The analysis of the correlation between connectivity and systemic risk is grounded in appropriate measures of systemic risk. Our paper is linked to the literature that analyzes synthetic measures of network topology and systemic risk in financial networks, both theoretically and empirically. A recent paper which analyzes a synthetic measure of network topology is ? which adapts input-output methodologies to banking networks. Papers which analyze theoretical measures of systemic risk are ? and ?, both using the Shapley value. From an empirical point of view recent papers analyzing measures of systemic risk are ? and ? among others. The first paper constructs a measure of conditional Value-at-Risk (VaR), while the second paper adapts measures of Granger causality to network topologies. Our paper is also related to the recent strand of the literature analyzing stability and evolution of banking networks: see among others ?, ?, ?). A connection can be established with the literature analyzing matching mechanisms in markets along the lines indicated by Shapley and Shubik (see for instance ?). A recent paper close to ours in the approach to the design of the matching mechanism in trading is ?. Finally our paper is related to the recent literature analyzing prudential regulations in systems of interconnected financial institutions. For a recent paper on this see ?.

3 Systemic importance and systemic risk

The 2007-8 crisis moved the attention of supervisory authorities from the too-big-to-fail to the too-interconnected-to-fail banks, the reason being the increasing degree of interlinkages which materialized in the banking system. In the past systemically important banks were identified based on concentration indices such as the Herfindahl-Index, nowadays systemically important banks are those who are highly interconnected with others. The number of bank interlinkages are indeed the ones who increase shock and risk transmission. The increase in complexity also expands the number of dimensions which can be monitored relative to the financial architecture. For this reason we consider various metrics describing the overall financial architecture and/or the positions of individual

\footnote{For a comprehensive survey see ?.}
banks in this architecture. All those measures can also be used to characterize the stability and the efficiency of the banking network.

At first we consider centrality measures which are somehow the most immediate evolution of traditional concentration indices. In graph theory and network analysis, the centrality of a vertex measures its relative importance within the graph. Several centrality measures have been proposed in the literature, each attempting to capture different aspects of importance within a network.

As noted by (see also ?), centrality measures can be catalogued within three main categories: degree-like (i.e., connectivity), closeness-like (i.e., proximity) and betweenness-like measures. We compute therefore measures capturing all parts of this taxonomy. Let the network be represented by an adjacency matrix $G$, where an element $g_{ij} \neq 0$ indicates the existence of a connection between banks $i$ and $j$. The most basic and intuitive centrality measure is degree centrality, which is defined as the number of links incident upon a node and has a direct bearing on the connectivity of the network. Degree centrality can be interpreted in terms of the immediate risk of a node of receiving losses (from debt default in this case). Since our network will be directed, a distinction must be made between in- and out-degree: the former indicating the number of connections “arriving” to a node, the latter the number of connections “leaving” a node. In matrix notation they can be easily represented respectively as follows:

$$c_{\text{in}}^i = i^\top G \quad (1)$$
$$c_{\text{out}}^i = G i \quad (2)$$

where $i$ is a column unit vector of appropriate dimension and a prime indicates the transposed of a vector.

A generalization of degree-like measures is eigenvector centrality, which has many variants (Katz centrality, PageRank, etc). Here we compute classic eigenvector centrality, which is given by the eigenvector corresponding to the largest eigenvalue of the matrix defining the network. This measure operates under the common-sense observation that links to nodes that have high centrality score themselves should carry more weight than links to low-scoring nodes.

While degree-like measures try to capture connectivity, closeness-like measures attempt to capture the importance of distance among nodes in the network. According to such measures a bank is important to the extent that it is “close” to other banks, hence more likely to transmit distress since it is relatively easy to go from it to many other banks. For node $i$, closeness centrality measures the shortest path between $i$ and all other nodes reachable from it, averaged across all other nodes\(^3\).

Finally, betweenness centrality assigns a high centrality score to nodes that lie in many shortest

\(^3\) If one defines the geodesic (i.e., minimum distance) matrix as $D$, then for a directed network in- and out-closeness can be defined analogously to degree measures, using $D$ instead of $G$. 
paths between all other pairs of nodes. A bank is important according to this criteria to the extent that it is critical in the flow of the network by virtue of being a “middleman”. Let $p_{ij}$ be the number of shortest paths between nodes $i$ and $j$ and $p_{ikj}$ be the number of shortest path between nodes $i$ and $i$ passing through node $k$; then betweenness centrality is given by:

$$c_{(bw)k} = \sum_i \sum_j \frac{p_{ikj}}{p_{ij}}$$

(3)

As a second set of systemic importance measures, following ? we consider metrics based on input-output matrices. Taking the interbank linkages as a point of departure one can derive expressions along the lines of the traditional ? input-output model. If a bank is hit by a shock it will transmit it to the rest of the system according to the coefficient of the input-output matrix, which in a banking context represents a transformation of the interbank exposure matrix. One advantage of these measures compared to the centrality measures is that systemically important banks emerge also in more sparse systems which do not necessarily feature a vertex to which many banks are connected. Furthermore, a broader picture of the balance sheet of banks is taken into consideration in the construction of such measures.

Following the notation from ? let the matrix $A_M$ represent the interbank exposure matrix, in which an element $ij$ indicates exposure (through lending) of bank $i$ to bank $j$. The Rasmussen-Hirschman (RH) backward and forward indices are computed respectively as:

$$h_{bj} = i^T (I - A)^{-1} i_j$$

(4)

$$h_{fj} = i^T (I - O)^{-1} i$$

(5)

where $A$ is the interbank exposure matrix with each element expressed as a share of the borrowing (column) bank’s total assets and $O$ is the interbank matrix with each element expressed as a share of the lending (row) bank’s total assets4. These measures capture all effects, both direct and indirect, of unitary (backward version) and system (forward version) liquidity shocks.

Using the same input-output formulation one can gauge the effect of shocks coming from the interbank positions themselves. For this the so-called “field of influence” (FoI) measures are computed. The column FoI captures the system effect of a unitary cut of interbank lending to bank $j$, whereas the row FoI assesses the systemic effect of a unitary cut of interbank lending by bank $j$5. In their un-normalized version they can be expressed respectively as:

---

4For details on the derivations see ?, which perform a normalization to the measures to express them relative to the mean of the system. The matrix $(I - A)^{-1}$ is normally referred to as the Leontief inverse.

5The two measures can be combined into a total FoI.
where $\mathbf{F}(i, j) = \mathbf{b}_i\mathbf{b}'_j$ is the field of influence matrix, with $\mathbf{b}_i$ and $\mathbf{b}'_j$ denoting respectively the $i^{th}$ column and the $j^{th}$ row of the Leontief inverse defined above.

A final input-measure considered, called the “total linkage effect”, aims to capture the effect of a complete cut-off of bank $j$ from the interbank market.

Our purpose in this paper is also that of providing a synthetic measure of systemic risk. To this end we borrow from the theoretical literature a measure that has been proposed and used extensively in recent papers (see for instance ? and ?), namely the Shapley value, a concept itself borrowed from the literature on cooperative and non-cooperative game theory. Given the total value assigned to a coalition the Shapley value measures the contribution of each individual to the aggregate value. This metric can be translated in the context of a banking network by defining the aggregate value as overall default risk and using the Shapley value to measure the contribution of each bank to the overall default probability. Formally, overall systemic risk can be computed as the ratio of assets from all defaulting banks subsequent to a shock to non-liquid assets:

$$\Phi = \frac{\sum \text{assets}_\Omega}{\sum \text{assets}_i},$$

where $\Omega \in i$ identifies banks defaulting on their interbank debts. The Shapley value is formally defined as follows. Define $O : 1, \ldots, n \rightarrow 1, \ldots, n$ to be a permutation that assigns to each position $k$ the player $O(k)$. Furthermore denote by $\delta(N)$ the set of all possible permutations with player set $N$. Given a permutation $O$, and denoting by $\Delta_i(O)$ the set of predecessors of player $i$ in the order $O$, the Shapley value can be expressed in the following way:

$$\Xi_i(v^\Psi) = \frac{1}{N!} \sum_{O \in \delta(N)} (v^\Psi(\Delta_i(O) \cup i) - v^\Psi(\Delta_i((O))))$$

where $v^\Psi(\Delta_i((O)))$ is the value obtained in permutation $O$ by the players preceding player $i$ and $v^\Psi(\Delta_i(O) \cup i)$ is the value obtained in the same permutation when including player $i$. That is, $\Xi_i(v^\Psi)$ gives the average marginal contribution of player $i$ over all permutations of player set $N$. Note that the index $\Psi$ denotes different possible shock scenarios, that is, banks’ contribution

---

6For a derivation of the formula we refer to ?.

7In the future we aim at incorporating to the analysis other measures coming from the empirical literature on systemic risk assessment.
to systemic risk is computed conditional on a shock vector to the banking system. In numerical simulations the computation of the Shapley value might be quite challenging. Since the number of permutations involved in calculating the Shapley value increases strongly with the number of banks, the analysis is subject to the curse of dimensionality. We follow the rest of the literature in approximating the Shapley value by the average contribution of banks to systemic risk over $k$ randomly sampled permutations as displayed in Equation 10:

$$\Omega_i(v^\Phi) \approx \hat{\Omega}_i(v^\Phi) = \frac{1}{k} \sum_{O \in \pi_k} (v^\Phi(Pre^i(O) \cup i) - v^\Phi(Pre^i(O)))$$

(10)

4 The Banking Network

The entire financial system is made up of $N$ banks. Let $N \in \{1, ..., n\}$ represent a finite set of individual banks, each of whom is identified with a node of the network. We define ex-ante for this population a network $g \in G$ as the set of links among heterogenous banks $N$, with $G$ being the set of all possible networks. An arc or a link between two banks $i$ and $j$ is denoted by $g_{ij}$ where $g_{ij} \in \mathbb{R}$. Here $g_{ij} \neq 0$ reflects the presence of a link (directed network), while $g_{ij} = 0$ reflects absence of it. Later on we shall specify the link $g_{ij}$ as either borrowing or lending from bank $i$ to bank $j$. Notice that each bank can borrow from some banks and lend to others at the same time. An important aspect is that cross-lending positions (hence the network links) result endogenously from the banks’ optimizing decision and the markets’ tâtonnement processes. As explained earlier the exposure to other banks in the interbank market constitutes a vehicle for risk transmission: as one bank defaults on its interbank debts, losses are directly transmitted to all lending banks. Formally it is useful to define $N^d(i; g) = \{k \in N \mid g_{ik} \neq 0\}$ as the set of banks with whom bank $i$ has a direct link in the network. The cardinality of this set is given by $\mu^d_i(g) = |N^d(i; g)|$, namely the number of banks with whom bank $i$ is directly linked in the network $g$. The $n \times n$ square adjacency matrix $G^{(t)}$ of the network $g$ describes the connections which arise after ($t$) iterations of the tâtonnement process. Given that our model features a directed weighted network, banks $i$ and $j$ are directly connected if $g_{ij} \neq 0$ or $g_{ji} \neq 0$.

Notice that interbank trading in our model occurs since banks end up with different optimal portfolios’ allocations, of which optimal interbank lending and borrowing represent the entry of the matrix $G$. The original source of heterogeneity is given by differences in returns to non-liquid assets: the underlying assumption is that banks have access to different investment opportunities of which some are more profitable than others.

Finally market equilibrium is achieved as follows. Prices in the interbank market and the market

---

8 For a derivation of Equation 10 and a proof of its unbiasedness as an estimator of the Shapley value see ? and ?.
for non-liquid assets are obtained through a sequential tâtonnement process. Central walrasian auctioneers (see also ? or ?) receive individual demand and supply of interbank lending and adjust prices until the distance between aggregate demand and supply has converged to zero: once a clearing price has been achieved, actual trade takes place. Traded quantities in our model are matched according to three different matching mechanisms which will be described later on.

4.1 Banking problem

To account for the fact that financial networks are made of purposeful agents we follow the approach of ? and have a system of banks maximizing profit subject to regulatory and balance sheet constraints. On the asset side, banks can hold cash, invest in non-liquid assets and lend to other banks. These are funded by means of equity, deposits and interbank borrowing. The balance sheet of bank $i$ is hence given by:

$$c_i + n_i p + \sum_{j=1}^{k} l_{ij} = d_i + \sum_{j=1}^{k} b_{ij} + e_i$$  \hspace{1cm} (11)

where $c_i$ represents cash holdings, $n_i$ denotes non-liquid assets, $l_{ij}$ ($b_{ij}$) stands for interbank lending (borrowing), $d_i$ for deposits and $e_i$ for equity.

When deciding their optimal balance sheet structure banks need to comply with two standard regulatory requirements, represented by the following two inequality constraints:

$$c_i \geq \alpha d_i$$  \hspace{1cm} (12)

$$cr_i = \frac{c_i + n_i p + \sum_{j=1}^{k} l_{ij} - \sum_{j=1}^{k} b_{ij}}{\omega_p n_i + \omega_l l_i} \geq \gamma + \tau$$  \hspace{1cm} (13)

Equation 12 represents a standard liquidity requirement which asks banks to hold at least a fraction $\alpha$ of their deposits in cash. Equation 13 represents a capital requirement, according to which the equity ratio of banks should be above a given level. The equity ratio is defined as the ratio of equity at market prices (where non-liquid assets $n_i$ are valued at their market price $p$) over risk weighted assets, where cash has a zero risk weight since it is considered riskless and $\omega_p$ and $\omega_l$ represent the risk weights on non-liquid assets and interbank lending respectively. Regulatory

---

9The sequential tâtonnement process takes place first in the interbank market for given returns and prices on non-liquid assets and in a second moment it takes place in the markets for non-liquid assets for given returns on interbank lending.

10As in all centralized tâtonnement processes this adjustment takes place in fictional times with no actual trading. Trading takes place only when price convergence has been achieved.

11Recall that banks in our model are risk averse, hence have concave objective functions and linear constraints. The convexity of the optimization problem and the assumption of an exponential aggregate supply function guarantees that individual and aggregate excess demand and supplies behave in both markets according to Liapunov convergence.

12Note that since banks cannot lend to nor borrow from themselves, we know that $l_{ii} = b_{ii} = 0 \forall i = 1, ..., N$. 

---

10
authorities require that this ratio be above the level $\gamma$ and it is further assumed that banks overfulfill this requirement by $\tau$, which represents a desired capital buffer banks choose to hold.

An important feature from real-world interbank matrices that we want to account for is that banks are very often both borrowers and lenders in the interbank market. To this end we assume that banks are risk averse and operationalize this by means of a CRRA profit function:

$$
\pi_i = \left( \frac{n_i r^a + l_i r^l - b_i r^b}{1 - \sigma_{(i)}} \right)^{1-\sigma_{(i)}}
$$

(14)

where $\sigma_{(i)}$ stands for the bank risk aversion, which could be different across banks or assumed to be homogeneous through institutions. $r^a$ represents the return on non-liquid assets, which is an important element introducing bank heterogeneity in the model. Interbank lending has an interest rate associated to it, $r^l$, which we take to be the risk free rate of the model. On the other hand, the interest rate for borrowing is bank-specific and can be decomposed as follows: $r^b_i = r^l + r^p_i$.

It is composed of the risk-free rate plus a premium that each bank must pay. Notice that the assumption of banks’ risk aversion is realistic also as it induces some degree of cautiousness as banks tend to trade less in the face of large adverse shocks. This feature is well in line with actual bank practices in both interbank as well as markets for non-liquid asset investment.

The problem of bank $i$ can be summarized as:

Maximize $\pi_i$

s.t. $Equation\ 12$, $Equation\ 13$, $Equation\ 11$

$ c_i, n_i, l_i, b_i \geq 0$

\( \text{(P)} \)

4.2 Interbank Market Clearing

The clearing in the interbank market occurs in two stages. In the first stage a standard tâtonnement process is applied\(^{14}\) in order to obtain an interbank interest rate that eliminates any excess demand/supply in the market. The goal of the second stage is to take the optimal vectors of demand and supply obtained in the first stage and match them in order to obtain the interbank matrix that summarizes bilateral exposures. The importance of this second stage resides in the fact that the final shape of the interbank network as summarized by the exposures matrix is an important element in the assessment of banks’ systemic importance. We entertain different matching mechanisms in order to see how they affect the topology of the network.

\(^{13}\)For details see ?. In their model the premium is given by the bank’s probability of default and the loss-given-default ratio, assumed to be the same for all banks. In our simulations we set the probability of default to 0.001 for all banks.

\(^{14}\)See ?.
4.2.1 Price Tâtonnement in the Interbank Market

For every bank \( i \) the solution to problem \( P \) will yield a vector containing optimal holdings of cash, non-liquid assets, interbank lending and borrowing. At his stage, it is of course not guaranteed that the funds supplied and demanded by all banks are mutually consistent, given the prevailing price \( r^l \). The market clearing required for this to happen is achieved by means of a standard tâtonnement process that occurs in fictitious time\(^15\).

For a given calibration of the model, which includes an initial level of the interbank interest rate, banks optimize and come up with optimal desired demands (\( b_i \)) and supplies (\( l_i \)) of funds\(^16\). These are submitted to a walrasian auctioneer who collects them in \( B = \sum_{i=1}^{N} b_i \) and \( L = \sum_{i=1}^{N} l_i \).

If \( B > L \) there is excess notional demand in the market and therefore \( r^l \) is increased to bring about equilibrium, whereas the opposite happens if \( B > L \). The highest yield on non-liquid assets\(^17\) is taken as the upper-bound on \( r^l \) (\( \bar{r}^l(0) \)) and zero is taken as the lower bound (\( r^l(0) \)). The average between the two is taken as the initial value that banks use to optimize: \( r^l(0) = \frac{\bar{r}^l(0) + r^l(0)}{2} \).

If, say, the optimization yields an excess supply of funds (i.e. \( L > B \)), the new (lower) interest rate is obtained by setting \( r^l(0) \) as the new upper bound: \( r^l(1) = \frac{r^l(0) + L}{2} \). This interest rate is then used in the new iteration period. This process continues until the change in interest rate is below an arbitrarily small threshold level\(^18\). A similar adjustment is undertaken in the opposite direction if \( B > L \).

The output of this stage is given by two vectors, namely \( l = [l_1 l_2 ... l_N] \) and \( b = [b_1 b_2 ... b_N] \), which collect the optimal lending and borrowing in the interbank market.

4.2.2 Matching algorithms

Using the results from the previous stage, the second stage aims at finding out how bank \( i \) distributes its lending (\( l_i = \sum_{i=1}^{N} l_{ij} \)) and/or borrowing (\( b_i = \sum_{i=1}^{N} b_{ij} \)) among its potential counterparts. The vectors \( l = [l_1 l_2 ... l_k] \) and \( b = [b_1 b_2 ... b_k] \) represent the row sum and column sum (respectively) of the matrix of interbank positions \( X \), with element \( x_{ij} \) indicating the exposure (through lending) of bank \( i \) to bank \( j \). The problem that the matching algorithms are aimed to solve is that of obtaining matrix \( X \) from vectors \( l \) and \( b \), using economic criteria and some additional constraints like for example the fact that the diagonal of matrix \( X \) is filled with zeros. We consider three matching mechanisms that when taken together are representative of a good spectrum of connectivity among banks:

(i) **Maximum Entropy.** Access to detailed data on bilateral exposures via lending/borrowing

---

\(^{15}\)Our problem is static so in every period \( t \) there is a new optimization taking place. The tâtonnement process can be thought of as taking place during a (potentially infinite) subdivision of the time period in iteration periods (see ?).

\(^{16}\)Note that contrary to previous studies, banks can be on both sides of the market simultaneously.

\(^{17}\)As will be explained below, the yield on non-liquid assets (\( r^l \)) is bank-specific and is drawn from a uniform distribution within specified bounds.

\(^{18}\)Equivalently, one can think of the process as stopping when \( |L-B| \leq \varepsilon \), where \( \varepsilon \) is an arbitrarily small threshold.
is a rare privilege. Nonetheless, the overall lending and borrowing in the interbank market by bank are publicly available data, that is, the vectors \( l \) and \( b \) are public information, at least for listed banks. The most common approach in the empirical literature is to use the so called maximum entropy method, which attempts to distribute lending and borrowing as evenly as possible among counterparties in conjunction with restrictions on the diagonal elements of the matrix to be estimated. Given the vectors \( l \) and \( b \), the matrix obtained by this method will yield the maximum density possible, that is, the market will be as complete as possible in the sense of.

To obtain the maximum entropy solution we follow and employ the RAS algorithm, which is a technique of biproportional matrix balancing developed in the context of input-output analysis for the purpose of matrix updating. The maximum entropy solution uses the relative entropy matrix as a prior, which assumes that the exposure of bank \( i \) to bank \( j \) is given by \( x_{ij} = l_i \ast b_j \) if \( i \neq j \) and equal to zero if \( i = j \) (see also ). In this way, from the vectors \( l \) and \( b \) we are able to reconstruct the matrix \( X \).

(ii) Closest matching (CMA). The second algorithm we explore is the closest matching, or minimum distance, algorithm. As noted in, since banks can ex-post charge different risk premia based on the bank-specific premium \( r^p \), they are indifferent among alternative counterparts. An efficient allocation can therefore be achieved by matching banks that present the minimum distance between the amounts offered and demanded.

The logic of the algorithm goes as follows. The vectors \( l \) and \( b \) are re-arranged in descending order such that the highest element is in the first position of the vector, the second highest in the second position and so on; denote by \( l_{(0)} \) and \( b_{(0)} \) the initial re-arrangement that is used to start the first iteration. The \( i^{th} \) element of \( l_{(0)} \) is then paired with the \( i^{th} \) element of \( b_{(0)} \), for \( i = 1, \ldots, N \), and the minimum is computed. The computation of the minimum will determine that either the \( i^{th} \) offering or demanding bank will be satisfied; to fix ideas assume that \( \min(l_i, b_i) = l_i \), in which case the supplier is satisfied and is hence eliminated from future iterations while the borrower is left with an un-net need for funds equivalent to \( b_i - l_i \), which remains on cue for the following iteration step. This pairing is performed for all possible pairs and at the end of the first iteration we will

\[19^{19}\text{Some examples of analyses using real world data are } \text{for Austria, } \text{for the FedWire payment system, } \text{for Italy and } \text{for Belgium, among others. More recently, } \text{present an analysis of the network of large European banks. For a summary of network approaches to interbank markets see } 19^{20}.\]

\[20^{20}\text{See for instance } \text{or } \text{among others.}\]

\[21^{21}\text{This method is usually criticized because it generates interbank matrices that are considerably less sparse (in network parlance, they are more “dense”) than real-world interbank matrices. For instance, } \text{explicitly compares an interbank matrix based on real data for Italy with matrices obtained by the maximum entropy method and concludes that the latter approach underestimates contagion risk. In our analysis, the maximum entropy matrix provides a good upper bound in terms of density of the interbank network.}\]

\[22^{22}\text{See } ? \text{ or } \text{.}\]

\[23^{23}\text{employ this way of matching banks in the interbank market, though in their model banks are either borrowers or lenders, which simplifies the workings of the algorithm.}\]

\[24^{24}\text{Some practical complications may arise, like for example a bank being paired against itself. For instance, the highest demander of funds might also be the highest supplier. These issues are dealt with in the computation, but are omitted here for expositional simplicity.}\]
be left with at most \( N/2 \) pairs to be considered in the second iteration. All un-net demand/supply left from the first iteration is again re-arranged in descending order in the vectors \( l_{(1)} \) and \( b_{(1)} \), which are paired in a similar fashion. Since the tâtonnement process form the previous sub-section guarantees that total demand is equal to total supply (i.e., \( \sum_{i=1}^{l} l_i = \sum_{i=1}^{b} b_i \)), it follows that the closest matching algorithm will eventually converge, leaving no bank with un-net demand or supply of funds. The closest matching procedure is compatible with pair-wise efficiency and has economic intuition and theory behind it. In his seminal treaty \( ? \) proposes four alternative elementary clearing mechanisms to specify how borrowing and lending takes place. The first two mechanisms, which are the ones that would apply to a framework like ours, are in fact two variations on pair-wise matching, one purely bilateral and another via a broker. Both of these mechanisms can be accommodated to our setting, depending on the story-telling one wishes to emphasize. The interbank matrix obtained by this method will have a considerably lower connectivity (i.e., will be less dense) than the one obtained via maximum entropy. It will provide in fact a lower bound of connectivity in our analysis.

(iii) Sequential matching with exposure limits (SMA). As a third alternative we entertain a modification of the closest matching algorithm\(^\text{25}\) designed to achieve a slightly higher density and building on real-world constraints on bank behavior. The inspiration for this is the Capital Requirements Regulation (CRR), which in its article 395 establishes limits on individual exposures as a percentage of capital of the bank being exposed by the transaction. For reasonable values of the exposure limit, this constraint forces banks to spread their exposure among more counterparts, thereby increasing the connectivity (density) of the interbank matrix. In our simulations we set the limit to be 100\% (i.e., banks cannot lend to a given counterpart more than 100\% of their equity capital)\(^\text{26}\).

4.3 Price Tâtonnement in the Market for Non-Liquid Assets

Once clearing has been achieved in both price and quantities in the interbank market, another clearing process takes place in the market for non-liquid assets. We rely on the idea of a sequential clearing process. Clearing takes place first in the interbank market for given prices and returns for non-liquid assets and subsequently it takes place in the market of non-liquid assets for given equilibrium returns in the interbank market. The clearing process in the market for non-liquid assets is modelled along the lines of ?.

Given the optimal banks’ portfolio decisions let us define each bank’s optimal supply (or demand)

\(^{25}\)As noted above, in the CMA the vectors of supply and demand are sorted and a pairwise matching is established. In the SMA vectors are also sorted and the matching is also done pairwise but in a different order: the bank ordered first in the supply vector goes sequentially through the demand vector transacting pairwise until it fulfils its desired supply, then the second supplier does the same and so on.

\(^{26}\)In reality, the CRR sets the limit to 25\%, but we need to accommodate the fact that our simulation will work with a number of banks several orders of magnitude smaller than real-world banking systems. With the limit implemented in our simulations we believe to capture the spirit of the regulation and at the same time make it workable in our setting.
of non-liquid assets with an $s_i$. Notice that this is also the amount that allows bank $i$ to fulfill the capital requirements. Since each $s_i$ is decreasing in $p$, the aggregate sales function, $S(p) = \sum_i s_i(p)$, is also decreasing in $p$. An equilibrium price is such that total excess demand equals supply, namely $S(p) = D(p)$. The price at which total aggregate sales are zero, namely $p = 1$ can certainly be considered one equilibrium price. We can define an aggregate demand function $\Theta : [p, 1] \to [p, 1]$; given this function an equilibrium price solves the following fixed point:

$$\Theta(p) = d^{-1}(s(p))$$

(15)

The price convergence process in this case is guaranteed by using the following inverse demand function$^{27}$:

$$p = \exp(-\beta \sum_i s_i),$$

(16)

where $\beta$ is a positive constant to scale the price responsiveness with respect to non-liquid assets sold, and $s_i$ is the amount of bank $i$’s non-liquid assets sold on the market. Integrating back the demand function in Equation 16 yields the following:

$$\frac{dp}{dt} = \beta S(p)$$

(17)

which states that prices will go up in presence of excess demand and downward in presence of excess supply. In the above differential equation $\beta$ represents the rate of adjustment of prices along the dynamic trajectory.

Numerically, price tâtonnement in the market for non-liquid assets takes place through an iterative process which can be described as follows. At the initial equilibrium the price is set to 1. Following a shock to the non-liquid asset portfolio of one specific bank a shift in the aggregate supply occurs. Bank $i$ starts selling non-liquid assets to satisfy its capital requirement and this results into $S(1) = s_i > 0$. At $S(1)$ the bid price, given by the inverse demand function, Equation 16, is given by $p(S(1))^{bid}$, while the offer price is one. Given this discrepancy we set the new price at the intermediate level between the two, $p(S(1))^{mid}$. The new price is lower than the initial equilibrium price. This determines a fall in the value of banks’ non-liquid asset portfolios. Once again to fulfill capital requirements banks are forced to sell other assets, a process which forces further price falls through the mechanism just described. The iterative process continues until demand and supply cross at $p^*$. Notice that convergence is guaranteed since we have a downward sloping market demand function given by equation Equation 16.

$^{27}$See also 7.
4.4 Equilibrium Definition

Definition. A competitive equilibrium in our model is defined as follows:

(i) A quadruple \((l_i, b_i, n_i, c_i)\) for each bank \(i\) that solves the optimization problem \(P\).
(ii) A price in the interbank market, \(r^I\), which clears demand and supply in the interbank market, namely satisfying \(B = L\), with \(B = \sum_{i=1}^{N} b_i\) and \(L = \sum_{i=1}^{N} l_i\).
(iii) A trading-matching algorithm for the interbank market.
(iv) A clearing price for the market of non-liquid assets that solves the fixed point: \(\Theta(p) = d^{-1}(s(p))\).

4.5 Risk Transmission Channels in the Model

Before proceeding with the simulation results it is useful at this stage to highlight the main channels of risk transmission in our model. There are two main channels of shock transmission. A direct one which goes through the lending exposure in the interbank market. When a bank is hit by a shock which makes it unable to repay interbank debt, default losses are transmitted to all exposed banks. Depending on the size of losses other banks might find themselves unable to fulfill their obligations in the interbank market. Notice that the increase of default losses in presence of risk averse banks might also increase their cautiousness when supplying funds to the interbank market: as losses increase endogenously concave utility induces banks to reduce the optimal amount of lending. Losses through network cascades therefore have also a further negative impact on the availability of liquidity in the interbank market. Lower liquidity will in turn also reduce investment in non-liquid assets. The second channel of shock and risk transmission is a traditional fire sale channel that works through pecuniary externalities. The channel, highlighted theoretically by ?, has been recently modelled within financial network models by ? and can be summarized in this way. A bank hit by a shock might be forced to sell non-liquid assets in order to meet capital requirements and/or VaR targets. If the sale of the assets is large enough, the market might feature a collapse in prices: this is the essence of the pecuniary externalities, namely the fact that individual banks’ decisions have an impact on market prices. In an environment in which banks’ balance sheets are measured based upon mark-to-market accounting, the fall in the price of non-liquid assets induces accounting losses to all banks exposed to this asset. Those banks might be forced themselves to sell non-liquid assets to meet capital requirements: this circle might turn into a spiralling chain of sales and losses due to the collapse in market prices. Three elements are crucial in determining the existence of fire sale externalities in our model. First, the presence of capital requirements affects market demand elasticities in a way that individual banks’ decisions about asset sales do end up affecting market prices. Second, the tâtonnement process described in the previous paragraph produces falls in asset prices whenever supply exceeds demand. Third, banks’ balance sheets are measured with a mark-to-market accounting procedure.
Both channels seem to have played and important role during the 2007 crisis. For instance, describe the origin of fire sale externalities in a model in which complex financial architecture also induces uncertainty, thereby amplifying financial panics. Describe instead a mechanism of loss transmission which occurred via direct exposure of banks in the money market. Our model merges both approaches and gains a full picture of the extent of the cascade that might occur in the wake of shocks.

4.6 Calibration and Baseline Scenario Results

In this section we present the baseline network configuration, which we analyze under three different matching algorithms for the traded quantities. The three matching algorithms considered are the ones described before, namely Maximum Entropy, Closest Matching and Sequential Matching with Exposure Limits. The main purpose of this section is threefold. First we wish to verify that our banking network resembles real existing ones. We will show that each of the matching algorithms produces a network which has some characteristics which resemble those observed in reality. Second, we wish to characterize the financial architecture of the three baseline configurations of our banking network. To this aim we employ and analyze several synthetic measures which are density, average path length, closeness, betweenness and eigenvector centrality. Third, we compute and analyze for all three configurations systemic risk via the Shapley values, network centrality metrics and input-output measures. Both measures will allow us to identify systemically important banks and to produce a risk map for each of the three matching algorithms considered.

In order to simulate the model we must calibrate all exogenous parameters, give starting values to endogenous parameters and set values for exogenous balance sheet items. The calibration tries to mimic values observed in real-world banking systems, either from existing empirical evidence or from regulatory/supervisory policy (most of the parameters are in fact representing the latter). Table 1 summarizes the calibration we employ.

Following [18], the number of banks is set to 20 to have a fair amount of players and yet keep the system manageable, especially for the computation of the Shapley value. The parameter from the liquidity requirement, $\alpha$, which governs the proportion of deposits that banks must hold in cash, is set to 10%, mimicking the cash reserve ratio in the U.S. The capital ratio requirement is set to 8%, following federal reserve bank regulatory agency definitions and also in line with Basel III. The buffer chosen by banks to over-fulfill the capital requirement is set to 1%, giving a margin of adjustment in the wake of shocks, i.e. following a shock to non-liquid assets is not necessarily the case that fire-sales will lead to unfulfillment of the capital requirement. Risk weights are set according to regulatory policy: $\omega_n$, the risk weight on non-liquid assets, is set to 1 in accordance with the weights applied in Basel II to commercial bank loans; $\omega_l$, the risk weight on interbank lending,
is set to 0.2, which is the actual risk weight used for interbank deposits in OECD countries.\footnote{Cash carries no risk so its risk-weight is equal to zero.} Following ?, the starting value for equities is set to 65 billions (with a variance of 10 billions), while deposits are set to 500 billions, which are figures in line with the balance sheet of Deutsche Bank at end 2012. The return on non-liquid assets is randomly drawn from a uniform distribution over the range 0-15%, whereas the vector of shocks to non-liquid assets, which is the starting point of the shock transmission process, is drawn from a multivariate normal distribution with a mean of 5, a variance of 25 and zero covariance. The variance is set high enough in order to capture the possibility of high stress scenarios. Finally, for the banks’ risk aversion parameter $\gamma$, we follow ? and set it to 2. Since bank heterogeneity is already introduced through equity and the returns on non-liquid assets, we keep the risk aversion parameter constant across banks.\footnote{The precise value of risk aversion is subject to a continuous debate, though a value greater than 1 seems to be well established in the literature (see ?).}

<table>
<thead>
<tr>
<th>Par./Var.</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Number of banks in the system</td>
<td>20</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Liquidity requirement ratio</td>
<td>0.10</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>Risk weight on non-liquid assets</td>
<td>1</td>
</tr>
<tr>
<td>$\omega_l$</td>
<td>Risk weight on interbank lending</td>
<td>0.20</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Capital requirement ratio</td>
<td>0.08</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Desired capital buffer</td>
<td>0.01</td>
</tr>
<tr>
<td>$d_i$</td>
<td>Bank deposits</td>
<td>500</td>
</tr>
<tr>
<td>$e_i$</td>
<td>Bank equity</td>
<td>$N(65, 10)$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Bank risk aversion</td>
<td>2</td>
</tr>
<tr>
<td>$r^u_i$</td>
<td>Return on non-liquid assets</td>
<td>$U(0, 0.15)$</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>Shocks to non-liquid assets</td>
<td>$N(5, 25 + I)$</td>
</tr>
</tbody>
</table>

Table 1: Baseline calibration

We start by analyzing the network configuration under the Maximum Entropy matching algorithm.\footnote{The equilibrium interbank rate in this simulation is 0.0201. There are 7 banks that only lend (banks 3, 4, 7, 8, 9, 12 and 14), 6 that only borrow (6, 11, 13, 15, 18 and 20) and 7 that both borrow and lend (1, 2, 5, 10, 16, 17 and 19).} The network has a classical configuration of evenly sparse links with few central nodes that provide most of the liquidity in the system.\footnote{Network charts are available upon request.} This network presents a density of 46\% (see Table 2 below), which is higher than the usually observed in country-specific studies on interbank networks, but is nonetheless in line with the density of the total assets exposure network of large European banks (48\%) presented in ?.

When the interbank matrix computed via the closest matching algorithm, there is a notable decrease in the amount of links, and given that the amounts transacted in the market remain the same, the amounts per transaction are higher. The density of the network is now 6.8\%, still above but
more in line with the evidence from country-specific studies of interbank markets (see for instance ? for the dutch case), and probably as low a density as we can get for the baseline scenario given the amount of banks in the system and the optimal quantities traded.

Finally, we have the the interbank matrix obtained via the sequential matching procedure. Imposing the limit of 100% of equity in exposure to any given counterparty roughly doubles the density of the network relative to the CMA case\textsuperscript{32}. The shape of the network already looks quite different and the higher number of links are of a reduced width, as imposed by the logic of the algorithm.

### 4.6.1 Synthetic Measures of Network Architecture

To appreciate further the difference among the three network configurations considered we also present some synthetic measures that characterize the financial architecture. The measures are density, average degree, path length, closeness, betweenness, eigenvector centrality, clustering and assortativity. Table 2 summarizes them for the three networks under consideration and for the baseline parameterization.

<table>
<thead>
<tr>
<th></th>
<th>RAS</th>
<th>CMA</th>
<th>SMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>0.46053</td>
<td>0.068421</td>
<td>0.12632</td>
</tr>
<tr>
<td>Degree (Av.)</td>
<td>8.75</td>
<td>1.3</td>
<td>2.4</td>
</tr>
<tr>
<td>Av. Path Length</td>
<td>1.13</td>
<td>2.9</td>
<td>1.92</td>
</tr>
<tr>
<td>Closeness (Av.)</td>
<td>0.099909</td>
<td>0.0026253</td>
<td>0.085331</td>
</tr>
<tr>
<td>Betweenness Cent. (Av.)</td>
<td>2.5</td>
<td>8.3</td>
<td>0.45</td>
</tr>
<tr>
<td>Eigenvector Cent. (Av.)</td>
<td>0.13891</td>
<td>0.12807</td>
<td>0.085327</td>
</tr>
<tr>
<td>Clustering Coeff. (Av.)</td>
<td>0.21182</td>
<td>0.030021</td>
<td>0.015669</td>
</tr>
<tr>
<td>Transitivity</td>
<td>3.7318</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Assortativity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>out-in degree</td>
<td>-0.5</td>
<td>0.0069078</td>
<td>-0.25926</td>
</tr>
<tr>
<td>in-out degree</td>
<td>-0.038339</td>
<td>0.17341</td>
<td>-0.28074</td>
</tr>
<tr>
<td>out-out degree</td>
<td>-0.34857</td>
<td>-0.2938</td>
<td>-0.63625</td>
</tr>
<tr>
<td>in-in degree</td>
<td>-0.37819</td>
<td>-0.22513</td>
<td>-0.68908</td>
</tr>
</tbody>
</table>

Table 2: Network characteristics - Baseline setting

The first two network metrics are closely related. The density of the network captures the share of existing links over the total amount of possible links and we already commented on it above, whereas the average degree gives the average number of connections per bank. As noted by ?, a simple relation of proportionality holds between the two measures, and they can be taken to represent the extent of diversification in the network. By construction, in the RAS network both density and average number of connections per bank are high, while the opposite holds for the...
CMA network, with the SMA lying somewhere in between.

The average path length is the mean shortest path between pairs of nodes. In the RAS network any given bank is almost one step away, for the SMA this number goes roughly to 2, whereas in the CMA network is 3. Overall, the average path length is small, in line with real-world interbank networks (see ?, ? or ? among others). This implies that exposure is not far away for the average bank in the network.

Closeness, betweenness and eigenvector centrality are averages for all nodes in the network. Closeness paints a similar picture to the average path length, since they are also related concepts. Betweenness on the other hand, shows a high score for the CMA network, which has some banks acting as gatekeepers due to the construction of the network (see chart above)?.

The clustering coefficient measures the tendency of neighbors of a given node to connect with each other, thereby generating a cluster of connections. The average clustering coefficient is, not surprisingly, relatively high for the RAS network, and is reduced notably for the CMA network and even more for the SMA network.

The assortativity coefficient aims at capturing the tendency of high-degree nodes to be linked to other high-degree nodes. As noted by ?, interbank networks tend to be dis-assortative, implying that high-degree nodes tend to connect to other high-degree nodes less frequently than would be expected under the assumption of a random rewiring of the network that preserves the nodes’ degrees. The RAS and SMA networks show in fact a dis-assortative behavior for all assortativity measures, whereas the same holds for two out of four of the combinations in the CMA network. These results are in line with their empirical counterparts (see for instance ? or ?).

4.6.2 Systemic Risk, Network Centrality and Systemically Important Banks

An essential prerequisite of prudential regulation consists in measuring systemic risk and identifying systemically important banks. To this aim and prior to the analysis of the prudential policy regulations we present some measures of systemic risk and some criteria for identifying systemically important banks for each of the three network configurations considered. Systemic risk is measured as the overall probability of default and the contribution of each bank to it is measured through the Shapley value. Other metrics of network centrality are used to identify systemically important banks.

Figure 4.1 presents systemic risk contributions by bank for the three networks, based on the Shapley Value methodology. The number of permutations considered for the computation of the Shapley Value was set to 1000. The clearing algorithm used is that of ?. For the cases considered, the lower the density of the network the higher the total systemic risk of the system. There seems to be a non-linearity though, since the difference in systemic risk between the RAS and SMA networks

\[ \text{For an empirical analysis using these measures for the European interbank market see ?}. \]
is negligible (for a difference in density of 35%), whereas the distance between the SMA and CMA networks is roughly two percentage points (for a difference in density of 6%). Let’s now examine which banks contribute the most to systemic risk. By jointly analyzing the data in Figure 4.1 and the banks’ optimal portfolio allocations as reported in Table 3 we find that the banks which contribute the most to systemic risks are the ones which borrow in the interbank market and invest in non-liquid assets. The more they borrow and the more they invest in non-liquid assets, the larger is their contribution to systemic risk. The rationale behind this is as follows. Banks which leverage more in the interbank market are clearly more exposed to the risk of default on interbank debts. The larger is the size of debt default the larger are the losses that banks transmit to their counterparts. Borrowing banks therefore contribute to systemic risk since they are the vehicle of network externalities. On the other side banks which invest more in non-liquid assets transmit risks since they are the vehicle of the pecuniary externalities. The higher is the fraction of non-liquid asset investment the higher is the negative impact that banks’ fire sales have on market prices. The higher is the collapse in the market prices the higher are the accounting losses experienced by all other banks which have invested in non-liquid assets.

Figure 4.1: Contribution to systemic risk (mean Shapley Value), by bank and by network

Table 4 presents the ranking of systemic importance for the different networks according to
network centrality measures. Table 5 presents the same but for input-output measures of systemic importance. Table 3 presents the optimal balance sheet structure in the baseline setting. While the contribution of each bank to systemic risk remained widely unchanged across the three network configurations considered, the metrics of network centrality vary significantly among them. Also, the relation between the various centrality measures across the three network configurations and the banks’ balance sheet characteristics is less stable (see also figures in subsection A.3 and subsection A.4) than for the Shapley values (see subsection A.5). For input-output metrics we generally observe that banks with higher ratios of liquid assets over total assets and with higher ratios of interbank lending over assets have higher ranking. This is largely true for all three network configurations considered. Intuitively banks which are relatively more exposed into the interbank market relative to other forms of investment are more important vehicles of risk. A similar relation between balance sheet characteristics and centrality ranking can be traced for the eigenvector centrality and for all three network configurations considered, while for other centrality measures it is not possible to identify univocal and stable relations.

5 Policy analysis

Recent guidelines on prudential regulations from Basel III include both ratios for capital requirements and ratios for liquidity requirements. For this reason we analyze the performance of our banking network model (as usual under the three different configurations for the three alternative matching algorithms) using the various network centrality and systemic risk statistics by varying two prudential policy coefficients, namely the liquidity requirement $\alpha$, and the capital requirement $\gamma$. In the simulations the number of permutations for the computation of the Shapley Value is set to 1000.

We start by examining how overall systemic risk and the contribution of each bank to it change when changing the two policy parameters. Clearly both overall systemic risk as well as the contribution of each bank to it decline when we increase the liquidity parameter $\alpha$ (see Figure 5.1, Figure B.9, Figure B.11, Figure B.13); notice that each panel also shows confidence bands for the simulated series). This decline is observed for all three network configurations considered. As banks must hold more liquidity for precautionary motives, their exposure in the interbank market declines. Banks leverage less (see Figure 5.9), the interbank interest rate increases due to the scarce supply of liquidity (see Figure 5.7) and banks’ investment in non-liquid assets declines as available liquidity falls (see Figure 5.15 and Figure 5.17). This clearly reduces the scope of the network externality for risk transmission.

Results are somehow more complex when we increase the capital requirement, $\gamma$. As this parameter increases, overall systemic risk (measured by the percentage of assets of defaulting banks over total assets) declines. Banks tend to leverage less and the interbank interest rate declines as
the demand of liquid funds has declined (see Figure 5.8 and Figure 5.10). This obviously reduces the overall scope for transmitting default losses. However, banks also reduce the overall amount of liquid assets while increasing the amount of non-liquid asset investment (see Figure 5.12 and Figure 5.16): the increase in equity indeed allows them to satisfy the higher capital requirements. The increase in the share of non-liquid assets increases the scope of risk transmission through fire sales. As for the contribution of each bank to overall systemic risk (see Shapley values in Figure B.10, Figure B.12, Figure B.14) while most banks tend to transmit less risk as \( \gamma \) increases, others instead tend to contribute more. Since all banks are less exposed to the interbank market the scope of loss
cascades through network linkages is reduced, on the other side some banks (particularly banks 7, 8, 9, 12, and 16) invest more in non-liquid assets. This exposes them to the swings in the market price for non-liquid assets and increases the probability that they will engage in fire sales. The pattern described is pretty much common to all three network configurations considered (RAS, CMS, SMA).

Finally we examine the evolution of the other network metrics when we change the liquidity and the capital requirement parameters. Four metrics, namely density, average degree, closeness and clustering, all have an inverse bell shaped dynamic in response to changes in $\alpha$. This can be
explained by examining the overall availability of interbank lending shown in Figure 5.19. As the liquidity requirement raises less banks supply liquidity in the interbank market and this reduces density and closeness in the interbank network. As the liquidity requirement raises, some banks increase their demand of liquid funds in the interbank market. The increased demand drives the interbank rate up and induces other banks to shift from investing funds in non-liquid assets to lend in the interbank market. This asset substitution effect increases again the available liquidity in the interbank market (as shown in Figure 5.19), which in turn increases again density, closeness and average degree in the interbank network. The same four metrics all have a bell shaped dynamic in
response to changes in $\gamma$. As the capital requirement increases more banks demand liquid funds in the interbank market to satisfy the equity requirement. This tends to increase density, closeness and average degree. Beyond a value of 0.12 for $\gamma$ instead most banks start to hoard liquid funds. This reduces the supply of liquidity and the network becomes more sparse (density, average degree and closeness all fall). The pattern described is pretty much similar for all three matching algorithms, although generally speaking all three metrics are higher for the RAS algorithm. The evolution of betweenness and eigenvector centrality in response to changes in $\alpha$ and $\gamma$ is less clear cut. Overall there seems to be an increase in response to both parameters and for all matching algorithms.
To sum up, increasing the liquidity requirement unequivocally reduces systemic risk as it reduces at the same time the extent of network links (as shown by the fall of all network centrality metrics) and the investment in non-liquid assets. As a consequence an increase in $\alpha$ reduces the scope of both network and pecuniary externalities rendering the network more stable. The fall in the overall non-liquid asset investment shows however that an increase in the liquidity requirement reduces system efficiency (measured here by the overall size of investment).
6 Concluding remarks

We analyzed a banking network model featuring risk transmission via two channels, network links’ externalities and pecuniary externalities. Banks in our model are risk averse and solve a concave optimal portfolio problem choosing the lending exposure in the interbank market and the investment in non-liquid assets. The individual optimization problems and the market clearing processes deliver a matrix of network links in the interbank market: each bank can be both a borrower toward some of the banks in the network and a lender toward others. Shocks to one bank are transmitted either through default on interbank debt or through price collapses of non-liquid assets triggered by fire sales. Clearing in the market takes place through a price tâtonnement iterative process and through a trading matching algorithm. We considered three alternative trading matching algorithms, namely Maximum Entropy, the Closest Matching and the Sequential Matching with Exposure Limits. Each one of them produces a different architecture for the banking network in terms of density, average degree, average path length, assortativity, closeness, betweenness and eigenvector centrality. This also implies that risk transmits differently in each of the three network configurations. We use our banking network also to assess the role of prudential regulations in reducing systemic risk. We find that increasing the liquidity requirement unequivocally reduces systemic risk and the contribution of each bank to it. As banks are forced to hoard liquidity they engage less in interbank lending: this reduces all network centrality measures (density, closeness, average degree, etc.) thereby reducing the scope for network externalities in the transmission of risk. As banks are required to hold more liquidity they also reduce their share of investment in non-liquid assets. This reduces also the scope for fire sale externalities. An increase in the capital requirements instead has mixed effects. Overall interbank liquidity falls and this reduces the extent of network links, but overall investment in non-liquid assets increases.
A Additional results for baseline scenario

A.1 Balance sheet characteristics
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-liquid assets</td>
<td>1212</td>
<td>656.7</td>
<td>222.2</td>
<td>207.6</td>
<td>612.4</td>
<td>717.4</td>
<td>354.1</td>
<td>233.4</td>
<td>0.0</td>
<td>623.3</td>
<td>709.9</td>
<td>281.6</td>
<td>721.4</td>
<td>245.9</td>
<td>714.8</td>
<td>643.0</td>
<td>609.3</td>
<td>742.9</td>
<td>680.4</td>
<td>706.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interbank lend.</td>
<td>47.4</td>
<td>26.3</td>
<td>285.0</td>
<td>228.8</td>
<td>45.6</td>
<td>0.0</td>
<td>261.9</td>
<td>274.8</td>
<td>517.0</td>
<td>39.1</td>
<td>0.0</td>
<td>255.8</td>
<td>0.0</td>
<td>265.8</td>
<td>0.0</td>
<td>24.0</td>
<td>48.2</td>
<td>0.0</td>
<td>17.2</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interbank borr.</td>
<td>152.5</td>
<td>172.2</td>
<td>0.0</td>
<td>0.0</td>
<td>151.9</td>
<td>197.9</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>156.2</td>
<td>186.8</td>
<td>0.0</td>
<td>193.7</td>
<td>0.0</td>
<td>198.0</td>
<td>0.0</td>
<td>157.6</td>
<td>205.5</td>
<td>187.4</td>
<td>186.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Assets (A)</td>
<td>709.6</td>
<td>733.0</td>
<td>557.2</td>
<td>586.5</td>
<td>709.1</td>
<td>767.4</td>
<td>566.0</td>
<td>558.3</td>
<td>567.0</td>
<td>714.3</td>
<td>759.9</td>
<td>547.4</td>
<td>771.4</td>
<td>561.7</td>
<td>764.6</td>
<td>717.0</td>
<td>707.5</td>
<td>792.9</td>
<td>790.6</td>
<td>756.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equity</td>
<td>57.1</td>
<td>60.8</td>
<td>57.2</td>
<td>56.5</td>
<td>57.1</td>
<td>69.5</td>
<td>66.0</td>
<td>58.3</td>
<td>67.0</td>
<td>34.1</td>
<td>73.2</td>
<td>47.4</td>
<td>77.6</td>
<td>61.7</td>
<td>66.6</td>
<td>59.5</td>
<td>56.8</td>
<td>87.4</td>
<td>63.2</td>
<td>69.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leverage</td>
<td>12.4</td>
<td>12.0</td>
<td>9.7</td>
<td>6.8</td>
<td>12.4</td>
<td>11.0</td>
<td>8.6</td>
<td>8.5</td>
<td>8.5</td>
<td>12.3</td>
<td>10.4</td>
<td>11.5</td>
<td>9.9</td>
<td>9.1</td>
<td>11.5</td>
<td>12.1</td>
<td>12.5</td>
<td>9.1</td>
<td>11.9</td>
<td>10.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liquid assets/A (%)</td>
<td>13.7</td>
<td>10.4</td>
<td>60.1</td>
<td>47.5</td>
<td>13.3</td>
<td>6.5</td>
<td>55.1</td>
<td>58.2</td>
<td>100.0</td>
<td>12.5</td>
<td>6.6</td>
<td>48.6</td>
<td>6.5</td>
<td>56.2</td>
<td>6.5</td>
<td>10.3</td>
<td>13.9</td>
<td>6.3</td>
<td>8.9</td>
<td>6.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interbank lend/A (%)</td>
<td>6.7</td>
<td>3.6</td>
<td>51.6</td>
<td>39.0</td>
<td>6.4</td>
<td>0.0</td>
<td>46.3</td>
<td>49.2</td>
<td>91.2</td>
<td>5.5</td>
<td>0.0</td>
<td>39.4</td>
<td>0.0</td>
<td>47.3</td>
<td>0.0</td>
<td>3.6</td>
<td>6.8</td>
<td>0.0</td>
<td>2.3</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interb./Liq. (%)</td>
<td>23.4</td>
<td>25.6</td>
<td>0.0</td>
<td>0.0</td>
<td>23.3</td>
<td>28.4</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>23.8</td>
<td>27.2</td>
<td>0.0</td>
<td>27.9</td>
<td>0.0</td>
<td>28.4</td>
<td>24.0</td>
<td>23.2</td>
<td>29.3</td>
<td>27.3</td>
<td>27.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nla/Dep. (%)</td>
<td>122.4</td>
<td>131.6</td>
<td>44.4</td>
<td>61.5</td>
<td>122.7</td>
<td>143.5</td>
<td>50.8</td>
<td>46.7</td>
<td>0.0</td>
<td>125.1</td>
<td>142.0</td>
<td>56.3</td>
<td>144.3</td>
<td>49.2</td>
<td>142.9</td>
<td>128.6</td>
<td>121.9</td>
<td>148.6</td>
<td>136.7</td>
<td>141.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nla/Equity (%)</td>
<td>1072.8</td>
<td>1079.5</td>
<td>364.6</td>
<td>355.8</td>
<td>1073.6</td>
<td>1032.6</td>
<td>385.1</td>
<td>400.7</td>
<td>0.0</td>
<td>1073.9</td>
<td>970.3</td>
<td>384.0</td>
<td>929.2</td>
<td>398.5</td>
<td>1072.4</td>
<td>1081.6</td>
<td>1072.6</td>
<td>849.6</td>
<td>1081.6</td>
<td>1014.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interb. lend./Equity (%)</td>
<td>83.1</td>
<td>43.2</td>
<td>498.5</td>
<td>264.7</td>
<td>79.9</td>
<td>0.0</td>
<td>396.8</td>
<td>471.7</td>
<td>771.6</td>
<td>67.2</td>
<td>0.0</td>
<td>452.2</td>
<td>0.0</td>
<td>430.7</td>
<td>0.0</td>
<td>40.4</td>
<td>84.8</td>
<td>0.0</td>
<td>27.2</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 3:** Optimal balance sheet items - Baseline setting
A.2 Systemic importance ranking
### Table 4: Systemic importance ranking by network centrality measures, bank and network - Baseline setting

<table>
<thead>
<tr>
<th></th>
<th>Network</th>
<th>Banks</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-degree</td>
<td>RAS</td>
<td>1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20</td>
</tr>
<tr>
<td></td>
<td>CMA</td>
<td>1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20</td>
</tr>
<tr>
<td>Out-degree</td>
<td>RAS</td>
<td>8 9 1 2 10 15 3 4 5 11 16 6 7 17 2 18 19 20 9</td>
</tr>
<tr>
<td></td>
<td>CMA</td>
<td>8 9 2 3 10 15 4 5 1 11 16 6 17 7 12 13 19 14 20</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>8 9 4 5 10 15 3 4 11 16 7 17 2 13 19 14 20 10</td>
</tr>
<tr>
<td>In-out degree</td>
<td>RAS</td>
<td>1 2 14 15 3 4 16 17 18 4 5 10 9 19 20 11 5 6 12 7</td>
</tr>
<tr>
<td></td>
<td>CMA</td>
<td>1 3 4 15 3 4 16 17 18 5 10 19 20 11 5 6 12 7 13</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>1 2 14 15 3 4 16 17 18 5 10 19 20 11 5 6 12 7 13</td>
</tr>
<tr>
<td>Closeness-in</td>
<td>RAS</td>
<td>3 12 14 15 3 4 16 17 18 5 6 8 9 10 20 12 13 14 15 7</td>
</tr>
<tr>
<td></td>
<td>CMA</td>
<td>3 12 14 15 3 4 16 17 18 5 6 8 9 10 20 12 13 14 15 7</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>3 12 14 15 3 4 16 17 18 5 6 8 9 10 20 12 13 14 15 7</td>
</tr>
<tr>
<td>Closeness-out</td>
<td>RAS</td>
<td>6 13 19 16 5 11 16 18 20 3 14 10 17 12 2 7 15 16 17</td>
</tr>
<tr>
<td></td>
<td>CMA</td>
<td>6 13 19 16 5 11 16 18 20 3 14 10 17 12 2 7 15 16 17</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>6 13 19 16 5 11 16 18 20 3 14 10 17 12 2 7 15 16 17</td>
</tr>
<tr>
<td>Betweenness</td>
<td>RAS</td>
<td>2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20</td>
</tr>
<tr>
<td></td>
<td>CMA</td>
<td>2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20</td>
</tr>
<tr>
<td>Eigenvector (left)</td>
<td>RAS</td>
<td>12 8 14 15 11 3 16 17 18 10 19 4 20 2 9 13 1 5 6</td>
</tr>
<tr>
<td></td>
<td>CMA</td>
<td>12 8 14 15 11 3 16 17 18 10 19 4 20 2 9 13 1 5 6</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>12 8 14 15 11 3 16 17 18 10 19 4 20 2 9 13 1 5 6</td>
</tr>
<tr>
<td>Eigenvector (right)</td>
<td>RAS</td>
<td>9 1 22 61 0 1 55 3 11 1 1 671 741 8 1 381 9 1 4 2 0</td>
</tr>
<tr>
<td></td>
<td>CMA</td>
<td>9 1 22 61 0 1 55 3 11 1 1 671 741 8 1 381 9 1 4 2 0</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>9 1 22 61 0 1 55 3 11 1 1 671 741 8 1 381 9 1 4 2 0</td>
</tr>
</tbody>
</table>

### Table 5: Systemic importance ranking by input-output measures, bank and network - Baseline setting

<table>
<thead>
<tr>
<th>IO measure</th>
<th>Network</th>
<th>Banks</th>
</tr>
</thead>
<tbody>
<tr>
<td>R.H. Backward</td>
<td>RAS</td>
<td>11 8 14 15 12 3 26 17 18 10 7 19 4 20 2 9 13 1 5 7</td>
</tr>
<tr>
<td></td>
<td>CMA</td>
<td>11 8 14 15 12 3 26 17 18 10 7 19 4 20 2 9 13 1 5 7</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>11 8 14 15 12 3 26 17 18 10 7 19 4 20 2 9 13 1 5 7</td>
</tr>
<tr>
<td>R.H. Forward</td>
<td>RAS</td>
<td>9 12 2 7 16 15 5 3 1 11 16 6 17 4 18 13 8 19 14 30</td>
</tr>
<tr>
<td></td>
<td>CMA</td>
<td>9 12 2 7 16 15 5 3 1 11 16 6 17 4 18 13 8 19 14 30</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>9 12 2 7 16 15 5 3 1 11 16 6 17 4 18 13 8 19 14 30</td>
</tr>
<tr>
<td>Row FoI</td>
<td>RAS</td>
<td>9 13 2 6 11 20 3 5 3 12 17 7 15 4 18 14 8 16 10 19</td>
</tr>
<tr>
<td></td>
<td>CMA</td>
<td>9 13 2 6 11 20 3 5 3 12 17 7 15 4 18 14 8 16 10 19</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>9 13 2 6 11 20 3 5 3 12 17 7 15 4 18 14 8 16 10 19</td>
</tr>
<tr>
<td>Column FoI</td>
<td>RAS</td>
<td>12 13 18 15 10 6 1 16 17 20 7 9 8 19 11 4 2 14</td>
</tr>
<tr>
<td></td>
<td>CMA</td>
<td>12 13 18 15 10 6 1 16 17 20 7 9 8 19 11 4 2 14</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>12 13 18 15 10 6 1 16 17 20 7 9 8 19 11 4 2 14</td>
</tr>
<tr>
<td>Total FoI</td>
<td>RAS</td>
<td>12 13 18 15 10 6 1 16 17 20 7 9 8 19 11 4 2 14</td>
</tr>
<tr>
<td></td>
<td>CMA</td>
<td>12 13 18 15 10 6 1 16 17 20 7 9 8 19 11 4 2 14</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>12 13 18 15 10 6 1 16 17 20 7 9 8 19 11 4 2 14</td>
</tr>
<tr>
<td>Total Linkage</td>
<td>RAS</td>
<td>10 8 9 2 16 14 5 3 1 11 19 7 17 4 15 18 12 10 13 20</td>
</tr>
<tr>
<td></td>
<td>CMA</td>
<td>10 8 9 2 16 14 5 3 1 11 19 7 17 4 15 18 12 10 13 20</td>
</tr>
<tr>
<td></td>
<td>SMA</td>
<td>10 8 9 2 16 14 5 3 1 11 19 7 17 4 15 18 12 10 13 20</td>
</tr>
</tbody>
</table>
A.3 Input-output measures and bank characteristics

Results are only presented when considering the RAS network (for other networks results are largely unchanged)

![Figure A.1: Rasmussen-Hirschman backward](image1)

![Figure A.2: Rasmussen-Hirschman forward](image2)

![Figure A.3: Column Field of Influence (FoI)](image3)

![Figure A.4: Row Field of Influence (FoI)](image4)
Figure A.5: Total Field of Influence (FoI)  
Figure A.6: Total Linkage Effect
A.4 Centrality measures and bank characteristics

Results presented for only a subset of measures and networks

Figure A.7: In-Out centrality (RAS)  
Figure A.8: In-Out centrality (CMA)

Figure A.9: In-Out centrality (SMA)  
Figure A.10: Betweenness centrality (CMA)
Figure A.11: Betweenness centrality (SMA)  Figure A.12: Eigenvector centrality (RAS)

Figure A.13: Eigenvector centrality (CMA)  Figure A.14: Eigenvector centrality (SMA)
A.5 Shapley Value and bank characteristics

Figure A.15: RAS network

Figure A.16: CMA network

Figure A.17: SMA network

A.6 Shapley Value and IO measures

$h_b$, $h_f$, $f_c$, $f_r$, $f_t$ and $t$ indicate respectively the R.H. backward, R.H. forward, column Field of Influence, row Field of Influence, total Field of Influence and Total Linkage Effect indices.
Figure A.18: RAS network  
Figure A.19: CMA network  
Figure A.20: SMA network
B  Additional results for comparative static analysis

Figure B.1: Closeness for different values of $\alpha$

Figure B.2: Closeness for different values of $\gamma$

Figure B.3: Betweenness cent. for different values of $\alpha$

Figure B.4: Betweenness cent. for different values of $\gamma$
Figure B.5: Eigenvector cent. for different values of $\alpha$

Figure B.6: Eigenvector cent. for different values of $\gamma$

Figure B.7: Clustering coeff. for different values of $\alpha$

Figure B.8: Clustering coeff. for different values of $\gamma$
Figure B.9: Contribution to systemic risk (Shapley Value) by bank for different values of $\alpha$ and for the RAS network.
Figure B.10: Contribution to systemic risk (Shapley Value) by bank for different values of $\gamma$ and for the RAS network.
Figure B.11: Contribution to systemic risk (Shapley Value) by bank for different values of $\alpha$ and for the CMA network.
Figure B.12: Contribution to systemic risk (Shapley Value) by bank for different values of $\gamma$ and for the CMA network
Figure B.13: Contribution to systemic risk (Shapley Value) by bank for different values of $\alpha$ and for the SMA network
Figure B.14: Contribution to systemic risk (Shapley Value) by bank for different values of $\gamma$ and for the SMA network